Maths and the making of the modern world

The maths behind Google and the Ipod

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Some common views on maths and mathematicians

Mathematics is completely useless

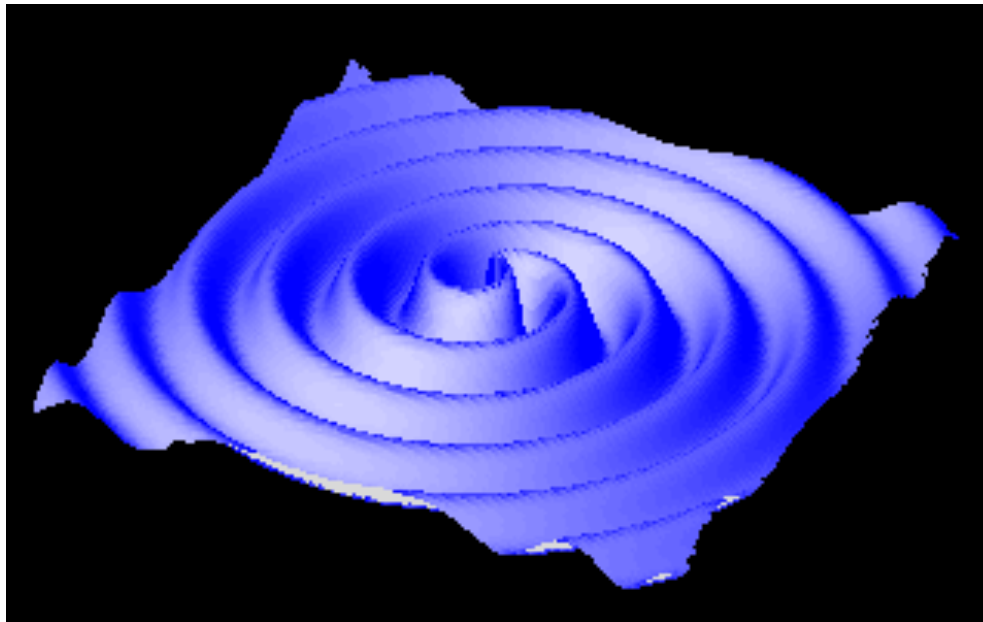Mathematicians are evil souless geeks

All Mathematicians are mad!

The truth is very different!

The modern world would not exist without maths

Maths lies at the heart of all modern technology

Spot the mathematician, and why are they important?

Maxwell and the discovery of electromagnetic waves

$$\nabla \times E = -\frac{\partial B}{\partial t} - M, \quad \nabla \times H = -\frac{\partial D}{\partial t} + J,$$

$$\nabla . D = \rho, \qquad \nabla . B = 0.$$
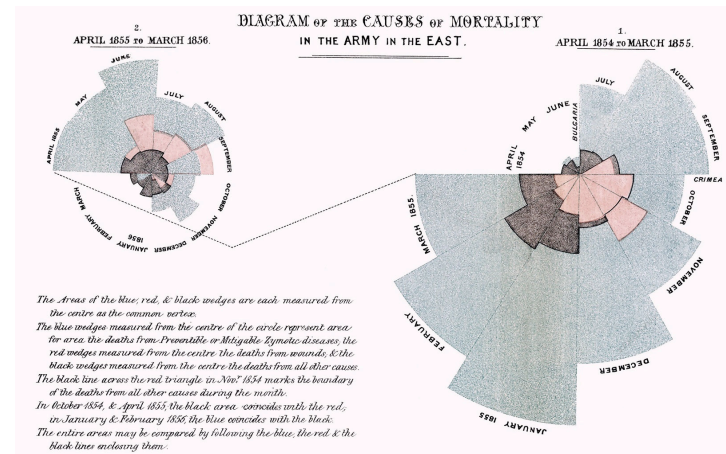
Electromagnetism, radio, WiFi,TV, radar, mobile phones, microwaves all come from the work of Maxwell!

The most famous ever female mathematician?

Florence Nightingale

Medical statistics

Mathematicians really have made the modern world possible

The key to the modern world is information!

The rate at which we receive data and store has grown incredibly

Morse Code: 2 Bytes per second

Teleprinter: 10 Bytes per second

Modem: 1 Kilobyte per second

Modern data: 1 Gigabyte per second

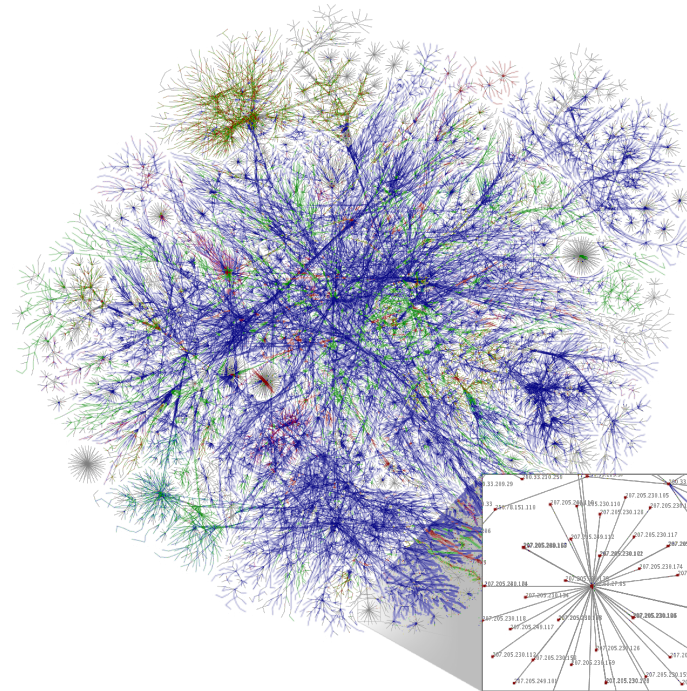Early computers: 1 kilo byte of memory

Now 1 Tera Byte of memory on a lap top

But this leads to challenging problems!!

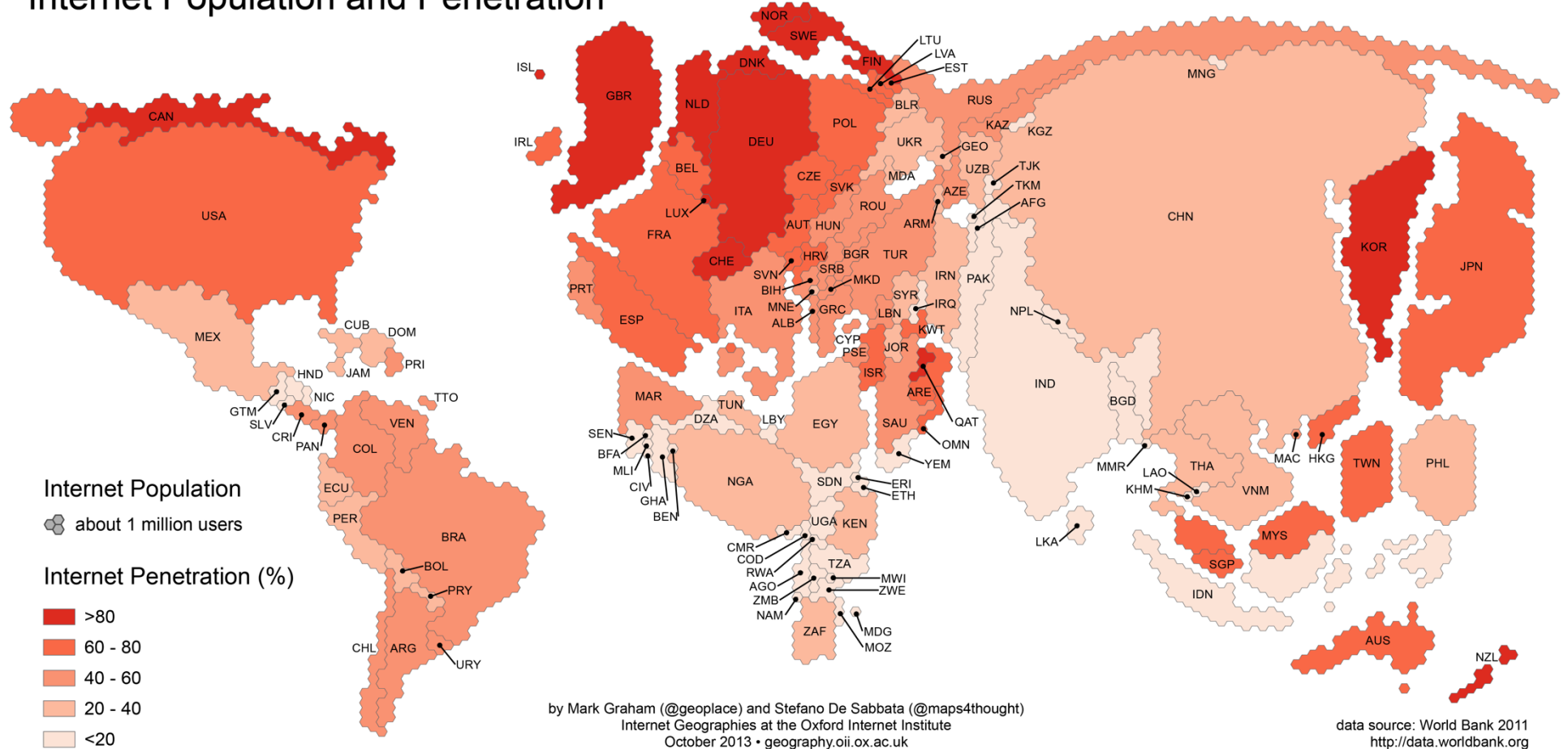For example, how do we control and search the internet for vaguely defined information?

Over a Zetta bytes = 10^{21} bytes of information and growing fast.

One petabyte of storage costs about £10K/month.

The Internet

Internet Population and Penetration

Internet Population

⬡ about 1 million users

Internet Penetration (%)

🟥	>80
🟥	60 - 80
🟧	40 - 60
🟧	20 - 40
⬜	<20

by Mark Graham (@geoplace) and Stefano De Sabbata (@maps4thought)
Internet Geographies at the Oxford Internet Institute
October 2013 • geography.oii.ox.ac.uk

data source: World Bank 2011
http://data.worldbank.org

Launched in 2004

2 Billion Registered users

1.5 Billion active

Huge amount of data stored as pictures

2.5 Billion Pieces Of Content And 500+ Terabytes Ingested Every Day

It is important that we store, transmit and search this information carefully and without making mistakes

Maths helps us to do this…

Storing information by telling the truth

Pick a number   0,1,2,3,...,7

Answer the following questions truthfully

Q1.   Is your number 4,5,6,7?

Q2.   Is your number 2,3,6,7?

Q3.   Is your number 1,3,5,7?

Binary numbers

0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

3 Bit Binary Number: x

x   represented by three digits a b c   eg. 101

a,b,c are 0 or 1

$$x = 4*a + 2*b + c$$

eg.

101 = 4+0+1 = 5

011 = 0+2+1 = 3

1, 0 are called bits of information

All information in a computer is made up of bits

Simplest information has ONE BIT

Do you like England?

YES   NO

Binary numbers 0..7 have 3 bits

Usually binary numbers have more than 3 bits

eg.   10011011   has 8 bits

10011011 = ????????????

A symbol of 8 bits is called a byte.

You can have 256 such symbols

Letters A,B,C, … are converted into 8 bit ASCII

Other languages eg. Japanese use 32 bit Unicode

暗黒
時代

Camera takes picture made up of PIXELS

8 BITS per pixel …..   256 range of intensity = 1 byte

1 000 000 Pixels per Picture

3 colours

Total 3 M Byte per picture

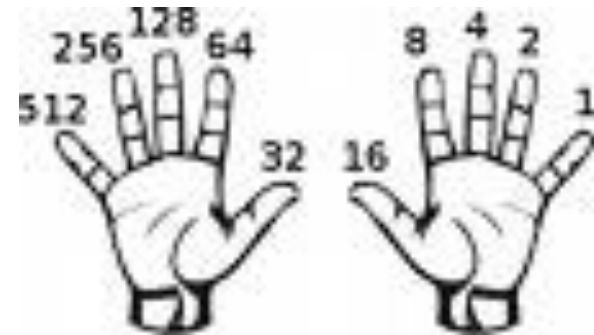**One bite memory**

How does a monster count to 25?

On their fingers!

Using binary you can count from 0 to 31 on one hand with

5 bit binary numbers

eg.   10110 = 16 + 4 + 2 = 22

11001 = 16 + 8 + 1 = 25

Sometimes we make mistakes

Mean to send 11100011

Make a mistake on one bit and send

11101011

Can we tell if we have made a mistake?

Answer the following questions.

Either tell the truth or lie at most once

Pick a number between 0 and 7

Q1   Is it   4,5,6,7?

Q2   Is it   2,3,6,7?

Q3   Is it   1,3,5,7?

Q4   Is it   1,2,4,7?

Can we find the liar?

```
0      0 0 0 0
1      0 0 1 1
2      0 1 0 1
3      0 1 1 0                    answer to last question
4      1 0 0 1
5      1 0 1 0
6      1 1 0 0
7      1 1 1 1
```

If all true there are an: even number of   1s

If one lie there is an: odd number of   1s

Last digit/question is called a parity bit   and tells us if we
have made a mistake

Barcodes use a parity bit

5 014016 150821 >

Once we spot an error we can either

Panic!!!!

.... OR ….

Ask for the information to be sent again

.... OR ….

We can try to correct it

Error correcting codes.

Used to store the numbers 0,1,2,3,4,5,6,7 and other data in such a way that any errors can not only be detected but corrected.

They work by asking extra questions to make the answers as different as possible so we can still tell the right answer even if it has mistakes in it

They are widely used in

- CDs

- Digital TV and Radio

- Mobile phones

- Satellites

Invented in the 1940s by Hamming in the Bell Labs

Using very fancy maths (Galois theory)

Answer the following questions .. You can either tell the truth or lie at most once

Choose a number 0,1,2,3,4,5,6,7

Q1   Is the number   4,5,6,7?

Q2   Is the number   2,3,6,7?

Q3   Is the number   1,3,5,7?

Q4   Is the number   1,3,4,6?

Q5   Is the number   1,2,5,6?

Q6   Is the number   2,3,4,5?

0	000 000
1	001 110
2	010 011
3	011 101
4	100 101
5	101 011
6	110 110
7	111 000

Binary number   Correcting number

Start with a binary number   110110

Telling the truth doesn't change the number   110110

Lying once changes the number by one digit   100110

Hamming Distance:

Take two binary numbers. How many digits do we have to change to turn one into the other?

0	000 000
1	001 110
2	010 011
3	011 101
4	100 101
5	101 011
6	110 110
7	111 000

All are a Hamming distance of 3 apart

Choose the closest number to the one you are sent. This must be correct.

Binary number   Correcting number

They are used a lot in iPODs
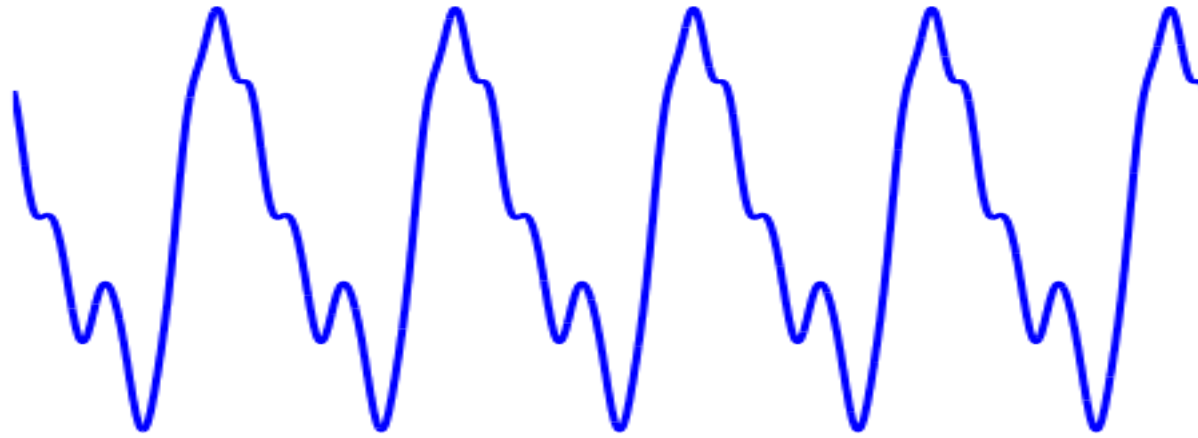
iPODs also compress the information.

For example

Instead of sending this message which has lots of vowels in it which we don't really need
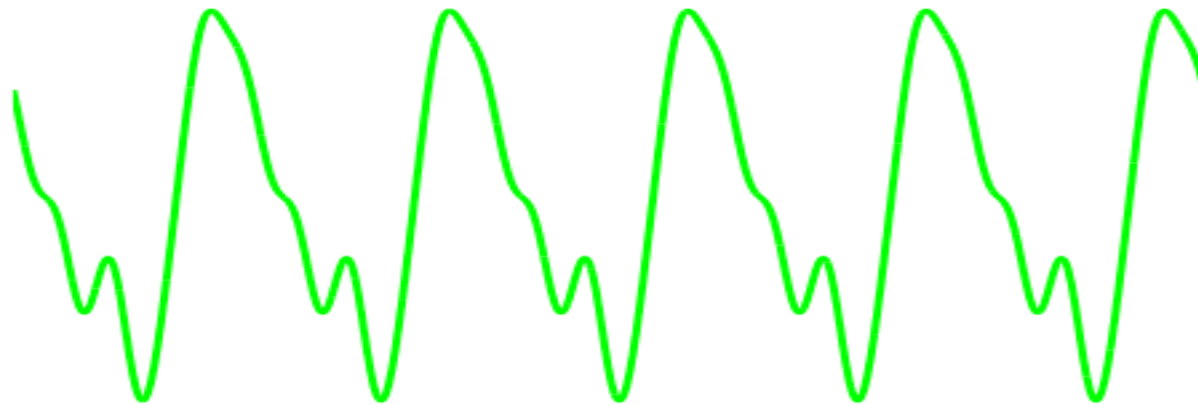
W cn snd ths mssg nstd whch ds nt hv ny vwls t ll

Nw try ths fr yrslf

MPEG file also compresses sound waveforms

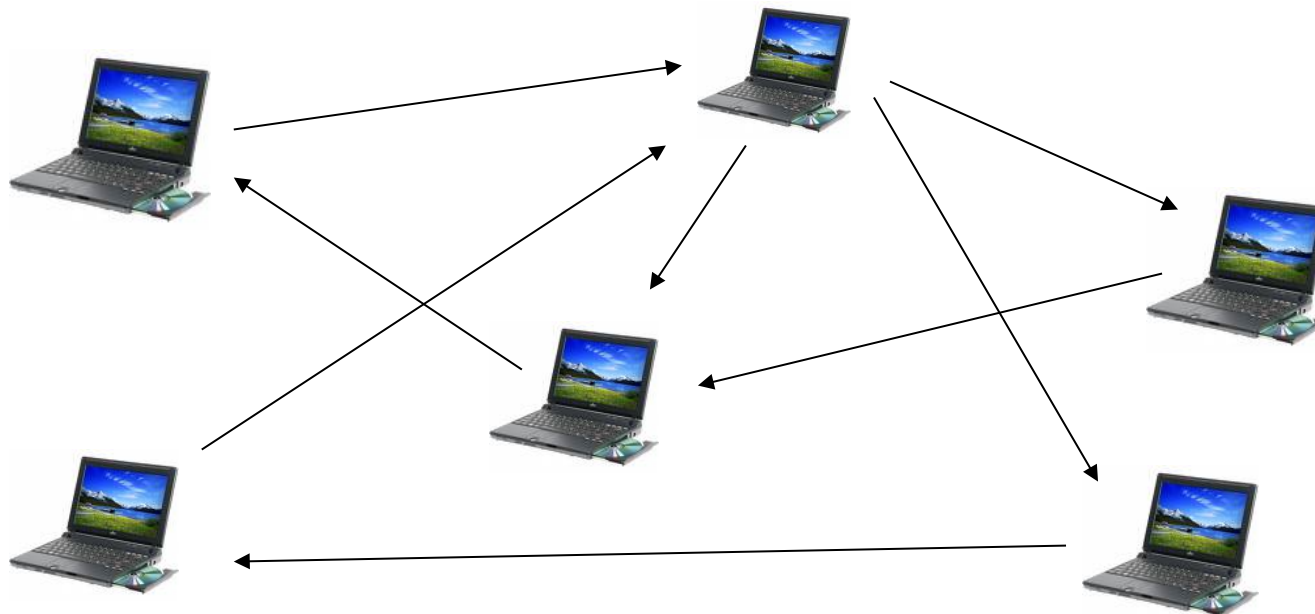Decompose into a sum of harmonics. Only store the first few of these
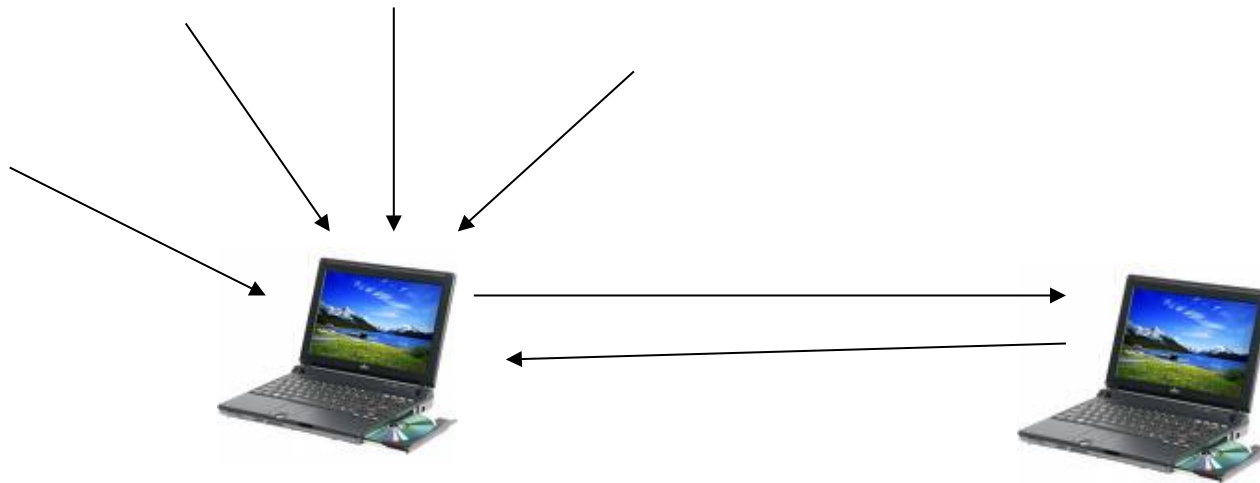
The Maths Behind Google

Google searches for information stored on many web-sites.

Web-sites are linked together by a network showing which web-site points to which other web-site

It RANKS web-sites in order of the importance of the information that they contain.

IDEA. A website is important if lots of other websites link to it.

A website is even more important if it is linked to by lots of important web-sites.

PAGE RANK

Each Web-site has a rank R

Divide R by the number N of web-sites that this web-site links, to get S=R/N

For each web-site, calculate R by adding up the values of S for every web-site that connects to it.

Now do the same for 1 000 000 0000 more computers

Need to calculate the unit eigenvector of an extremely large matrix!

And make many $$$$$$$$$$$$$$$$$$$