

## Limbering-Up Exercises in $\beta$ -reduction

$\beta$ -reduce each of the following by one step.

1.  $\lambda x.x$
2.  $x(\lambda x.x)(\lambda x.x)$
3.  $(\lambda x.x)(\lambda x.x)$
4.  $(\lambda x.x)(\lambda x.x)(\lambda x.x)$
5.  $(\lambda x.xx)(\lambda x.xx)$
6.  $x((\lambda x.xy)z)y$
7.  $(\lambda x.xy)xy$
8.  $(\lambda x.(\lambda y.xy))y$
9.  $(\lambda x.(\lambda y.xy))(\lambda y.y)$
10.  $(\lambda x.(\lambda y.xy)x)z$
11.  $(\lambda x.y)(\lambda x.xx)(\lambda x.xx)$
12.  $(\lambda x.y)((\lambda x.xx)(\lambda x.xx))$
13.  $(\lambda x.x)(\lambda x.x)((\lambda x.xx)(\lambda x.xx))$
14.  $(\lambda x.xxx)(\lambda x.xxx)$
15.  $(\lambda xy.yx)(\lambda z.z)$
16.  $(\lambda xy.yxy)(\lambda xy.yxy)(\lambda xy.yxy)$
17.  $(\lambda xy.x(x(xy)))MN$  where  $M$  and  $N$  are any  $\lambda$ -terms  $x, y \notin FV(NM)$
18.  $S(\lambda xy.x(x(xy)))$  where  $S = \lambda n.\lambda xy.x(nxy)$
19.  $SKK$ , where  $S = \lambda xyz.xz(yz)$  and  $K = \lambda xy.x$
20.  $YM$ , where  $M$  is any  $\lambda$ -term and  $Y = (\lambda zx.x(zzx))(\lambda zx.x(zzx))$ ,  $x, z \notin FV(M)$

Now reduce as them far as possible (meaning until a normal form is reached, or you are convinced there is no normal form).

Bonus question: find the normal form for

$$(\lambda xy.x(x(x(x(x(x(xy))))))))(\lambda xy.x(x(x(x(x(xy))))))$$