Answers to Limbering-Up Exercises in β-reduction

1. normal form

2. normal form (application associates to left)

3. $\succ_1 \lambda x.x$, normal form

4. $\succ_1 (\lambda x.x)(\lambda x.x) \succ \lambda x.x$, normal form

5. $\succ_1 (\lambda x.xx)(\lambda x.xx)$, no normal form

6. $\succ_1 x(zy)y$, normal form

7. $\succ_1 xyy$, normal form

8. $=_\alpha (\lambda x.(\lambda z.xz))y \succ_1 \lambda z.yz$, normal form

9. $\succ_1 \lambda y.(\lambda y.y)y \succ \lambda y.y$, normal form

10. $\succ_1 (\lambda x.xx)z$ or $\succ_1 (\lambda y.zy)z$, both $\succ zz$, normal form

11. $\succ_1 y(\lambda x.xx)$, normal form

12. $\succ_1 y$ or $\succ_1 (\lambda x.y)((\lambda x.xx)(\lambda x.xx))$, both $\succ y$, normal form

13. $\succ_1 (\lambda x.x)((\lambda x.xx)(\lambda x.xx))$ or $\succ_1 (\lambda x.x)(\lambda x.x)((\lambda x.xx)(\lambda x.xx))$, both $\succ (\lambda x.xx)(\lambda x.xx)$, no normal form

14. $\succ_1 (\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx)$, no normal form

15. $\succ_1 \lambda y.y(\lambda z.z)$, normal form

16. $\succ_1 (\lambda y.y(\lambda xy.yxy)y)(\lambda xy.yxy) \succ (\lambda xy.yxy)(\lambda xy.yxy)(\lambda xy.yxy)$, no normal form

17. $\succ_1 (\lambda y.M(M(My)))N \succ M(M(MN))$, can't tell if normal form (depends on N and M)

18. $\succ_1 \lambda xy.x((\lambda xy.x(x(xy)))xy) \succ \lambda xy.x(x(x(xy)))$, normal form

19. $\succ_1 (\lambda yz.(\lambda xy.x)z(yz))(\lambda xy.x) \succ (\lambda yz.(\lambda y.z)(yz))(\lambda xy.x) \succ (\lambda yz.z)(\lambda xy.x) \succ \lambda z.z$, normal form

20. $\succ_1 (\lambda x.x((\lambda zx.x(zzx))(\lambda zx.x(zzx))x))M \succ M(((\lambda zx.x(zzx))(\lambda zx.x(zzx)))M) = M(YM)$, no normal form

Bonus question: $\lambda xy.x^{16777216}y$.