GRAVITY MODELS AND GENERATED TRAFFIC

A Note*

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The Road Research Laboratory report Research on Road Traffic[1] sets out (pages 130–135) a number of possible methods for estimating future traffic movements; one section covers the problem of estimating generated traffic. The same approach is adopted in the more recent Road Research Laboratory Technical Paper No. 75, The Economic Assessment of Road Improvement Schemes[2].

The argument briefly is as follows. The traffic flow from one zone of an urban area, or from one urban area, to another is given by the general gravity model formula

\[ t_{ij} = KA_i A_j f(d_{ij}) \]

where \( t_{ij} \) = the number of journeys made from i to j

\( A_i, A_j \) = measures of the strength of the attractiveness of i and j

\( K \) = a constant

\( f(d_{ij}) \) = some function measuring the areal separation of the zones—travel time, distance or cost (cost perhaps the most desirable but also the most difficult to specify, requiring as it does an estimate of time costs as well as out-of-pocket expenses). This function is normally expressed in the form \( d_{ij}^n \).

(1) can then be written in logs as

(2) \[ \log t_{ij} = \log K + \log (A_i A_j) - n \log d_{ij}. \]

Differentiating this with respect to \( d_{ij} \) we obtain an expression for \( n \) equal to

\[ \frac{d}{d_{ij}} \log t_{ij} \]

This has the same form as the familiar own-price elasticity co-efficient, \( \frac{p}{q} \frac{dp}{dq} \), and is treated as an own-journey-time elasticity co-efficient.

The value of \( n \) is obtained from cross section data. In urban transportation studies, for example, the value is that which minimises the divergence between the observed and the predicted frequency distributions of trip lengths.

Empirical studies of interurban flows carried out in the U.S.A. and Sweden indicate a value of \( n \) in the range 2 to 3.5. In the more recent R.R.L. publication it is stated that the value of \( n \) usually lies between 1 and 3 (for urban traffic it usually lies between 1 and 2; for interurban between 2 and 3).

The R.R.L. report therefore continues: “Applying this figure to the approximate formula suggests that the percentage increase in traffic is 2 to 3.5 times the percentage decrease in journey times.” The purpose of this note is to examine the validity of this

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conclusion, and more generally to try to relate trip generation and distribution models to economic demand models.

Additional journeys made between two points as a result of a reduction in travel time or cost are of two kinds:

(a) diverted journeys—defined as journeys between \( i \) and \( j \) which before the cost change were made from \( i \) to some other zone.

(b) generated journeys—defined as journeys induced by the fall in costs.

Analysis of the effects of an improvement has therefore to be made in two stages:

(1) Examine the change in the total number of journeys made, \( \Delta O_i \) (trip origins), caused by the \( \Delta t_{ij} \).

(2) Examine the way in which the supplemented journey origins \( (O_i + \Delta O_i) \) are distributed (or redistributed) as a result of the \( \Delta t_{ij} \).

In the urban transport studies the first stage is effectively ignored. In these studies the trip distribution model is used as a "closed" model to distribute an exogenously given number of trip origins. The "closed" distribution model can be specified as follows:

\[
t_{ij} = \frac{t_{iAf}(d_{ij})}{\sum_j t_{iAf}(d_{ij})}
\]

A separate study is made of the number of trips generated, and several more or less sophisticated forecasting methods have been developed.\(^1\)

Response to the cost change will vary according to journey purpose; except in the long run, the number of work or school journeys may be regarded as fixed, the only possible margin of adjustment being through the effect on the participation rate. The effect of the cost change on the volume of traffic between \( i \) and \( j \) will therefore depend on the composition of the existing traffic flow between them.

On the second point we need to examine the nature of the \( n \) co-efficient in equation (2). The value of \( n \) is derived from a collection of cross-section data at the existing set of relative prices; it has therefore nothing to tell us about the way in which the volume of journeys between any two points will react either to general or to particular price changes. Estimation of particular elasticity co-efficients would, of course, require time series data. To put it in another way, the own-price elasticity co-efficient for a particular journey between \( i \) and \( j \) has no necessary connection with the slope co-efficient of the curve relating the number of journeys to length or time of journey derived at a particular set of relative prices. The price elasticity of demand, which is what we are interested in when we look at the effect of a price change on the number of journeys made, depends on the availability of substitutes; there may or may not be close substitutes available for a journey from \( i \) to \( j \). In the case of work journeys, for example, the short-run price elasticity of demand for travel between zones is probably very low. If we regard the demand for transport as being complementary to the demand for employment, then even quite a large change in transport costs will in general produce a very small proportional change in the net return from employment. For shopping trips the price elasticity may be higher, as transport costs form a higher proportion of total outlay.

The slope of the trip length distribution function may therefore alter as a result of relative price changes. For example, parking charges may be sharply increased; this will increase the cost of short journeys relative to longer ones. The effect will probably be to alter the slope of the trip distribution function as shown in the diagram from a position \( t_1 \) in year \( t \) to \( t_2 \) in year \( t + 20 \).

It is recognised that the value of \( n \) varies between different trip purposes; it would be useful to know how it varied between income groups, to see whether income changes themselves produced changes in the deterrence function.

There is a methodological parallel between the distance exponent of the gravity model and the \( V \) in the familiar quantity theory equation \( MV = PT \). Ex-post the \( n \) is a definitional device—its value being that which minimises divergences between the observed and the synthesised trip length distribution—given the interzonal distances, observed zonal trip generation, observed zonal attractors, and the observed matrix of interzonal flows. In the same way the \( V \) in the quantity theory equation closes the circle of observed money stock and money value of national income. The behavioural or predictive content of the models is in the assumed stability of \( n \) on the one hand or of \( V \) on the other (or, à la Friedman, in the predictability of \( V \)). The underlying assumption of predictability of \( V \) should therefore be a matter for empirical testing.

The usefulness of the gravity and other interactance models can be tested, first with respect to the accuracy with which they can reproduce trip-making patterns in the base year, and secondly with respect to the stability or predictability of their behavioural parameters. Both have been the subject of empirical tests—the first more frequently than the second, since there are relatively few urban areas with more than one set of origin-destination data.

An article by Heathcote and Pyers in the *Highway Research Board Record* 114[3] contains some examples of testing procedures. Of particular interest is an attempt to examine the temporal stability of the model parameters. Unfortunately the conclusions are not clear-cut because of variation in the computation procedures. The data used was taken from origin and destination surveys carried out in Washington in 1948 and 1955. An attempt is made to distribute 1955 journeys on the basis of 1948
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travel time factors (similar to the exponent in the simple gravity model equation (1)).
The authors assert that an acceptable level of accuracy is obtained. But the discussion
is confused by talk of adjustments to socio-economic variables. Referring back to the
original model (1), $K$ is no longer assumed constant. Instead it is defined by the
authors as follows (page 23):

"$K_{ij} =$ specific zone-to-zone adjustment factor to allow for the incorporation of
the effect on travel patterns of defined social or economic linkages not
otherwise accounted for in the gravity model formulation".

It is not clear whether, or if so in what way, the procedure of adjusting $K$ factors
is being used to adjust for errors in travel time factor values.

A test of the gravity model should be in two stages, using data from two or more
origin and destination surveys in a single urban area:

Stage 1: Derive an exponent value for each set of survey data which minimises
the divergence between observed and predicted trip length distributions, and
compare the two.

Stage 2: Use the exponent value derived from the first survey to forecast travel
patterns in the second survey year, using data on trip origins and trip ends ob-
tained in the second survey. Comparison of these with the travel patterns fore-
cast by the exponent derived from the second survey (in terms of root mean
squared errors) would indicate the loss of accuracy involved in assuming a
constant exponent value.

An assessment of the overall utility of the trip forecasting procedure would require
checks on the accuracy of forecast travel times and on the accuracy of forecast
generation rates and destinations of zonal trips.

To summarise:

(1) The use of the gravity model as an “open” model with the exponent acting in
effect as an elasticity co-efficient to estimate the effect of relative price changes is
invalid.

(2) The validity of the model as a “closed” model depends on the assumption of
the stability of $n$; a priori reasoning suggests that this assumption is questionable.

REFERENCES

[2] Road Research Laboratory: The Economic Assessment of Road Improvement Schemes. Road Research