OPTIMAL SUBSIDIES FOR PUBLIC TRANSIT

By Raymond Jackson*

The severe traffic congestion in urban areas has been blamed on the lack of an efficient pricing scheme. The automobile owner's decision to drive is based on his own private cost; and, since each car can join in the traffic and experience only average costs and delays, the driver's private cost equals average social cost. No driver gives any consideration to marginal social cost, which includes the additional direct expenses and time delays imposed on others. The result is over-utilisation of the road networks.

In theory it is necessary to impose a congestion toll during peak periods in order to equate marginal social cost and private cost. Complex highway systems with unlimited as well as limited access have discouraged the implementation of this optimal plan, so "second best" solutions must be considered. One "second best" solution is to subsidise public transit during the peak period. This will enable lower fares to be charged, and these lower fares may induce some travellers to switch modes from highway travel to public transit. Though public transit will be provided below cost, the benefits from reduced highway congestion may result in a net increase in welfare.

The first section of this paper develops a model to estimate the optimal fare subsidy for public transit. The model is extended in the second section to estimate optimal subsidies for increasing the speed of public transit rather than lowering fares.

In a recent paper in this Journal Roger Sherman [4] derived rules for determining when subsidies are justified under conditions of congestion interdependence (e.g. bus lines and autos using the same roads). The model developed here assumes congestion independence, a constraint still applicable to several forms of urban transit. It thus presents more clearly the "second best" nature of the problem, estimates the optimal subsidies instead of just indicating when a subsidy is desirable, and makes it possible to calculate the net welfare effect of the subsidy to see whether there is a significant improvement in allocative efficiency.

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If a fare subsidy for public transit is to be justified on "second-best" grounds, it must reduce the economic costs of highway congestion. The current equilibrium in the highway sector is shown in Figure 1. The demand for highway travel as a function of the cost per car passenger mile is given by $D_H$, and the marginal and average social

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cost schedules are represented by $MC_H$ and $AC_H$. When only private costs are considered by the driver there is a non-optimal road use of $H_1$ car passenger miles at an average social cost (private cost) of $AC_{H_1}$ and marginal social cost of $MC_{H_1}$. The optimal level of travel occurs at $H_1^*$ where marginal social cost is $MC_{H_1}^*$ and

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1. The use of car passenger miles follows the procedure used by Sherman [4]. Vehicle-miles is often used [3] [6] and can be converted to car passenger miles by multiplying by the average number of passengers per vehicle. Walters employs a figure of 2 [6, p. 201] and Sherman cites a value of 1.5 [4, p. 27].

2. This divergence between private cost and marginal social cost also leads to problems in evaluating road improvements. See J. M. Thomson [5] for a model that estimates these benefits in the presence of continued congestion.
average social cost is $AC_{H1}^s$. The social value of the extra $H_1-H_1^*$ car passenger miles is calculated as the area under the demand curve from $H_1^*$ to $H_1$, while the social cost is given by the area under the marginal social cost schedule $MC_H$ over this interval. The net loss in welfare, or the congestion cost, can be estimated by the shaded triangle $W_{H1}$. A congestion toll equal to $MC_{H1}^s-AC_{H1}^s$ per mile would reduce $W_{H1}$ to zero by reducing car passenger miles to $H_1^*$. The "second best" solution of a fare subsidy creates welfare losses in the public transit sector, and by this means lowers but does not eliminate congestion costs in the highway sector.

A fare subsidy shifts the demand curve for highway travel from $D_H$ to $D_H'$, lowering the number of car passenger miles at the new equilibrium point in Figure 1 from $H_1$ to $H_2$. There is still a congestion cost or welfare loss in the highway sector which could be eliminated by a congestion toll, since $H_2$ rather than $H_2^*$ miles are travelled, but the fare subsidy reduces the congestion cost from area $W_{H1}$ to $W_{H2}$. Social welfare has been increased in the highway sector by $W_{H1}-W_{H2}$, but the net effect of the subsidy must include the welfare loss now created in the transit sector. In Figure 2 the demand schedule for public transit is shown as $D_T$. We shall assume that (1) public transit operates under constant costs per passenger mile within the relevant range; (2) current fares are non-subsidised and cover the costs of operation; (3) transit riders impose no external costs on others; and (4) as there are separate
roadbeds or rights-of-way, transit operations do not impose costs on highway traffic. Under these conditions the average cost per mile is constant and private cost, marginal social cost and average social cost in public transit are all equal. In Figure 2 the current cost per mile, including direct costs and the value of time, is $AC_T$, and the number of passenger-miles is $T_1$. A government subsidy $S$ per transit passenger-mile for the peak travel period reduces the fare by an equal amount, lowering the cost to the user from $AC_T$ to $AC_T-S$. As a private cost of $AC_T-S$ the number of passenger miles increases from $T_1$ to $T_2$. The increase in welfare from this greater transit use is the area under the demand curve from $T_1$ to $T_2$, while the real increase in costs is the area under $AC_T$ for the interval. The net change in welfare is negative; its value is indicated in Figure 2 by the shaded triangle $W_T$. The net gain in welfare $W$ from the fare subsidy, considering both the highway and public transit sectors, can now be estimated as:

$$W = W_{H_1} - W_{H_2} - W_T$$ (1)

An optimal fare subsidy maximises the net welfare gain, or alternatively minimises the sum of the remaining welfare losses of $W_{H_2}$ in the highway sector and $W_T$ in the transit sector. An estimate of this optimal transit subsidy $S^*$ can be derived from the characteristics of the demand and cost variables in each sector. The current congestion cost $W_{H_1}$ can be expressed as:

$$W_{H_1} = \frac{1}{2}(H_1-H_1^*) (MC_{H_1}-AC_{H_1})$$ (2)

The term $H_1-H_1^*$ must be related to demand and cost elasticities in the highway sector. Following A. A. Walters [6, p. 189], the elasticity of the average cost schedule $\varepsilon_H$ is equal to:

$$\varepsilon_H = \frac{\Delta AC_H/AC_{H_1}}{\Delta H/H_1}$$ (3)

permitting marginal social cost to be related to average social cost by $MC_{H_1} = AC_{H_1}(1+\varepsilon_H)$. Setting $\Delta AC_H = AC_{H_1} - AC_{H_1}^*$, $\Delta H = H_1 - H_1^*$, and using $MC_{H_1} = \varepsilon_H AC_{H_1}$, equation (3) can express $H_1-H_1^*$ as

$$H_1 - H_1^* = \left[ \frac{AC_{H_1}-AC_{H_1}^*}{\varepsilon_H AC_{H_1}} \right] H_1$$ (4)

The term $H_1-H_1^*$ is now expressed as a function of the cost elasticity $\varepsilon_H$ and will now be related to the elasticity of demand for highway travel. Denoting $\eta_{HH}$ as the absolute value of the demand elasticity, we have

$$\eta_{HH} = \frac{-(H_1-H_1^*)/H_1}{(AC_{H_1}-MC_{H_1}^*)/AC_{H_1}}$$ (5)

Setting $MC_{H_1} = AC_{H_1}^*(1+\varepsilon_H)$ in the expression for $\eta_{HH}$ and solving for $H_1 - H_1^*$ yields

$$H_1 - H_1^* = -\eta_{HH} \left[ \frac{AC_{H_1}-AC_{H_1}^*(1+\varepsilon_H)}{AC_{H_1}} \right] H_1$$ (6)

Equating both expressions for $H_1 - H_1^*$ from (4) and (6) yields the efficient average cost per mile $AC_{H_1}^*$ as

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3In the following analysis all demand and cost elasticities are understood as absolute values.
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\[ AC^*_H = \frac{1 + \varepsilon_H \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)} AC_H. \]  
(7)

Substituting (7) into (4) allows \( H_1 - H^*_H \) to be written as

\[ H_1 - H^*_H = \frac{\varepsilon_H \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)} H_1. \]  
(8)

Using this expression for \( H_1 - H^*_H \) and \( MC_{H_1} - AC_{H_1} = \varepsilon_H AC_H \) in the original formula for \( W_{H_1} \) shown in (2) yields an estimate of the congestion loss in the absence of a fare subsidy as

\[ W_{H_1} = \frac{1}{2} \left[ \frac{\varepsilon_H^2 \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)} \right] AC_H H_1. \]  
(9)

A transit subsidy shifts the demand curve \( D_H \) to \( D_H' \); and, assuming \( D_H' \) has the same elasticity with respect to its own price, the congestion loss in the highway sector after the subsidy \( W_{H_2} \) is similarly derived as

\[ W_{H_2} = \frac{1}{2} \left[ \frac{\varepsilon_H^2 \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)} \right] AC_H H_2. \]  
(10)

The value of \( W_{H_2} \) can be related to the size of the fare subsidy \( S \) through the cross-elasticity of demand between highway travel and public transit. If \( \eta_{HT} \) denotes the cross-elasticity, it is defined as

\[ \eta_{HT} = \frac{\Delta H / H_1}{S / AC_T} \]  
(11)

where \( S \) is the transit subsidy and \( AC_T \) the current cost per mile, including direct costs and the value of time, to the rider. From \( \eta_{HT} \) the reduction in highway travel is \( \Delta H = \eta_{HT} \left( S \right. \left./ AC_T \right) \) \( H_1 \), and since \( \Delta H = H_1 - H_2 \) the final level of highway travel is \( H_2 = H_1 \left(1 - \eta_{HT} \frac{S}{AC_T} \right) \). The average cost per mile \( AC_{H_2} \) associated with \( H_2 \) is found from the definition of \( \varepsilon_H \) yielding \( AC_{H_2} = AC_{H_1} \left(1 - \varepsilon_H \frac{\Delta H}{H_1} \right) \). Once again using \( \Delta H = \eta_{HT} \left( S \right. \left./ AC_T \right) H_1 \), the average cost per mile after the fare subsidy is \( AC_{H_2} = AC_{H_1} \left(1 - \varepsilon_H \eta_{HT} \frac{S}{AC_T} \right) \). Substituting these expressions for \( H_2 \) and \( AC_{H_2} \) into the estimate of \( W_{H_2} \) yields

\[ W_{H_2} = \frac{1}{2} \left[ \frac{\varepsilon_H^2 \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)} \right] \left[ 1 - \eta_{HT} \frac{S}{AC_T} \right] \left[ 1 - \varepsilon_H \eta_{HT} \frac{S}{AC_T} \right] AC_H H_1. \]  
(12)

There is an improvement in economic welfare in the highway sector of \( W_{H_1} - W_{H_2} \), which can now be estimated by

\[ W_{H_1} - W_{H_2} = \frac{1}{2} N \left[ \eta_{HT} \frac{S}{AC_T} + \varepsilon_H \eta_{HT} \frac{S}{AC_T} - \varepsilon_H^2 \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \right] AC_H H_1 \]  
(13)

with

\[ N = \frac{\varepsilon_H^2 \eta_{HH}}{1 + \varepsilon_H \eta_{HH}(1 + \varepsilon_H)}. \]

The welfare loss in the transit sector shown in Figure 2 as \( W_T \) must be subtracted.
from this welfare gain. An estimate of \( W_T \) is

\[
W_T = \frac{1}{4} S(T_2 - T_1)
\]

where \( T_2 - T_1 \) is the increase in transit use induced by a reduction in fare of \( S \) per mile. This increased travel is composed of expanded transit use (1) by those already committed to public transit (non-drivers) and (2) by those switching to public transit from the use of their automobiles. To connect the fare subsidy to increased use by non-drivers, a demand elasticity \( \eta_{TT} \) is defined as the ratio of the percentage increase in transit travel accounted for by non-drivers to the percentage decrease in the cost per mile or \( \eta_{TT} = \frac{\Delta T/T_1}{S/AC_T} \). Recalling \( \eta_{HT} \) as the cross-elasticity of demand, the subsidy reduces highway use by \( \eta_{HT}(S/AC_T)H_1 \) car passenger miles and, we shall assume, increases transit passenger miles by the same amount. This transfer from car to transit passenger miles may not be exact if former drivers decide to use subsidised transit more intensively than they previously drove their automobiles. Though it is unlikely that a commuter would take more trips, he might decide to seek employment at a place of work further from home when transit is subsidised. For the purposes of this study we shall assume the transfer equality, but it is most likely a slight underestimate of the additional transit travel of ex-drivers.

The total increase in public transit travel of \( T_2 - T_1 \) due to the subsidy is the sum from both sources with

\[
T_2 - T_1 = \eta_{TT} \frac{S}{AC_T} T_1 + \eta_{HT} \frac{S}{AC_T} H_1.
\]

Substituting this expression for \( T_2 - T_1 \) into \( W_T \), the welfare loss in the transit sector is

\[
W_T = \frac{1}{2} \left[ \eta_{TT} \left( \frac{S}{AC_T} \right)^2 + \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \frac{H_1}{T_1} \right] AC_T T_1.
\]

Subtracting \( W_T \) of (15) from the welfare gain in the highway sector shown in (13) yields the net gain \( W \) from a transit subsidy \( S \) as:

\[
W = \frac{1}{2} N \left[ \eta_{HT} - \frac{S}{AC_T} + \epsilon_H \eta_{HT} \frac{S}{AC_T} - \epsilon_H \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \right] AC_T H_1
\]

\[
- \frac{1}{2} \left[ \eta_{HT} \left( \frac{S}{AC_T} \right)^2 + \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \frac{H_1}{T_1} \right] AC_T T_1.
\]

Setting the differential of \( W \) with respect to \( S/AC_T \) equal to zero results in a welfare maximising fare subsidy for public transit of

\[
\frac{S^*}{AC_T} = \frac{KN\eta_{HT}(1 + \epsilon_H)}{2(\eta_{HT} + \eta_{HT} \frac{H_1}{T_1} + K\epsilon_H \eta_{HT})}
\]

where

\[
K = \frac{AC_T H_1}{AC_T T_1}
\]

and \( N = \frac{\epsilon_H^2 \eta_{HH}}{1 + \epsilon_H \eta_{HH}} \).

Note that the optimal fare subsidy is zero when the roads are uncongested (\( \epsilon_H = 0 \)), when the cross elasticity of demand is zero (\( \eta_{HT} = 0 \)) and when the elasticity of demand for highway travel with respect to its own cost is zero (\( \eta_{HH} = 0 \)). The optimal subsidy is lower as \( \eta_{TT} \) increases, or as non-drivers become more responsive.
to transit fare reductions, and is also lower as the ratio of relative costs $AC_{H_1}/AC_T$ declines.

Some exploratory calculations of optimal fare subsidies are presented in Table 1, using estimates of demand elasticities made by Thomson [5] and by Kraft and Domencich [1] and an estimate of the relative costs of travel done by Meyer, Kain and Wohl [2]. Thomson [5, p. 301] suggests that the demand elasticity for highway travel does not differ significantly from unity; therefore we set $\eta_{HT} = 1.0$. Kraft and Domencich cite a range of fare elasticities for public transit of 0.17 to 0.32 [1, pp. 464-465]. Their study also suggests low cross-elasticities between auto and transit trips; hence the above elasticities should relate to non-drivers or those committed to public transit, and are therefore a measure of $\eta_{RT}$. To strengthen the case for a fare subsidy a figure of $\eta_{RT} = 0.2$ was chosen, recalling that a high $\eta_{RT}$ means that a fare reduction creates considerable additional transit usage with no compensating reduction in highway congestion. Though Kraft and Domencich found cross-elasticities of demand close to zero, we shall consider this response to a fare reduction to be a short-run phenomenon and calculations of $S^*$ in Table 1 use values of $\eta_{HT}$ from 0.2 to 0.6. Meyer, Kain and Wohl calculated the over-all cost per passenger trip by automobile through high density streets at $1.20$, while rapid-transit and subway have an over-all cost of $0.80$ for the same trip, implying $AC_{H_1}/AC_T = 120/80$ [2, pp. 302-306]. A minimum elasticity of the average social cost schedule in the highway sector of 0.2 has been suggested by A. Walters [6, p. 201], and Table 1 considers the range from $\varepsilon_H = 0.2$ to $\varepsilon_H = 1.20$, implying that the marginal social cost per car passenger mile of highway travel is from 20 per cent to 120 per cent above private cost. For the purpose of this example a highway-oriented society is considered, with car passenger miles four times greater than public transit mileage, setting $H_1/T_1 = 4.0$. In summary, Table 1 shows $S^*/AC_T$ the optimal fare subsidy as a fraction of the over-all cost to the user per mile of public transit, with $\eta_{HT} = 1.0$, $\eta_{RT} = 0.2$, $AC_{H_1}/AC_T = 120/80$ and $H_1/T_1 = 4.0$ with $\eta_{HT}$ ranging from 0.2 to 0.6 and $\varepsilon_H = 0.2$ to $\varepsilon_H = 1.20$. The table also shows the ratio of the remaining welfare costs as a fraction of the original highway congestion.

### Table 1

<table>
<thead>
<tr>
<th>Cross-Elasticity of Demand</th>
<th>Divergence of Marginal Social Cost and Private Cost</th>
<th>$\varepsilon_H = 0.20$</th>
<th>$\varepsilon_H = 0.60$</th>
<th>$\varepsilon_H = 0.80$</th>
<th>$\varepsilon_H = 1.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^* / AC_T$</td>
<td>$WH_1 + W_T$</td>
<td>$WH_1 + W_T$</td>
<td>$WH_1 + W_T$</td>
<td>$WH_1 + W_T$</td>
<td>$WH_1 + W_T$</td>
</tr>
<tr>
<td>$\eta_{HT} = 0.2$</td>
<td>0.023</td>
<td>0.997</td>
<td>0.171</td>
<td>0.972</td>
<td>0.270</td>
</tr>
<tr>
<td>$\eta_{HT} = 0.4$</td>
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<td>0.923</td>
<td>0.185</td>
<td>0.990</td>
<td>0.283</td>
</tr>
<tr>
<td>$\eta_{HT} = 0.6$</td>
<td>0.027</td>
<td>0.990</td>
<td>0.186</td>
<td>0.910</td>
<td>0.278</td>
</tr>
</tbody>
</table>
cost, or $W_{H_2} + W_T/W_{H_1}$. For each $S^*/AC_T$ the value of $W_{H_2}$ is found from equation (12) and $W_T$ from (15), and the sum is divided by $W_{H_1}$ of equation (9). This ratio is an indication of the effectiveness of fare subsidies in reducing the level of allocative inefficiency generated by highway congestion. Optimal fare subsidies for public transit, shown in Table 1, suggest that fare subsidies are not warranted when the divergence between marginal social cost and private cost in the highway sector is 20 per cent ($\varepsilon_H = 0.20$). The optimal subsidy reduces the average cost per mile to the transit user by less than 3 per cent and the remaining welfare loss is about 99 per cent of the original loss due to highway congestion, whether $\eta_{HT} = 0.2$ or $\eta_{HT} = 0.6$. As $\varepsilon_H$ increases significant subsidies are indicated. Where $\varepsilon_H = 0.8$ and $\eta_{TH} = 0.4$ the optimal subsidy reduces $AC_T$ by 28.3 per cent, bringing the remaining loss to 89.8 per cent of the original congestion cost, or a decline of about 10 per cent. Significant subsidies and marked improvements in allocative efficiency are possible when marginal social cost is 120 per cent above average social cost. The optimal subsidy lowers $AC_T$ by over 40 per cent, and when $\eta_{HT} = 0.6$ the remaining welfare costs are only 71 per cent of the original level. Though the above calculations are only an illustration of the use of the model, they do suggest that subsidising transit fares in a highway-oriented transport network should not be considered unless $\varepsilon_H$ is at least 0.80 and the cross-elasticity of demand $\eta_{HT}$ is greater than 0.2.

OPTIMAL SUBSIDIES TO INCREASE TRANSIT SPEED

Road users can be switched to public transit by subsidising improvements in transit service rather than in fares. The cost per mile to the rider $AC_T$ includes both a charge covering operating costs of the system and the value of the required time. Increased transit speed reduces this cost per mile, since a value can be placed on travel time. If at current service standards a rider must spend $t$ minutes per mile in the system and time is valued at $w$ dollars per minute, the time cost per mile is $wt$. In addition to the value of time, a fare charge $C$ equal to operating costs is collected, resulting in a cost per mile to the rider of $AC_T = wt + C$. An increase in speed paid for by a subsidy reducing the required time from $t$ to $t + \Delta t$ lowers $AC_T$ by $w\Delta t$ per mile. At this greater speed the cost per mile to the transit rider is $AC_T - S$ where $S = w\Delta t$. The welfare benefit in the highway sector derived from the speed subsidy can still be understood as the $W_{H_1} - W_{H_2}$ of equation (13), except that $S = w\Delta t$, the reduction in cost due to faster service instead of lower fares.

The value of this service improvement in the transit sector is illustrated in Figure 3. Initially, the cost per mile is $AC_T$ with $T_1$ passenger miles of use. Increasing the speed by $\Delta t$ lowers the cost per mile to $AC_T - S$ and results in $T_2 - T_1$ additional passenger-miles of service. The increment to welfare is represented in Figure 3 by the shaded area: the rectangular portion from zero to $T_1$ is the cost-saving through reduced travel time on current passenger-miles $T_1$, and the remaining triangular

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*The fare subsidy $S/ACT$ is the required reduction in over-all cost per transit mile. If fares account for only 50 per cent of this over-all rider cost, an optimal subsidy of 3 per cent must actually reduce fares by 6 per cent. To determine the optimal percentage reduction in fares the values of $S^*/ACT$ in Table 1 must be divided by the fraction of over-all cost $AC_T$ accounted for by transit charges.
area from $T_1$ to $T_2$ is the increase in welfare to the new riders generated by the faster service. Algebraically this gain is equal to $ST_1 + \frac{1}{2}S(T_2 - T_1)$. From this sum must be subtracted the cost of providing the improved service. With $C$ the current operating cost per mile for time $t$ now covered by fares, $\Delta C$ denotes the additional cost per mile of reducing the service time by $\Delta t$. Subtracting the incremental cost of improved service of $\Delta C$, $T_2$ from the benefit described above yields the net welfare change in the transit sector $W_T$. In Figure 3 the incremental cost $\Delta C$ is added back to $AC_T - S$ to obtain the real net cost per transit passenger mile. Subtracting the indicated shaded area of welfare gain from the rectangle $\Delta C$, $T_2$ yields the area of welfare loss shown as $W_T$. Algebraically $W_T$ can be estimated as
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\[ W_T = S \cdot T_1 + \frac{1}{2} S(T_2 - T_1) - \Delta C \cdot T_2 \text{ where } S = \omega \Delta t. \] \hfill (18)

In determining the optimal subsidy for transit speed the decrease in travel time \( \Delta t \) must be related to the increase in operating costs per mile \( \Delta C \). This can be done by defining a cost elasticity with respect to speed \( \varepsilon_s \) as

\[ \varepsilon_s = \frac{\Delta C/C}{\Delta t/t} \] \hfill (19)

which connects the percentage increase in the cost per mile required by a percentage reduction in travel time. From this expression for \( \varepsilon_s \) we can write \( \Delta C = \varepsilon_s \frac{(\omega \Delta t)C}{\omega t} \).

or in more useful terms \( \Delta C = \varepsilon_s \frac{(\omega \Delta t)C}{\omega t} \). Note that \( S = \omega \Delta t \) and \( \omega t \) is the current time cost per mile. Using the symbol \( \alpha \) to denote the fraction of total transit cost \( AC_T \) accounted for by the value of time, \( \omega t = \alpha AC_T \) and \( \Delta t/t \) can be expressed as \( S/\alpha AC_T \). This gives the final expression for \( \Delta C \) used in the model as

\[ \Delta C = \frac{\varepsilon_s}{\alpha} \frac{S}{AC_T} C \] \hfill (20)

Substituting this expression for \( \Delta C \) into (18) and using

\[ T_2 = T_1 \left(1 + \eta_{TT} \frac{S}{AC_T} + \eta_{HT} \frac{S}{AC_T} \frac{H_1}{T_1} \right) \]

derived from (14) yields a net welfare change in the transit sector for the speed subsidy \( \Delta C \) of

\[ W_T = \left( -\frac{S}{AC_T} \right) AC_T T_1 + \frac{1}{2} \left[ \eta_{TT} \left( \frac{S}{AC_T} \right)^2 + \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \frac{H_1}{T_1} \right] AC_T T_1 \]

\[ -\frac{\varepsilon_s}{\alpha} \frac{C}{AC_T} \left[ \frac{S}{AC_T} + \eta_{TT} \left( -\frac{S}{AC_T} \right)^2 + \eta_{HT} \left( \frac{S}{AC_T} \right)^2 \frac{H_1}{T_1} \right] AC_T T_1 \] \hfill (21)

Adding the net increase in the highway sector of \( W_{H_1} - W_{H_2} \) to \( W_T \) and differentiating with respect to \( S/AC_T \) gives the first-order conditions for a welfare maximising subsidy as:

\[ S^* = \frac{1 + \frac{1}{2} NK \eta_{HT} (1 + \varepsilon_s) - \frac{\varepsilon_s}{\alpha} \frac{C}{AC_T}}{NK \varepsilon_s \eta_{HT} + \frac{2 \varepsilon_s}{\alpha} \frac{C}{AC_T} \left[ \eta_{TT} + \eta_{HT} \frac{H_1}{T_1} \right] - \eta_{TT} - \eta_{HT} \frac{H_1}{T_1}} \] \hfill (22)

Once \( S^*/AC_T \) is calculated the optimal increase in speed is found from

\[ \frac{\Delta t^*}{t} = \frac{S^*}{\alpha AC_T}, \] and the required increase in operating costs \( \Delta C^*/C = \varepsilon_s \frac{\Delta t^*}{t} \).

In order to determine whether "second-best" considerations call for a subsidy for faster public transit one must first determine the optimal unsubsidised speed. The correct speed from a welfare standpoint, leaving aside "second-best" problems, is reached when the marginal reduction in time costs equals the marginal increase in operating costs, or \( \omega \Delta t = \Delta C \). Substituting \( \Delta C = \varepsilon_s \frac{(\omega \Delta t)C}{\omega t} \) into this equilibrium
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condition and dividing through by $AC_T$ yields

$\varepsilon_S = \frac{u/t}{AC_T}$ or $\varepsilon_S = \frac{\alpha}{C/AC_T}$

since $\alpha = u/t/AC_T$. Disregarding “second-best” considerations, the optimal transit speed is reached when the cost elasticity $\varepsilon_S$ is equal to the ratio of the fraction of over-all rider cost per mile accounted for by the value of time and the fraction accounted for by operating fees. If $\varepsilon_S$ is less than this ratio an increase in transit speed is indicated, and riders should be willing to pay for the added cost per mile. When $\varepsilon_S$ is equal to or greater than this ratio, further increases in speed can be justified only for the purpose of reducing highway congestion.

Optimal increases in transit speed $\Delta t^*/t$ derived from equation (22) are shown in Table 2, and then the required increase in operating costs $\Delta C^*/C$. As in the previous calculations the values used for demand elasticities, relative costs and relative travel use are $\eta_H = 1.0$, $\eta_T = 0.2$, $AC_H/AC_T = 120/80$ and $H/T = 4.0$. It is also assumed that the value of time currently accounts for 60 per cent of the cost of using public transit, $\alpha = u/t/AC_T = 0.60$, and operating costs covered by fares account for the remainder, $C/AC_T = 0.40$. The cross-elasticity $\eta_HT$ is allowed to range from 0-2 to 0-6, and $\varepsilon_H$ ranges from 0-2 to 1-2, implying a divergence between marginal social cost and private cost in the highway sector of from 20 per cent to 120 per cent. Values of 1.5 and 1.75 are considered for $\varepsilon_S$, the cost elasticity of transit speed. A starting point of $\varepsilon_S = 1.5$ is chosen, since the speed of public transit should be increased till $\varepsilon_S = \frac{\alpha}{C/AC_T} = \frac{0.60}{0.40} = 1.5$ in the absence of “second-best” benefits.

Table 2 also shows the remaining final measure of inefficiency of $W_{H2} + W_T$ as a fraction of the original congestion cost in the highway sector of $W_{H1}$.

As expected, the case for speed subsidies improves with wider divergence between marginal social cost and private cost in the highway sector. Where $\varepsilon_H = 0.20$, the optimal increase in speed is small when $\varepsilon_S = 1.5$ and zero when $\varepsilon_S = 1.75$. When $\varepsilon_H = 0.80$ significant increases in speed are suggested, but the remaining level of economic inefficiency measured by $W_{H2} + W_T/W_{H1}$ is still high. Given the value $\varepsilon_H = 1.20$, substantial subsidies for increased speed are justified, but the improvement in economic efficiency still appears modest when $\varepsilon_S = 1.75$.

CONCLUSION

The model presented in this paper incorporates “second-best” considerations in determining optimal fare subsidies for public transit and optimal subsidies for increased transit speed. Such subsidies may improve allocative efficiency, though no significant improvement is apparent unless marginal social cost per car passenger mile is at least 80 per cent above private cost in the highway sector. This tentative finding is based on estimates of demand elasticities, and of the relative costs of transit

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3Public pressure for greater transit speed should be expected as the value placed on travel time $w$ increases. Since $\alpha = u/t/AC_T$ increased $w$ leads to an increase in $\alpha$ resulting in $\varepsilon_S < \frac{\alpha}{C/AC_T}$.

When this occurs an increase in transit speed is economically justifiable and need not be subsidised.
Table 2
Optimal Reduction in Transit Time ($\Delta t^*/t$), the Required Subsidy ($\Delta e^*/e$) and the Remaining Welfare Loss in the Highway and Transit Sectors as a Fraction of the Original Congestion Cost ($W_{H_2} + W_T/W_{H_1}$)

<table>
<thead>
<tr>
<th>Cross-Elasticity of Demand</th>
<th>Divergence of Marginal Social Cost and Private Cost</th>
<th>$\varepsilon_H = 0.20$</th>
<th>$\varepsilon_H = 0.80$</th>
<th>$\varepsilon_H = 1.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_S = 1.5$</td>
<td>$\Delta t^*$</td>
<td>$\Delta C^*$</td>
<td>$W_{H_2} + W_T$</td>
</tr>
<tr>
<td>$\gamma_{HT} = 0.2$</td>
<td>0.038</td>
<td>0.058</td>
<td>0.997</td>
<td>0.449</td>
</tr>
<tr>
<td>$\gamma_{HT} = 0.4$</td>
<td>0.042</td>
<td>0.064</td>
<td>0.993</td>
<td>0.471</td>
</tr>
<tr>
<td>$\gamma_{HT} = 0.6$</td>
<td>0.044</td>
<td>0.066</td>
<td>0.990</td>
<td>0.463</td>
</tr>
<tr>
<td>$\varepsilon_S = 1.75$</td>
<td>$\gamma_{HT} = 0.2$</td>
<td>0</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{HT} = 0.4$</td>
<td>0</td>
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</tr>
<tr>
<td></td>
<td>$\gamma_{HT} = 0.6$</td>
<td>0</td>
<td>0</td>
<td>—</td>
</tr>
</tbody>
</table>
OPTIMAL SUBSIDIES FOR PUBLIC TRANSIT

Raymond Jackson

and highway travel, made in recent studies. Constant costs are assumed in the transit sector, but the model can be modified to deal with decreasing marginal costs if this seems warranted.

The analysis has considered only the case where there is independence in congestion costs between transit and highway travel, but as a result the model (1) clearly shows "second-best" gains and losses in each sector; (2) permits a calculation of the optimal fare or speed subsidy rather than simply indicating whether a subsidy is justified or not; (3) provides a measure of the effectiveness of the subsidy in alleviating the degree of allocative efficiency caused by highway congestion; and (4) still should be applicable to various forms of urban transit based on rail or separate rights of way. Congestion interdependence would reduce the welfare gain of a transit subsidy, and calculations made with this model yield an upper limit for an optimal subsidy under this condition.

REFERENCES


