CHOICE OF TRAVEL MODE FOR THE JOURNEY TO WORK

Some Findings

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INTRODUCTION

The worst urban traffic congestion usually occurs during the periods of travel to and from work, although there is some evidence that in London "peak" conditions are spreading throughout the day. The situation is worse in the typical nineteenth century core-oriented cities, such as those in northern industrial areas. While this pattern of urban form persists — and with current town planning philosophies there is good reason to think it will, although probably more shopping will be done in the suburbs — heavy morning and evening flows of commuters to and from the centres of towns will continue. Congestion exists partly because many car owners find it more convenient to travel to work by car than by public transport, even in congested conditions. As car ownership increases, an increasing proportion of present "captive riders" on public transport will have the choice of car or public transport. While public transport vehicles share the same road as private vehicles, the shift of commuters from public transport to private car will increase congestion and will delay both types of vehicles. But this shift will happen as long as commuters continue to find a relative advantage in driving their own cars. It seems likely that, under present policies, this relative advantage will persist even as conditions worsen.

It has been estimated that in Leeds (where this study was made), to maintain approximately the same numbers of commuters travelling by car as now, the proportion of car owners using their cars to drive to work would have to be reduced from about 70 per cent now to about 20 per cent in 40 years' time. Given continuing congestion, some reduction would come about by "natural" processes, i.e. by the sort of "feedback" process which somehow ensures that total breakdown of traffic is a rare occurrence. But the conditions under which these natural processes operate are very costly, particularly in time lost by congestion, and it may well be that specific policy action — increasing either the disincentives of car usage or the attractiveness of public transport — could achieve the same result at less cost.

Various ad hoc measures have been proposed from time to time. Buchanan mentioned four main alternatives: a system of permits or licences to control the entry of vehicles; a system of pricing the use of road space; parking policy; subsidising public transport. The acceptability, and cost, of such measures would depend on how much of each or any would be required — this might be expressed in terms of a target percentage of potential car commuters diverted to public transport. Evaluation of possible measures is difficult because very little is known quantitatively about what
influences people's choices of travel mode for the journey to work. This point is brought out in a timely, but not entirely coincidental, way in Dr. Sharp's article in the first issue of this journal, an article which discusses the "supply" side of the general problem - i.e. the effect of a transfer of commuters on speeds and times of different vehicles.

The research to be described here is an attempt to develop and calibrate a model to explain, and subsequently predict, the choice of travel mode of car owners on their journey to work, in terms of factors which can be directly related to policy variables and in terms of a plausible hypothesis about people's behaviour. The research was carried out at Leeds University from 1964 to 1966, under a grant from Leeds City Transport Department, which felt the need for guidelines in thinking about the long-range development of public transport facilities. It was seen as important for a modal choice model to offer a means of estimating the proportion of car-owning commuters that would be diverted to any proposed new transport system.

Since comprehensive transportation study models are continually improving, one might argue that a model for predicting modal choice alone, out of the context of the other travel decisions people make (as represented by generation, distribution and assignment sub-models), would not be particularly useful. If we were examining all trips, or trips in general in urban areas, this point would have some validity. But the journey to work is a special case, because people in work nearly all make two trips per day, and their numbers can be fairly closely and simply related to population, so that the generation sub-model is extremely simple. With predominantly core-oriented cities, like Leeds and Manchester, the destinations of journeys to work are determined, so that the distribution sub-model is by-passed. Nearly all such cities have a largely radial road system and a radial public transport system, so that the route assignment is usually determined. To a large extent this is an oversimplification; but it suggests that in examining this particular situation the problems of ignoring sub-models other than modal choice may only be of secondary importance. Although not seen as a primary task of the research, it is to be hoped that the modal choice model outlined here may form the basis for a new sub-model in the typical transportation study model: modal split is often considered the least satisfactory part of the package.

The quantifying, or calibration, of the model to be described here was carried out by the little known technique of discriminant analysis, which was first used on this type of problem by Warner at Northwestern University and during the last few years has been taken up in a somewhat modified form for modal choice forecasting on the North East Corridor Project. It is noteworthy that a number of projects have started in the U.K. In the last twelve months, some sponsored by public authorities, to investigate further the application of discriminant analysis and other techniques for modal choice in the urban journey to work.

It will be shown later that in this particular case a straightforward multiple regression approach yields equivalent results. For some researchers this approach may be more convenient, since multiple regression programs are available for most computing installations. But the discriminant analysis approach derives direct from the nature of the problem and from a plausible hypothesis of behaviour, and the prediction of probability of choice of mode can be seen to come direct from the form of
the discriminant function. In this article the development of the model is followed through in discriminant analysis terms, and reference is made to the multiple regression equivalent where appropriate.

Before turning to the model, we examine the context of other work in this general field.

**Context**

Almost all previous work concerned with explaining modal choice, or levels of usage of different modes of travel, within urban areas has been carried out in the United States, and can be classified in three ways. Firstly, there is the type of study which relates the use of public transport throughout a city to such characteristics as size, density and age of the city, and to population characteristics such as income, car ownership, and so on. Secondly, there are models of mode choice developed by North American transportation consultants as part of their comprehensive travel forecasting procedures: typically the aim is to predict public transport and private car use for all trips made between any pair of the zones into which the urban area is divided. Factors such as relative travel times and costs for pairs of zones are often taken into account. Thirdly, some researchers have developed models to explain and predict individual choice of mode, taking account of individual travel and household characteristics.

Studies of the first type tend to be of little use for short or medium-term policy — for instance, for increasing use of public transport by improvements or by increasing the disincentives of car use. A study by Schnore relates the use of public transport in different cities to factors such as size, density and age of the cities; a more comprehensive study by Adams develops regression equations explaining city-wide use of public transport in terms of five independent variables — population over five years old, quality of service, income, land use distribution, and urbanised land area in square miles. A recent article by Beesley and Kain has looked at the use of public transport in Leeds in relation to the number of cars per 1,000 population and to urban population density. The article is primarily a discussion of car ownership, but considers the use of public transport as a simple function of population density and income.

Empirical relationships such as these embody no a priori causal hypothesis about people's travel behaviour. It is in fact highly likely that the quality of public transport service, the routing and structure of services, and aspects of car travel such as the provision of freeways and parking facilities are closely associated with some of the independent variables mentioned above (size, density, urban land area, etc.). This is to be expected where there is, and has been, across the country some general implicit agreement on the development of transport facilities in urban areas. But if some new policy decision were adopted, such as the spending of large sums of money on public transport development (as by a recent decision in the United States), these implicit relationships between public transport service and highway facilities and the size, density, etc., of cities would change. Since it is reasonable to suppose that, in the first instance, the use of public transport depends on the former, more immediate, variables rather than on the latter, "global", variables, associations such as those measured by Schnore will no longer be valid. Where one is concerned only to
explain the status quo, this does not matter; but where one is concerned to explain and predict what will happen in new circumstances, or to suggest ways of implementing change, it is important that all variables which are likely to change, or can be changed, be incorporated into a model which has some a priori causal basis – i.e. involves some plausible hypothesis about the behaviour of travellers.

Most of the comprehensive travel forecasting models (the "transportation study" models) currently in use by consultants for the analysis of travel within urban areas embody modal split as an integral part of the model.* It has long been disputed "where modal split should come" in these models – whether the sequence of trip generation, distribution, modal split or assignment is appropriate. This question matters not only from the viewpoint of estimating as closely as possible the behaviour of the individual, but because the position of the modal choice procedure in the forecasting model determines the method of analysis and prediction used. It is also important because it affects the validity of other studies of modal choice which are based to a greater extent on hypotheses of individual behaviour (see below), since these mainly concern the choice of mode on the assumption that the trip itself, its purpose and its destination have already been decided. This problem will not be taken up here; we confine ourselves to a discussion of the modal choice procedure most commonly advocated. This procedure comes after the distribution stage, and its development owes much to Traffic Research Corporation of Toronto.

The procedure, or model, is based on a family of "diversion curves". These curves enable one to predict what proportion of the trips made for a particular purpose between any pair of zones will be by public transport, given the ratio of travel times by public transport to private car (for that zone pair), the cost ratio, the service ratio (a measure of the walking and waiting times involved), and the economic status of the travellers.

Rather than attempting to use linear, logarithmic or other versions of the variables in a regression equation to explain the public transport share of trips, the approach has been to stratify by four ranges of cost ratio, five ranges of economic status, and four ranges of service ratio. Then, on the basis of existing transport study data, a curve relating "public transport share of trips" to overall travel time ratio is fitted for each of the eighty sub-classifications. One purpose of stratifying in this way is to facilitate the application of diversion curves to particular zone pairs: instead of having to make some awkward judgment about the prevailing service ratio or income in a given zone half a mile square, one only has to classify the zone in up to five ways. But there is a problem in using zones for the grouping of observations about travel time and service ratios, particularly since TRC found that modal choice is fairly sensitive to service ratios. Furthermore, one would expect as much variation of travel time ratios, service ratios and perhaps cost ratios among individual observations within one zone as among the mean values of these variables between different pairs of zones. This is recognised by TRC and is one of the strongest arguments against attempting to use diversion curves and zonal analysis for predicting the use of public transport for particular routes or corridors, or for investigating policy changes on a "micro" basis. Nevertheless, the transportation model as devel-

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* It is not appropriate to digress into an exposition of the basic model here; there are many descriptions and discussions, among which the reader is referred to 9 and 10.

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oped by TRC and others is undoubtedly the best available for predicting travel within urban areas for aggregated groups of zones, and it is based on reasonable hypotheses of behaviour. The modal choice procedure has been tested by the Bureau of Public Roads on Washington, D.C., data,12 and has given good results, particularly for journeys to work.

On the individual choice of mode, Warner's work4 is interesting and probably the most comprehensive. He uses multiple regression and discriminant analysis techniques to obtain probability functions which predict the probability that a traveller with given travel time, cost and other characteristics will choose a particular mode for both work and non-work trips to the Central Business District (CBD). He uses information gathered by the Cook County Highway Department from interviews in an outer suburban area of Chicago concerning trips to the Loop, Chicago's CBD. The distances travelled by respondents all lay within a small range, since interviews were conducted within a defined suburban area; certain important variables, such as walking and waiting times, were omitted. But by direct use of individual travel and household data Warner avoids the zonal aggregation inherent in the procedures used by TRC and other consultants. His method is also attractive because it relates travel behaviour to explanatory variables which are as direct as any in their presumed influence; one can consequently have more confidence in predictions from this model than from many other models. Warner did not develop the model beyond predictions based on individuals; he did not, for instance, suggest a means for predicting modal choice on an area or corridor basis. His methodology, however, forms a basis for the work described in this paper.

The remaining work to be commented on is less sophisticated statistically and mainly concerns the valuation of time spent travelling, which will be discussed at length later on. Here we are concerned with the contribution made by this work to the modal split problem; the connection arises, of course, because the choice between two modes of travel frequently involves a trade-off of time against money – i.e. one mode is cheaper but takes longer than the other. Moses and Williamson14 develop an economic model to predict people's choice of mode, based on indifference curves and rates of substitution between working time, travelling time and leisure time. It ignores some factors which might seem important, such as the demand for car use in the household, the use of the car for work, and the relative inconveniences of each mode expressed in walking and waiting times. Using the marginal wage rate to represent the value of time spent in travelling to work, they predict the fares (or negative fares) needed on public transport to cause different proportions of commuters to use it. These are calculated for the sample of commuters in the Cook County survey. But the predictions are not empirically based, since the value of time used is not derived from empirical analysis but is assumed from the theory.

Instead of assuming a value of time and using this to predict modal choice, Beesley15 sets out to find what valuation of time best explains the observed choices of a sample of commuters working in the Ministry of Transport. His approach is to compare, within one mode, those people who chose a cost saving at the expense of extra time with those who chose a time saving at extra cost, and to find that trade-off between time and cost which explains the observed choices with the minimum discrepancy. Beesley is the first to admit that the results are only exploratory – the sample is extremely small, and it is argued below that the method cannot be relied on to give
good results. Conceptually the method has much in common with that used by Warner. It is discussed at greater length below.

This review is not exhaustive; its purpose is to indicate some of the approaches to the modal choice problem. Two criteria have been consistently invoked in discussing previous work: first, that any theory of modal choice should be able to take account of planning or policy variables as influences on modal choice, and secondly, that any hypotheses implicit in such theories should be plausible in relation to individual travel behaviour.

In this article we develop a model similar to those of Warner and Beesley; we use Beesley’s method of solution to show how one can derive, from first principles, a discriminant analysis approach to the quantification of the model parameters, which subsequently yields a probability model similar to Warner’s. We start with a discussion of the chosen variables and then turn to the data and results of the Leeds survey. The general form of the model and its technical analysis are to be found in Appendix A.

CHOICE OF VARIABLES

The model is expressed in terms of the disutility of alternative modes of travel; this disutility is measured in several “dimensions” (time, cost, etc.). Where \( Z \) is the relative disutility of one mode compared with the other, the form of the model as derived in Appendix A is

\[
Z = \lambda_1 x_1 + \lambda_2 x_2 + \ldots + \lambda_n x_n
\]

and \( x_i \), \( i = 1 \) to \( n \) are relative measures of these dimensions. The task of discriminant analysis is to find the best values of \( \lambda_i \), \( i = 1 \) to \( n \), to explain the observed choices of a sample of commuters.

The number of variables that can be put in to measure the relative attractiveness of different modes is not limited by the model itself; it is more likely to be limited by the information that can be conveniently obtained by questionnaire from a sample of users of the two modes. In deciding on a list of variables about which to obtain information from a sample of car-owning commuters, some using car and some using public transport, account was taken of previous work done in the area and of what had previously been found to be important. In TRC’s study\textsuperscript{13} five variables were found to be important:

- ratio of overall travel times by each mode;
- cost ratio;
- excess travel time ratio (walking and waiting times);
- economic status of traveller;
- trip purpose.

Since we are concerned with one trip purpose only, the journey to work, this list reduces effectively to four variables.

Cost

It can be argued that there is a further interrelationship between the effects of cost and income, which is not self-evident from these findings. From the axiom that

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higher income travellers are better able to afford to pay a premium to satisfy their preferences than are low income travellers, it follows that if the cost of travelling by car is greater than by public transport the effect of higher income is to lessen the effect of cost as a deterrent to using car. This would suggest that as long as public transport is cheaper than car, the probability of going by car ought to be positively related to income. (The same applies to public transport if it is more expensive.) If relative costs are very different, the effect will depend on the size of the traveller's income; if costs are more nearly the same, the effect will be less, and one may wonder whether it will be significant.

Costs have no meaning for a traveller except in the context of his income and what he can spend: one might therefore argue that relative costs should always be expressed as a function of income. The problem is, what function?

If relative costs are expressed as a ratio, there is no way of expressing income in a way that is dimensionally correct. Note, in passing, that this is an argument against using cost ratios at all. If relative costs were expressed as a difference, one form would be the ratio of cost difference to income. But this presupposes that the effect of cost is directly proportional to the traveller's income. Too little is known about people's preferences for us to say with any certainty what a better form might be. One might argue that more important demands on the household's income leave a smaller sum subject to decisions about expenditure on commuter travel, and this smaller sum is not directly proportional to income. But one might also advance the possibility that, if higher income commuters spend more money in total on commuting than lower income people, the marginal utility of extra increments of cost could be much more nearly the same. Then cost differences between modes could justifiably be entered as one variable, and one would expect that income expressed as another variable would turn out in the course of the analysis to be insignificant. It was decided to test both ways of handling costs and income and also a further one, in which cost by mode 1/income is one variable, and cost by mode 2/income is a second variable.

There is an argument for incorporating income as a straight variable anyway. This is that the effect of income levels is not only financial, since those with high incomes might feel they should conform to certain patterns of social behaviour which are seen as appropriate to that economic status, and not using public transport could be an important part of the pattern. If this is a significant effect, then, quite separately from the above arguments relating to the expression of cost effects, discriminant analysis would be expected to yield a positive weighting coefficient for income in an expression measuring the relative disutility of public transport. But where costs and income are expressed independently it might be difficult to separate the financial effect from the social effect in the weighting coefficient for income.

In the formulation in which "mode 1 cost/income" and "mode 2 cost/income" are expressed as separate variables, we should expect the weighting coefficients for these two factors to be the same, or insignificantly different, since money is the same commodity however it is spent. The cost of running a car is not, however, an immediately evident sum; and if significant differences do exist between these coefficients, they might well be due to an inappropriate figure for car mileage cost being used to calculate total car costs. This figure of car mileage cost is crucial and will have an important effect on the results, yet very little is known about how people actually do cost out the running of their cars. In a model which is trying to explain and pre-
dict people's behaviour, it may be hazardous to use an "engineering" figure for mileage cost without question, since people might well perceive running costs quite differently. It is their perceptions that are important in influencing what they do. It will be seen later that it is possible to experiment with different car costs and find that cost which gives the best discrimination, other things being equal. If the difference between the best discrimination so achieved and the discrimination for other car costs is significant, this will be as good a guide as any to how people implicitly cost out their cars.

To start with, it was decided to use fourpence a mile as the variable cost per mile for all cars. This figure was a compromise between the Commercial Motor figures and the Road Research Laboratory operating cost formulae. Unfortunately, data on different car sizes in the sample had not been gathered. Previous work suggests that there is a choice of ways of expressing relative costs (when income is expressed separately): cost ratio, as TRC and other consultants express it; log of cost ratio, implied by Warner's methodology; and a cost difference, which is perhaps the most intuitively reasonable. It was decided to test all three forms, as well as the different ways of combining income mentioned above.

**Time**

Not much evidence is needed to support the inclusion of travel time as an important factor. TRC's addition of excess travel time suggests that people may value walking and waiting time differently from "in-vehicle" time. Goldberg reports that analysis of a survey in Paris showed transfer time was treated as double actual time, and waiting time as treble. There may therefore be a case for separating walking time from waiting time. But this would again cause an increase in the number of variables, and in the absence of any further evidence it was decided to collect together all off-vehicle time (walking, waiting and transfer). Some researchers (for instance, Wilson) have also counted the time spent in a car looking for a parking space. There is no doubt that such time might well be frustrating and therefore have a higher implicit cost than the ordinary driving time, but it is both hazardous and difficult to estimate. It was not considered here.

Given that relative excess travel time is worth including, it remains to decide whether the whole journey should be further characterised by relative overall travel time or by relative in-vehicle time, i.e. overall travel time less excess travel time. The argument in favour of the latter is that excess travel time and in-vehicle time are fairly independent, while excess travel time is a constituent part of overall travel time and therefore not independent. But its dependence is acceptable if account is taken of it in prediction schemes. The main case for using overall travel time is that people usually see a journey primarily in these terms, rather than in terms of in-vehicle time. At the time of the design of the research there was no evidence for this, but an interesting retrospective justification of it was developed on the basis of the survey data.

Travel times, like costs, can be expressed in different ways. The main ones considered are: overall and excess travel time differences, ratios, and logs of ratios. No other ways were considered, as there is neither evidence nor intuitive argument to support any. Although there is, in terms of the relative disutility model, an a priori case in favour of differences, it is difficult to justify this method on a separate intuitive
basis. Take the three men, A, B, and C, whose travel times in minutes by car and 
public transport are as follows:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time by public transport</td>
<td>50</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>Time by car</td>
<td>25</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Travel time ratio</td>
<td>2</td>
<td>1.25</td>
<td>2</td>
</tr>
<tr>
<td>Travel time difference</td>
<td>25</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

If it is the ratio of travel times that is important for people in deciding what to do, 
then the car is equally preferable for A and C, but less preferable for B, other things 
being equal. If differences are important, the car is equally preferable for C and B, 
but more so for A. Which is more likely to express how people actually make compar-
isons? Because TRC's modal split procedures (based on ratios) actually "work" it 
does not necessarily follow that this is the only valid approach. Yet if the assumptions 
about travel time differences inherent in the process of valuing time savings are valid, 
is it merely an artefact that ratios seem to work?

Evidence by Wabe in a recent paper\textsuperscript{22} seems to suggest that people's valuation of 
time varies according to the length of work journey. His analysis does not, however, 
support the time ratio measure; that would require the value of time to be directly 
proportional to the total journey time, whereas he found that the variations in the 
value of time were wider than this. On the other hand, it might be argued that the 
apparent time-cost trade-off is too closely involved with the rent gradient and the 
price of household space, which he considers concurrently, to permit this conclusion. 

It was decided to test the following alternative forms, the second of the fourth pair 
being included because of an anticipated oversensitivity of excess travel time ratio, yet 
allowing that the ratio of overall travel times (rather than differences) may be a 
good explanatory variable.

1. overall travel time ratio;
   excess travel time ratio.
2. overall travel time difference;
   excess travel time difference.
3. \log (overall travel time ratio);
   \log (excess travel time ratio).
4. overall travel time ratio;
   excess travel time difference.

Other Variables
There are no other quantifiable variables which express the characteristics of travel 
by each mode. Comfort, safety and reliability may indeed be important factors affect-
ing use of public transport, but they cannot easily be quantified, and are more open 
to argument about differential perception than are any of the previous factors. Re-
results obtained without consideration of comfort, safety and reliability will be valid 
only as long as these do not change. For instance, if buses were suddenly made more 
comfortable and attractive this model would not be able to explain or predict any 
resulting modal change.

It is interesting that other researchers have recently devised a way of handling the 
problem which might help to show the order of magnitude of these effects. This

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involves asking the sample to indicate by some means (for instance, on a five-point scale) how they feel about the comfort or reliability of a public transport service, or to ask attitude questions designed to elicit non-quantifiable feelings about bus travel and car travel such as might influence modal choice. Then the sample is sub-divided according to the answers given to one or more of these questions, and the discriminant analysis run on each sub-sample. Significant differences between weighting coefficients, or the constant term, will indicate the effects of different attitudes or feelings corresponding to the sub-divisions of the sample. This was not done for the work reported here.

It is as well to acknowledge at this stage a further factor which by an oversight was not taken account of, and which future research could well examine: the carrying of passengers in the commuter’s car. Their existence might well make him less inclined to give up his car, particularly if he were receiving payment. But this latter practice, being illegal in this country, might prove difficult to find out about.

At a personal level, it may be that age is an important factor, particularly in determining how far people are prepared to walk. It was not considered sufficiently important to incorporate here; no other researchers have taken account of it in modal split work.

Since the chosen approach to modal split concerns only the choice between private car and public transport (either bus or train) for the journey to work, there are certain other factors which may incur disutility if one or other of these modes is used, but which would not be important if the choice between two public transport modes were being examined. Although some of the more academic work on modal split has considered factors relating to alternative uses for the car at home as a possible influence on modal choice, none of the operational planning models has done so. If the commuter drives his car to work, he deprives the rest of the family of the use of the car for the whole day. The importance of this depends partly on how many other driving licences there are in the family. An arbitrary measure was chosen for the disutility that a commuter would incur if he used his car to drive to work: car demand ratio, which is the ratio of the number of driving licences in the household to the number of cars in the household. This was not further analysed: it might be argued, for instance, that a wife with a driving licence has more “pull” than a son or daughter, but this difference was not thought sufficiently important.

In the U.K. we are at a stage where the proportion of the working population which uses cars for the work journey is still small compared to the United States (even though, in relation to the size and scale of British city streets, it seems a large absolute number). Consequently it is to be expected that those who need or have occasion to use their cars for work will form a higher proportion of Britain’s car commuters than of U.S. car commuters. However, there was felt to be a distinction between the commuter who had an overriding need to use his car for work (traveller, service man, etc.) and one who had occasional need to use his car for work but could, if necessary, organise his working day according to whether or not he had brought his car in. For the latter the sensitivity of modal choice to general conditions of travel would be much more like that of the man who never had occasion to use his car for work than that of the man who had an overriding need to do so. In these circumstances, the (occasional) use of car for work could be seen as another factor – not an overriding one – influencing his choice of mode. If he had an occasional use for his
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car during work, then the relative disutility of travel by bus would be somewhat higher. Thus, if this factor is given some arbitrary positive score, a priori we should expect the discriminant analysis to yield a positive weighting factor in an expression for relative disutility of bus travel.

It was also felt that, other things being equal, the commuter with a firm's car would feel more constrained to drive to work in it, irrespective of whether or not he used it for work, than if he owned it himself.

The following seven factors, and the versions of them described above, formed the basis for the early runs of the discriminant analysis. Subsequent versions of factors, and sub-groupings of them, suggested themselves on the basis of the early runs, and are described, together with their results, later on.

Relative overall travel time;
Relative excess travel time;
Relative cost
Income combined in different ways;
Car demand ratio;
Use of car for work;
Ownership of car by firm.

DATA AND RESULTS

For the model, information is needed from users of both car and public transport who have choices (i.e. they must be car owners) about times and cost of travel by both the chosen and the alternative modes, and about personal and household information – income, number of driving licences and cars in the household, ownership of the (relevant) car, and whether there is a non-overriding need to have the car available at work. An inexpensive method is to carry out the survey at people's places of work, particularly since we are studying only the journey to work (as Beesley did at the Ministry of Transport).

The sampling could be a problem here: if one large workplace is chosen, or a number of workplaces which are very close together, there may be a predominance of one, or a few, particular distances from parking place to workplace, or of one or a few distances from bus stop (or station) to workplace. To obviate this, care was taken in choosing the firms for this survey to achieve a spread in terms of their nearness to the core of the central business district of Leeds, and in terms of available parking facilities. A few of the 40 firms and organisations which co-operated provided free or low-price contract parking for their employees, but most did not. It was also possible to achieve a spread in terms of nearness to the main rail and bus termini. Thus the firms were chosen not at random, but to achieve a representative distribution of walking distances with the central business district (CBD). Within firms, it could be assumed that the other variables were randomly distributed, and not biased with regard to the totality of car-owning commercial and office employees in the CBD. Further, there is no reason to suppose that the distribution of incomes among car-owners within the 40 firms approached was not representative of the distribution of car-owners' incomes within the whole CBD.

In each firm, after personal visits to its head, a liaison man was appointed. He was responsible for ascertaining the number of car owners in the firm, and for cir-
culturating the questionnaires; when completed, these were returned direct in reply-paid envelopes. The instructions were that those people who used their cars necessarily in the course of their jobs were to be excluded (as having no feasible choice of mode); and married women whose husbands owned cars were excluded, except where there were second cars of which they had exclusive use.

Of the 993 questionnaires sent out, a whole block of 152 were found to have been misplaced by one of the organisations and never distributed. It was confirmed that no bias would be introduced by excluding these from the calculation of response. Of the remaining 841, a total of 688 were returned, giving a response rate of 81.8 per cent. Of these, 18 were excluded because the firm in question sent them out to suburban sub-offices and 31 because their chosen modes were as passengers in other people's cars, and we were not examining this particular choice. Of the remaining 639 there were 542 who had a choice between car and bus, and 97 between car and train.

Direct questions on income have been known to reduce significantly the response obtained on surveys.* This was possibly due to the way in which the question was phrased in many surveys – it is an important and sometimes embarrassing question for a respondent. The question we asked was:

"We would like some indication of your income, as this is an important piece of information for the research. However, many people are understandably reluctant to give this, so please do not feel obliged to if you prefer not to."

"If you are willing to give this information, would you please indicate in which of the ranges below your gross annual income lies?"

- less than £600
- £600 to £900
- £900 to £1,200
- £1,200 to £1,500
- £1,500 to £1,800
- £1,800 to £2,400
- over £2,400"

In fact over 96 per cent of those returning questionnaires (82 per cent of the total) did answer it.

It is perhaps worth mentioning how the time questions were phrased. We had had some unfortunate experiences on previous surveys from asking questions like "When do you usually leave your home in the morning?" In the present case, all questions referred to what the respondent actually did on the morning of the day he answered the questionnaire (using the chosen mode of that day) and what he would have done on that morning if for some reason he had come by the other mode. Account was taken of the possibility of an untypical choice on the day in question by asking whether what he had done today was what he "normally" did. If not, a further question was asked, the answer to which would decide whether he should be transferred to the group of users of the other mode for analysis purposes. For instance, if a man came by bus though he did not use the bus normally, he would be transferred to the car mode if his car had gone in to be serviced on that day. If

*F.R. Wilson (reference 21, page 456) records some difficulty in his workplace surveys; this was tackled by issuing a further similar questionnaire with the income question omitted. The subsequent response was about 60 per cent.
his wife needed the car on that day, however, then he would remain a bus user, since the fact of his wife’s potential use would be taken account of by the number of driving licences in the family, which enters the analysis explicitly.

**Simple Checks on the Data**

In coding the questionnaires for punching, care was taken to check all the cost information. Minor discrepancies were ignored, but gross errors in, for instance, bus and train fares were corrected, on the grounds that when the traveller next used that mode he would find out exactly how much it cost him, and he might alter his relative usage of that mode.

There is a more fundamental problem in people’s reporting of time. The first issue is people’s perceptions of the time of walking and waiting. It can be argued that, if a person reports 5 minutes where it is actually 3, it feels like 5 minutes to him, and whatever decisions he makes will be made on the basis of that perception. Against this it can be argued that, where the perception refers to the mode not being used, the next use of that mode might cause him to revise his estimate. Even requiring people to think about the time taken might cause them to estimate it more accurately when they next travel, and the answer given then might be different. In order to explore this problem, linear regressions were carried out for a sub-sample of respondents of the reported walking times on corresponding distances as measured on a map— for walking time from home to bus stop, and for walking time from car parking place to workplace. It was striking that extremely good correlations were achieved in each case (0.942 and 0.906 respectively); the regression line suggested that in each case a good rule of thumb was a minute for a hundred yards and a minute over. During coding, trivial discrepancies with the regression estimate were in every case ignored; for gross discrepancies, a time midway between the reported time and the regression estimate was taken, to prevent emotionally exaggerated reports (which are perhaps not quite the basis for a decision) having an over-important effect.

Waiting times for buses can be approached in a similar way. This is rather more awkward: although a good estimate of mean waiting time is half the average interval between buses where intervals are 10 minutes or less and the service is regular, considerable variations from the scheduled intervals do occur during the peak in Leeds. A decision rule was adopted that reported waiting times were to be corrected only where the mode was the alternative, and only where an estimate was more than three times the scheduled interval: it was then reduced by half.

A further problem concerns people’s estimates of (clock) times for the beginning and end of the whole journey, from which the overall travel time is calculated. This is particularly so for the journey by the alternative mode. It is not the problem of the time by a clock so much as one of a respondent’s conscious or unconscious overestimate of the time that the alternative mode would take, by which he can feel justified in having chosen the mode he did. Yet there is a distinction between overestimation as an unconscious way of characterising some inconvenience or unattractiveness of the alternative mode, and over-estimation as a conscious rationalisation. To incorporate the former is valid, since it is part of the man’s perceptions of the alternative mode, affecting his choice. But unconscious overestimates may be altered if the alternative mode is actually used. As the two types of over-estimate cannot normally be differentiated, it was decided to accept all times for coding as stated,
and to determine subsequently whether there was any significant difference between the way bus users perceive journey times by car and bus and the way car users perceive them.

This subsequent analysis consisted of attempting to fit different types of relationships between speed, time and distance, by means of simple linear regression, for each mode and within the group of users of each mode. Any differences in the perception of say—bus travel by bus users and by car users would emerge as significant differences between the regression parameters obtained by regressing — say — average bus speed on log (distance) for bus users, and then again for car users. The three types of relationships tested were: time on distance, time on log (distance), and average speed on log (distance); each was done twice, once for overall travel time, once for net in-vehicle time (overall travel time less excess travel time), and each of these was done for each mode and each group of mode users, making a total of 24 regressions.

All but three of the correlation coefficients lay between 0.60 and 0.82. There was little to choose between the three types of relationships as far as the goodness of correlation was concerned, although the speed v. log (distance) relationship was marginally the best. It was interesting that the regressions based on net in-vehicle time were not significantly better than those based on overall travel time. This is perhaps surprising, since different travellers have more in common in the conditions of speed and congestion in the traffic than in walking and waiting times at each end of the journey. It suggests that people perceive the overall travel time of their journey to some extent independently of variations in the amount of walking and waiting time.

In all three types of relationships, there was remarkable agreement between car users and bus users on the speed and time of car travel — in all cases the differences between regression coefficients are not significant. But bus travel in the perception of car users is about 20 per cent slower than in that of bus users, and the implied difference between the regression coefficients is significant at the 5 per cent level. Of this 20 per cent difference, it seems that about half is attributable to a difference in the walking and waiting times: bus users either know their time tables better or live (or work) nearer bus stops. But half the difference occurs on “in-vehicle” time, implying that there is a genuine difference in perception between car users and bus users of about 10 per cent in the actual speed of bus travel.

**Analysis**

With a basic model of the form

\[ z = \lambda_1 x_1 + \lambda_2 x_2 + \ldots + \lambda_n x_n \]

where \( z \) is the relative disutility of the public transport mode, \( \lambda \) are weighting coefficients, and \( x \) are (relative) measures of factors, such as time and cost, the task of the analysis was to provide answers to the following questions:

- Which type of formulation (ratios, differences, etc., of time and cost) provides the best explanation of the observed modal choices — i.e. which measure of relativeness most appropriately represents people’s comparisons of two modes?
- What can be said about how people cost the running of their cars?
- What is the relative importance of the various factors in affecting choice of mode?
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How well can the discriminant function explain the choice of mode, in terms of the “best” set of factors discovered?

What other relationships are there between factors, such as might affect the validity of predictions?

What can be said about how people value time?

Are there significant differences in the results obtained between car-bus and car-train choices?

Measures of Relativeness

The first piece of analysis was designed to compare the ratios method of expressing relative times and costs (as used by transportation study consultants), logs of ratios (Warner), and differences (Beesley, Moses and Williamson, etc.). (Note that no substantial case is argued by any writer in favour of one method over another.) The discriminant analysis programs were run on the car-bus sample of 542 with overall travel times, excess travel times, and costs expressed in these three different ways (set 1, set 2, set 3, respectively):

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall travel time ratio</td>
<td>Log (overall travel time ratio)</td>
<td>Overall travel time difference</td>
</tr>
<tr>
<td>Excess travel time ratio</td>
<td>Log (excess travel time ratio)</td>
<td>Excess travel time difference</td>
</tr>
<tr>
<td>Cost ratio</td>
<td>Log (cost ratio)</td>
<td>Cost difference</td>
</tr>
</tbody>
</table>

In all three cases a further four factors were included: income, car demand ratio, use of car for work, and ownership of car by firm.

The cost for car travel was half the daily parking charge plus one-way mileage at 4d a mile.

The multiple correlation coefficients in each case were:

**Car-bus data**

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>46.2%</td>
<td>50.2%</td>
<td>52.3%</td>
</tr>
</tbody>
</table>

Set 1 and Set 3 were also run on the car-train sample, with these results:

**Car-train data**

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>56.0%</td>
<td>57.6%</td>
</tr>
</tbody>
</table>

It seems that the “differences” formulation of factors was the best of the three tried. This was further confirmed by the results of two additional sets, in which the costs of each mode, divided by income, were entered as separate factors:

<table>
<thead>
<tr>
<th>Set 4</th>
<th>Set 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall travel time ratio</td>
<td>Overall travel time difference</td>
</tr>
<tr>
<td>Excess travel time ratio</td>
<td>Excess travel time difference</td>
</tr>
<tr>
<td>Public transport cost/income</td>
<td>Public transport cost/income</td>
</tr>
<tr>
<td>Car cost/income</td>
<td>Car cost/income</td>
</tr>
<tr>
<td>Car demand ratio</td>
<td>Car demand ratio</td>
</tr>
<tr>
<td>Use of car for work</td>
<td>Use of car for work</td>
</tr>
<tr>
<td>Ownership of car by firm</td>
<td>Ownership of car by firm</td>
</tr>
</tbody>
</table>
The multiple correlation coefficients were:

**Car-bus data**
- Set 4: 47.3%
- Set 5: 52.3%

**Car-train data**
- Set 4: 57.1%
- Set 5: 58.0%

On the theory that misclassification of travellers can be measured about a notional threshold halfway between the means, a further indication of discrimination within the sample itself is a measure of the proportions misclassified, weighted for the different numbers of the car and bus-using populations. These were:

**Car-bus data**
- Set 1: 26.9%
- Set 2: 24.0%
- Set 3: 23.6%

As a result of this it was decided to use differences for all the further analyses.*

**Car Mileage Costs**

There were certain results from the above set of runs which suggested that 4d. a mile was not an appropriate figure for the calculation of car mileage costs. It has been remarked before that, since a model of this type is seeking to explain people’s decision-making behaviour, it is important to recognise that perceptions, on which decisions are based, may be different from reality. This is specially important where the reality is not evident, as with the cost of running a car.

In Set 4 and Set 5, it was noted that the coefficients for car cost/income and bus cost/income were very different:

---

*The important question is whether the differences between correlation coefficients for the different sets of factors are significant or not. As mentioned in Appendix A on the model, Fisher’s transformation may be used to investigate whether differences between correlation coefficients are significant: at 52 per cent, the 95 per cent confidence interval appears to be about ± 6 per cent. It is not clear whether this is applicable here; the alternative “jack-knife” technique was also used, running one set of factors (set 3) on 90 per cent of the data ten times, leaving out a different 10 per cent each time. The estimated variance of $R$ comes out as 1.403 at 52 per cent, giving a standard error of 1.185, or 1.19. On this basis, differences greater than about 2.3 per cent are significant at the 5 per cent level. We would conclude, then, that the difference between set 3 and set 2 was almost significant, and that between set 2 and set 1 was certainly significant. For the car-train data, set 3 is better than set 1, but the difference is only significant at the 10 per cent level. As between set 4 and set 5, the differences for the car-bus data are conclusive, but again those for the car-train data are less so.

If the jack-knife estimate of the standard error of $R$ is acceptable, one can say that use of ratios is definitely inferior to the other two methods, and that differences seem preferable to logs of ratios (although the evidence is not entirely conclusive) at a 5 per cent level of significance. The remainder of the analysis uses differences alone; little more work was done using logs of ratios, as the computing time available was limited.

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Car-bus data

\[
\begin{array}{c|c|c}
& Bus cost/income & Car cost/income \\
(\lambda_3) & (\lambda_4) & \lambda_3/\lambda_4 \\
\hline
\text{Set 4} & 0.1519 & -0.0628 & 2.42 \\
(\text{t-value}) & (4.14) & (3.90) & \\
\text{i.e., } \lambda/\text{standard error of } \lambda & & & \\
\text{Set 5} & 0.1522 & -0.0471 & 3.23 \\
(\text{t-value}) & (4.03) & (3.83) & \\
\end{array}
\]

Since there is no reason, a priori, why the two types of cost should have such very different coefficients, these results suggest, even though the t-values are not very high, that the figure for car mileage cost of 4d a mile, while approximating to the engineering cost, is not how people actually perceive car running costs. The fact that $\lambda_4$ is substantially less than $\lambda_3$ suggests that the car cost is overstated. But since parking charges enter the total cost calculation one cannot infer that car mileage cost is overestimated by any one of the four ratios of $\lambda_3$ to $\lambda_4$. It was decided, therefore, to design a new set of runs of the discriminant programs, experimenting with different car mileage costs, and the results are described below.

It is to be noted particularly that Set 4 gave as good a discrimination as Set 3, the best of the previous three tried, and that for the car-train data Set 4 is somewhat better than Set 3. In the car-bus case this suggests that the possible "constraints" in Set 4 of treating costs as a proportion of income must be complemented by some other constraint in Set 3, which can only be that of combining two costs into a cost difference (at 4d. a mile) instead of leaving them separate.

In the search for a value of car mileage cost that seems to best represent people's perceptions, the various factor sets were each run several times with different car mileage costs. Discrimination should be at its maximum when the "constraint" of using a cost difference, instead of treating car and bus costs as separate factors, was at a minimum: this would be when the figure inserted for car mileage cost most nearly corresponded with the mean value imputed by commuters in the sample. For those factor sets where bus or train costs and car costs are already separate, one would not expect the use of the "optimum" car mileage cost to make such a large difference to the discrimination achieved, since there is no constraint of using differences whose effect is thereby reduced. It was expected that the optimum car mileage cost would be less, rather than more, than 4d. a mile: consequently, up to 6 different car costs, ranging from 1d. to 3\text{4d.} by 1\text{4d.} intervals, were tried.

The correlation coefficients obtained for Set 3 were:

Car-bus data

\[
\begin{array}{|c|c|}
\hline
\text{Car costs per mile} & R \\
\hline
1\text{d.} & 52.5\% \\
1\text{4d.} & 53.0\% \\
2\text{d.} & 53.4\% \\
2\text{4d.} & 53.1\% \\
3\text{d.} & 52.9\% \\
3\text{4d.} & 52.6\% \\
4\text{d.} & 52.3\% \text{ (as before)} \\
\hline
\end{array}
\]

A maximum occurs within the range, at 2d. a mile; but the differences between the correlation coefficients are not, by the previous criterion, particularly significant.
In Set 4 the differences between correlation coefficients were even smaller, as expected, and the ratio of the two cost coefficients, on the car-bus data, was about unity at 1½d. a mile:

**Car-bus data**

<table>
<thead>
<tr>
<th>Car costs per mile</th>
<th>( \lambda_3/\lambda_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d.</td>
<td>0.827</td>
</tr>
<tr>
<td>1½d.</td>
<td>1.079</td>
</tr>
<tr>
<td>2d.</td>
<td>1.338</td>
</tr>
<tr>
<td>2½d.</td>
<td>1.601</td>
</tr>
<tr>
<td>3d.</td>
<td>1.869</td>
</tr>
<tr>
<td>4d.</td>
<td>2.421 (as before)</td>
</tr>
</tbody>
</table>

The standard errors of \( \lambda_3 \) and \( \lambda_4 \) are not sufficiently low for us to conclude that 1½d. is definitely the best value, although the coefficients themselves are clearly significant (from their t-values). For Set 2 optimum car mileage cost was 2d. a mile, but the difference between the correlation coefficients for car costs of 2d. and 2½d. was only 0.1 per cent. On the car-train data the following results were achieved:

**Car-train data**

<table>
<thead>
<tr>
<th>Car costs per mile</th>
<th>( R )</th>
<th>( \lambda_3/\lambda_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d.</td>
<td>55.1%</td>
<td>0.422</td>
</tr>
<tr>
<td>1½d.</td>
<td>56.0%</td>
<td>0.621</td>
</tr>
<tr>
<td>2d.</td>
<td>57.0%</td>
<td>0.816</td>
</tr>
<tr>
<td>2½d.</td>
<td>57.7%</td>
<td>1.011</td>
</tr>
<tr>
<td>3d.</td>
<td>57.8%</td>
<td>1.208</td>
</tr>
<tr>
<td>3½d.</td>
<td>57.7%</td>
<td>1.402</td>
</tr>
<tr>
<td>4d.</td>
<td>57.6%</td>
<td>1.596</td>
</tr>
</tbody>
</table>

Maxima occur at 3d. a mile and 2½d. a mile respectively.

Thus for the car-bus sample of 542 the differences are not significant, although nearly the same optimum mileage cost is given by two quite different methods of organising the factors. To continue the analysis one must choose some figure, and, while its validity is open to question, the figure of 2d. a mile was chosen as the most likely estimate for the car-bus data. From the car-train sample of 97 one might prefer 2½d. per mile: there may well be a real difference here, since the mean income of the car-train sample was exactly £200 more than the mean income of the car-bus sample (£1,602 as against £1,402), and, on the argument that wealthier people tend to run more expensive and more thirsty cars, the mileage costs may indeed be perceived to be more in the car-train sample. There may, however, be no real difference between the true values. The smaller car-train sample merits less weight to results obtained from it, although the correlation is markedly better.

Perhaps the strongest case for accepting these figures is that 2d. to 2½d. a mile was almost exactly the petrol cost of cars averaging 25-35 m.p.g., and this might represent the perceived cost for most motorists. It is regrettable that the original questionnaire did not ask people how much they thought their cars cost per mile to run or what kinds of cars they had; this would have enabled perceived costs to be related to actual costs for different sizes of car.
Further Exploration of Set 3

To continue the analysis using Set 3 at 2d. a mile, the full results were:

Car-bus data. Sample size 542 (376 car users, 166 bus users). R = 53.2%.

\[
\begin{align*}
\text{Relative disutility of bus travel (in } z) &= \frac{t}{\lambda} \text{, i.e. } \lambda, \\
0.0536 &\text{ (overall travel time difference in minutes)} \\
+ 0.0066 &\text{ (excess travel time difference in minutes)} \\
+ 0.06911 &\text{ (cost difference, with 2d. a mile car cost)} \\
- 0.535 &\text{ (income in } \mathcal{L} \text{ p.a., divided by 1,000)} \\
- 0.333 &\text{ (car demand ratio: } \frac{\text{driving licences in households}}{\text{number of cars in households}}) \\
+ 0.620 &\text{ (use of car for work, either 3 or 0)} \\
+ 0.323 &\text{ (ownership of car by firm, either 1 or 0)} \\
\text{Constant term: 0.461}
\end{align*}
\]

From this, the first three factors and the sixth are clearly significant; car demand ratio and ownership of car by firm are not significant. The \( t \)-value for the income coefficient is 2.51, which suggests its effect is significant, but its magnitude is in some doubt. Its sign is the only implausible one: it suggests that increasing income will cause the relative disutility of bus travel to fall – i.e. will increase the likelihood of travelling by bus. This result is probably due to the very small difference between the mean incomes for the two modes: \( \mathcal{L}1,395 \) for car users and \( \mathcal{L}1,416 \) for bus users (compare also the mean incomes in the car-train sample: \( \mathcal{L}1,677 \) for car users, and \( \mathcal{L}1,441 \) for train users). Both these findings, the insignificant income coefficient and the small difference between incomes, can probably be explained as follows. In the original analysis, performed with mileage cost at 4d. a mile, the mean cost differences (bus cost less car cost) were –12.8od. for car users and –14.23d. for bus users. But at the optimising car mileage cost of 2d. a mile the mean cost differences are –1.04d. for car users and –3.75d. for bus users. It is argued above in the section on Choice of Variables that – social effects apart – income will only have a significant effect on modal choice if the costs of travel by the two modes are very different. It seems therefore that the insignificant income coefficient is plausible if car mileage is costed at 2d. a mile or thereabouts. This result thus adds more weight to the conclusions favouring 2d. a mile as an optimum. Putting the argument differently, it seems that, since we already have evidence that 2d. a mile is the appropriate figure, and with this figure there is little to choose between the modes on a cost basis, the fact that income appears to have an insignificant effect suggests that the “social” effect of income cannot be important either, particularly as the coefficient has a negative sign.

The cost difference coefficient is itself significant, however; it seems that, while there are differences in cost between one mode and the other, and these differences are different between car users and between bus users, they are not large enough for a person’s income level to have much effect on how he decides to satisfy his preferences, at the margin at least.

There is, however, one possible explanation of this finding which need not imply that income is unimportant. Since bus fares increase almost uniformly with distance, and car mileage cost is reckoned on a linear basis, one might expect the cost difference to increase as distance to the CBD increased (more or less, since there are parking charges to take account of as well). If incomes increased with distance out from the CBD (a plausible hypothesis, given the form of Leeds), income would be associated
with cost difference, and any dependence of the effect of cost difference on income in the discriminant analysis would be hidden by this pre-existing association. The correlation matrix for Set 3, however, shows the following:

Correlation between cost difference and income: for car users: 0.0169
for bus users: 0.0068

In both cases the correlation is insignificant. But the cost differences themselves are so small that slight variations in the reporting of one cost or the other might render the correlation insignificant when in reality some association does exist. Now, there is some evidence for an association between income and distance. Correlations were carried out between income and log (distance travelled to CBD) for car users and bus users:

For car-bus sample: correlation for car users: 0.192 sample 376
easily significant at 1% level
correlation for bus users: 0.132 sample 166
not significant at 5% level

One might therefore suppose that income has a longer-term effect on modal choice, through its effect on where people live.

However, once the choice of place of residence has been made, income appears to have an insignificant effect. Nevertheless, such cost differences as do exist in individual cases do influence choice of mode – but these differences do not seem large enough for different income levels to affect how far people can satisfy their preferences. Warner* also reaches the conclusion that, once the decision has been made to buy a car and to live in a certain district, income has little effect on modal choice. None of this indicates what would happen were the cost of travelling to change radically; in addition to the alterations in real income, people's residential decisions would no longer be in equilibrium with the costs of travel, and some of the assumptions behind our analysis would no longer be valid.

As regards the other factors, since excess travel time is itself a component of overall travel time, it is valued at \((0.0966 + 0.0556)/0.0556\), i.e. about 2 \(\frac{1}{2}\) to 3 times overall travel time. This means that walking and waiting time is valued at about 2 \(\frac{1}{2}\) to 3 times the time spent in a vehicle. Goldberger in Paris\(^{20}\) found that people treated transfer time as though it were double actual time, and waiting time as though it were treble. Our findings seem roughly consistent with this.

Since the overall travel time difference and cost difference coefficients represent a relative trade-off between these two factors, the imputed value of time, based on Set 3 at 2d. a mile, is 0.0556/0.0911 pence per minute, i.e. 3s. 1d. per hour. With a mean income of £1,400 per annum, and a working year of 2,000 hours, this value of time is 21.4 per cent of the wage rate. The value of time will be discussed more extensively later.*

To see how much of the discrimination is accounted for by fewer than the original

*See Appendix B (pages 312–3) for a discussion of the statistical significance of the results.
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seven factors, further runs were made of the discriminant program with one, two and then three of the four significant factors, with results as follows:

Car-bus data at 2d. a mile

Overall travel time difference alone 38.1%
Ditto and cost difference 42.5%
Value of time 4s. 7d. per hour, or 33.6% of the wage rate
Both, and excess travel time difference 46.9%
Value of time 2s. 10d. per hour, or 20.6% of the wage rate

We can assume that the "use of car for work" factor is almost entirely responsible for raising the discrimination to 53.2 per cent in Set 3.

Additional Car-train Results

Analysis of the separate car-train sample using Set 3 yielded somewhat similar levels of significance:

Car-train data. Sample size 97 (68 car users, 29 train users). R = 57.7%

\[
\begin{align*}
\text{Relative disutility of train travel (}=z) & \quad t = \frac{z}{s.z.(\chi)} \\
0.0607 \text{ (overall travel time difference)} & \quad 3.69 \\
+ 0.0793 \text{ (excess travel time difference)} & \quad 2.72 \\
+ 0.0739 \text{ (cost difference at 2d. a mile)} & \quad 3.15 \\
- 0.087 \text{ (income in £ p.a., divided by 1,000)} & \quad 0.25 \\
+ 0.093 \text{ (car demand ratio)} & \quad 0.32 \\
+ 0.248 \text{ (use of car for work, 3 or 0)} & \quad 1.36 \\
+ 0.843 \text{ (ownership of car by firm)} & \quad 1.39 \\
\text{Constant term: 1.285}
\end{align*}
\]

The first three factors are clearly significant, the fourth and fifth clearly not, while the last two are not significant at the 5 per cent level. This accords reasonably well with the car-bus findings, although the insignificant income coefficient is a little more difficult to explain here, since with car mileage costs at 2d. a mile the mean cost differences are rather larger (−8.59d. for car users, −12.76d. for train users). The mean income of train users is £236 less than that of the car users, which is consistent with larger cost differences; but the insignificant coefficient cannot have come about because of a small difference between mean incomes, as one might argue in the car-bus case. Note also that the "use of car for work" factor is insignificant. Some of all this may be explained by certain important differences between the car-bus and car-train samples. The mean distance travelled by the former was 5.84 miles, by the latter 11.32 miles. Most of the former live within the Leeds boundary; most of the latter live in the rural hinterland of the city, or in dormitory towns like Harrogate, Ilkley, Garforth, etc. If a person lives in Leeds, it is difficult for him not to be reasonably near a bus route. Outside the city, however, there are comparatively few bus routes (and these are slow), or railway routes (these are quite fast), and the choice of residential location may well depend on whether the commuter is likely to want to use the rail service. This in turn may depend on whether he is wealthy enough to travel by car a longish distance each day, or whether he needs his car for work, or on other factors; thus the fact that "use of car for work" is not explicitly significant here does not preclude the possibility of its having influenced residential location in the first place, affecting current overall and excess travel times. The same
might be true for income. It is not surprising, therefore, to find in the correlation matrices for the car-train sample that income and the use of car for work are both correlated at the 1 per cent level with overall travel time difference.

The value of time is 4s. 1d. per hour, which with an average income of £1,600 p.a. is 25.6 per cent of the wage rate. Walking and waiting times are estimated at \((0.0793 + 0.0607)/0.0607\), or about 2.31 in-vehicle times. This compares with \(2\frac{1}{2}\) to 3 times for the car-bus sample. Since the car-train sample typically has higher excess travel time than the car-bus sample, and since the higher excess travel times for train journeys are due entirely to longer walking times (waiting times for trains rarely exceeded 5 minutes), one might conclude that walking time is worth less than waiting time.

**Further Results**

It was suggested during discussion of the results that no allowance had been made for the possibility that people valued time differently by different modes, since the time variables had always been *differences* of overall and excess travel times. There may indeed have been a "constraint" here similar to that of a cost difference at 4d. a mile. A new set of runs was prepared, in which the overall travel time difference was represented as (bus overall travel time \(-a \times \text{car overall travel time}\)), and the cost difference as (bus cost \(-\text{car parking cost} - \text{mileage cost at } b \text{ pence per mile}\)), where \(a\) took values from 0.2 to 1.5 and \(b\) took values from 1.5d. to 4.0d. per mile. The table below shows the subset of values of \(a\) and \(b\) around which maximum \(R\) was obtained, using the following factors:

- bus overall travel time \(-a \times \text{car overall travel time}\)
- excess travel time difference
- bus cost \(-\text{car cost using } b \text{ pence per mile}\)
- use of car for work
- income
- car demand ratio

<table>
<thead>
<tr>
<th>(a)</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td></td>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>2.5</td>
<td>53.771</td>
<td>53.878</td>
<td>53.835</td>
</tr>
<tr>
<td>3.0</td>
<td>53.909</td>
<td>53.855</td>
<td>53.866</td>
</tr>
<tr>
<td>3.5</td>
<td>53.863</td>
<td>53.863</td>
<td>53.640</td>
</tr>
<tr>
<td>4.0</td>
<td>53.720</td>
<td>53.696</td>
<td>53.482</td>
</tr>
</tbody>
</table>

No other local maxima occurred in the range of \(a\)'s and \(b\)'s tried. A maximum occurs where \(a = 0.4, b = 3.0\), i.e. where car time is worth 0.4 of bus time and mileage cost is 3d. per mile. But the differences between adjacent correlation coefficients are extremely small, as before, and do not permit a radical change on this evidence alone in the form of the best discriminant function. The values of time obtained from the results for \(a = 0.4\) and \(b = 3.0\) were: value of bus time 48.10d. per hour (33.7 per cent of the wage rate), and value of car time 13.10d. per hour (13.5 per cent of the wage rate). The average of these is 39.3d. or 23.6 per cent of the wage rate, which corresponds well with previous estimates. The implication of the two different values is that people are prepared to spend rather more to save a given amount of time on a bus than to save the same amount of time in a car – which is plausible.
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An alternative approach is to enter bus time, car time, car mileage and (bus cost—
car parking cost) as separate variables along with the remaining variables found by
previous analysis to be significant. In this run, the components of excess travel time
were disaggregated as well. The results may be written:

\[ z = 0.0787 \text{ (bus time - 0.327 car time)} + 0.0924 \text{ (bus cost - car parking cost - car mileage at 3.21d per mile)} + 0.379 \text{ (use of car for work)} + 0.0526 \text{ (bus walking time + 2.53 bus waiting time - 2.23 car walking time)} \]

Thus the direct estimates of \( a \) and \( b \) are 0.327 and 3.21d per mile. Values of time are:
for bus, 4s. 3d. per hour (30.5\% of the wage rate); and for car, 1s. 4d. per hour (9.4\% of wage rate). The average is 2s. 1od. or 20.0\% of the wage rate.

These results would seem to suggest that bus times and car times really are differ-
ently valued; but splitting up times and costs in this way introduces a severe multi-
collinearity between certain of the variables. We must conclude, therefore, that,
while there is some evidence that bus time has a larger value than car time (which is
plausible), it is not possible to assign to each mode values that are different and
reliable. The subsequent discussion on prediction and planning must, therefore,
assume the same value for each mode, as implied in the original Set 3 formulation.

Values of Travelling Time

Summarising the estimates of value of time, so far, we have:

Car-bus data

(i) from Set 3 at 2d. a mile
(ii) from the set of factors:
    overall travel time difference
    cost difference at 2d. a mile
(iii) from the set of factors:
    overall travel time difference
    excess travel time difference
    cost difference at 2d. a mile
(iv) from the results in the last section, with car time
    worth 0.4 bus time, and 3d. a mile car cost:
    car time
    bus time
    average
(v) from the results with times and costs
    disaggregated into separate factors
    as above:
    car time
    bus time
    average
(vi) from Set 4 at 2d. a mile

<table>
<thead>
<tr>
<th>Per hour</th>
<th>% of wage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>3s. 1d.</td>
<td>21.4</td>
</tr>
<tr>
<td>4s. 7d.</td>
<td>33.0</td>
</tr>
<tr>
<td>2s. 1od.</td>
<td>20.6</td>
</tr>
<tr>
<td>1s. 1od.</td>
<td>13.5</td>
</tr>
<tr>
<td>4s. 10d.</td>
<td>33.7</td>
</tr>
<tr>
<td>3s. 3d.</td>
<td>23.6</td>
</tr>
<tr>
<td>1s. 4d.</td>
<td>9.4</td>
</tr>
<tr>
<td>4s. 3d.</td>
<td>30.5</td>
</tr>
<tr>
<td>2s. 1od.</td>
<td>20.0</td>
</tr>
<tr>
<td>n.a.</td>
<td>24.9</td>
</tr>
</tbody>
</table>

Car-train data

(i) from Set 3 at 24d. a mile
(ii) from Set 3 at 2d. a mile
(iii) from Set 4 at 24d. a mile

<table>
<thead>
<tr>
<th>Per hour</th>
<th>% of wage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>4s. 1d.</td>
<td>25.6</td>
</tr>
<tr>
<td>3s. 8d.</td>
<td>23.0</td>
</tr>
<tr>
<td>n.a.</td>
<td>34.3</td>
</tr>
</tbody>
</table>

It is also worth reporting that, at an early stage of analysis, the car-bus sample
was split into four income ranges (number in each range in brackets):
Discriminant analysis was run on each sub-sample, using factors "overall travel time difference" and "cost difference" alone, with mileage costs varying from 1d. to 4d. a mile in each case, and additionally with the other significant factors. The following optimum costs and resulting values of time were obtained with two factors:

**Car-bus data**

<table>
<thead>
<tr>
<th>optimum car mileage cost</th>
<th>R</th>
<th>t-value λ₁</th>
<th>t-value λ₂</th>
<th>as per cent of mean income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range 1: less than £900</td>
<td>(93)</td>
<td>2.46</td>
<td>2.53</td>
<td>2s. 4d.</td>
</tr>
<tr>
<td>Range 2: £900 - £1200</td>
<td>(149)</td>
<td>2.15</td>
<td>2.39</td>
<td>1s. 1d.</td>
</tr>
<tr>
<td>Range 3: £1200 - £1800</td>
<td>(186)</td>
<td>2.22</td>
<td>2.44</td>
<td>1s. 3d.</td>
</tr>
<tr>
<td>Range 4: over £1800</td>
<td>(114)</td>
<td>2.38</td>
<td>2.61</td>
<td>1s. 10d.</td>
</tr>
</tbody>
</table>

Maximum discrimination occurs at car mileage costs which increase as incomes increase - which is plausible, since one might expect people with higher incomes to run more thirsty cars. Except in range 3, where the t-value for the second coefficient is lower than in the other ranges and thus casts doubt on the estimated value of time, the values of time are nearly a constant proportion (one third) of income. Part of the purpose of running the analysis on these two variables alone was to provide a comparison with Beesley's results on Ministry of Transport employees; there is close correspondence with his finding that the value of time is about one-third of the wage rate, as also with our earlier results when excess travel time and the use of car for work were omitted. Now the 7-variable analysis of Set 3 showed that these two factors are also significant in influencing modal choice. Thus, where analysis does not explicitly include these factors, they will still have a hidden effect through the factors that are used. Consequently, for deriving a value of travelling time we should have more confidence in results based on an analysis which includes all the significant factors explicitly.

The results from running the analysis on each sub-sample with excess travel time and use of car for work (the other significant factors) gave values of time as follows:

<table>
<thead>
<tr>
<th>optimum car mileage cost</th>
<th>R</th>
<th>t-value λ₁</th>
<th>t-value λ₂</th>
<th>as per cent of mean income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range 1: 1.8d.</td>
<td>49.6%</td>
<td>2.44</td>
<td>2.85</td>
<td>2s. 9d.</td>
</tr>
<tr>
<td>Range 2: 2.1d.</td>
<td>51.1%</td>
<td>2.97</td>
<td>3.17</td>
<td>2s. 7d.</td>
</tr>
<tr>
<td>Range 3: 2.1d.</td>
<td>54.3%</td>
<td>2.71</td>
<td>1.31</td>
<td>5s. 2d.</td>
</tr>
<tr>
<td>Range 4: 2.5d.</td>
<td>61.7%</td>
<td>3.07</td>
<td>2.35</td>
<td>4s. 10d.</td>
</tr>
</tbody>
</table>

The effect of taking explicit account of the other significant factors can be clearly seen. The resulting trade-offs between travelling time and cost lie in the same range (20–25% of wage rate) as that using the whole sample, and this percentage is roughly
constant over a broad range of incomes. (Note that, as with the two-variable analysis, the value of time for Range 3 is unreliable).

Our general conclusions about the value of time are: firstly, it lies in the range 20–25% of the commuter's income; secondly, this proportion is roughly constant over a wide range of incomes; thirdly, there is insufficient evidence to conclude that bus times and car times have significantly different values.

Conclusions

Summarising this section, it has been shown that, as a measure of relativeness of time and cost, ratios are significantly inferior to logs of ratios and differences; and that differences are consistently better than logs of ratios, in both the car-bus and the car-train samples, but by an amount significant only at the 10 per cent level. Using differences, one may conclude that overall travel time difference, excess travel time difference, cost difference, and the possibility of use of car at work are all important in influencing modal choice; income is an insignificant factor, most probably because of the very small cost differences between modes (at 2d. a mile car cost), as also manifested by a small difference between mean incomes for both sets of mode users. It was also found that walking and waiting times are worth between two and three times in-vehicle times; that an average value of time on both modes lies between about 3s. and 3s. 6d. per hour, or between 21 per cent and 25 per cent of wage rates (or, leaving out two significant factors, about one-third of wage rates, corresponding closely with Beesley's results). The early analysis suggested that 2d. a mile was the car mileage cost which best fitted people's observed behaviour, but there was a large margin of error in this estimate. Subsequent analysis suggested that, if car time could be valued differently from bus time, the former was worth between 40 per cent and 50 per cent of the latter, and the car mileage cost was between 3d. and 3.5d. per mile. The car-train results were very similar to those from the car-bus sample; values of time were slightly higher, partly as a result of a questionable estimate of car mileage cost of 24d. a mile. The "use of car for work" factor was insignificant in the car-train sample, though it may have had a prior influence on residential location and thus influenced modal choice indirectly through travel times and costs.

It was found that conditions of homoscedasticity were not strictly met (see Appendix B); but there were no large intercorrelations between factors such as might upset the validity of predictions based on the discriminant function.

PREDICTION AND PLANNING

The following discussion of prediction and planning employs the results from the Set 3 combination of factors, at a car mileage cost of 2d. a mile.

Prediction of modal choice is best regarded as assigning probability of choice of one mode, for a given value of the discriminant function, rather than as a deterministic classification into one mode or the other. The method is described in Appendix A. It gives us a way of estimating the proportion of car owners who will commute to the CBD of Leeds by car after different sets of public transport improvements or
parking restrictions. It also provides a basis for estimating the car-using proportion among car owners living in the catchment area of a proposed public transport system. Changes in the behaviour of car-owning commuters to the CBD can be estimated by analysing the effect on the sample, on the assumption that the sample is representative of all car owners working in the CBD. It need hardly be emphasised that the quantitative estimates themselves are tentative; the results of further work in the area, already initiated, will help to confirm (or otherwise) the general method and results. There will always remain, however, the usual doubts about predictions using models with a multivariate statistical base, and only actual experiments on the ground can go some way towards dispelling them.

The effect of a policy change on an individual’s probability of choosing a car depends on his existing \( z \) value, and may also depend on his value of the relevant \( x \)-factor. Calculating the effect on the sample therefore requires the calculation of a new \( z \) value and a new probability for each person in the sample. The new proportion of the sample likely to travel by car is then the mean of the probabilities of choice of car for everyone in the sample*

\[
\hat{p}(z) = \frac{n_2/n_1 \cdot e^{z+\hat{t}}}{1 + n_2/n_1 \cdot e^{z+\hat{t}}}
\]

expresses the possibility of choice of car for a given \( Z \) (and \( t \) is the constant term).

The changes in percentage diversion in Leeds resulting from the policy changes detailed below are somewhat notional, however, since secondary effects would also occur and would usually reduce the effect of the change. For instance, if parking charges were increased and the car-using proportion fell to – say – 50 per cent, there would be less car traffic on the roads at peak periods, so that car travel would, on average, be slightly faster. The speed of bus travel would also increase, but probably not so much. Consequently there would be a “feedback” effect which would reduce the effect of the initial change, some of those who switched to bus coming back again to car with the improvement in conditions of car travel. The competition for parking places would be lower, too, so that some car walking times would probably be less. The final equilibrium affected by a change is difficult to predict without some knowledge of the “supply” side – the relationship between conditions of travel and the volumes of traffic approaching and entering the city at the relevant times – such as that proposed by Sharp.\(^3\) A framework for the examination of the interaction of supply and demand in this context was recently proposed by the author.\(^3\) \(^4\)

*If one has confidence in the normality and homoscedasticity of the sample, an alternative method for treating changes in variables which are the same for everybody (e.g. absolute changes in cost) consists simply of calculating a new mean probability \( \hat{p}' \), where

\[
\hat{p}' = \frac{ke^{\hat{z}}}{1 + k' e^{\hat{z}}}
\]

and \( \hat{z} \) is the change in everyone’s \( z \) value resulting from the variable change. In our sample these two methods were found to give insignificantly different results in all cases where the second method could be applied.

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<table>
<thead>
<tr>
<th>Policy change from present situation</th>
<th>% of cars owners using their cars</th>
<th>Change from present %</th>
<th>% diversion to public transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>No change</td>
<td>69</td>
<td>—</td>
<td>31</td>
</tr>
<tr>
<td>All bus fares up by 6d.</td>
<td>77</td>
<td>+ 8</td>
<td>23</td>
</tr>
<tr>
<td>Buses free</td>
<td>52</td>
<td>— 17</td>
<td>48</td>
</tr>
<tr>
<td>All parking to cost 1s. per day more</td>
<td>57</td>
<td>— 12</td>
<td>43</td>
</tr>
<tr>
<td>All parking to cost 3s. per day more</td>
<td>41</td>
<td>— 28</td>
<td>59</td>
</tr>
<tr>
<td>Bus frequencies doubled</td>
<td>66</td>
<td>— 3</td>
<td>34</td>
</tr>
</tbody>
</table>

All walking times from car parking place to workplaces increased by 5 mins.  
Cars 10 minutes faster  
Buses 10 minutes faster  
61                                           61                                           61                                           61
— 8                           — 8                           — 8                           — 8
39                                           39                                           39                                           39
22                                           22                                           22                                           22
40                                           40                                           40                                           40

It is interesting to examine the possible combinations of changes in policy which, according to the model, could reduce the car-using proportion of car-owning commuters in Leeds to 20 per cent, the proportion required by the city's policy for the year 2010. For this purpose we are ignoring the secondary effects referred to above. This is not unreasonable, since the total number of car-using commuters will not be markedly different from what it is today. There will be an improved system of urban motorways, but with the growth of other miscellaneous traffic it is likely that general conditions of car travel would not be very different from today, provided that the diversion of 80 per cent to public transport is achieved.

In theory, one would expect this diversion to be achieved by any combination of changes in the following factors which together will reduce the general level of z-values (relative advantage of private over public transport) by the required amount:

- Reductions in: time spent in the public transport vehicle;
  - walking and waiting time by public transport;
  - cost by public transport.

- Increases in: time spent in the car;
  - walking time involved in the car journey;
  - cost of travel and parking by car.

It is estimated that, at 1966 incomes and prices, an 80 per cent diversion could be achieved by combining changes in one of the following ways:

(a) increasing all parking charges by 3s., and ensuring that all those bringing cars have to walk, on average, 6 minutes longer from their parking places than now, and making public transport faster by fifteen minutes; or

(b) increasing all parking charges by 2s., introducing road pricing in the peak hours so as to charge 1s. per day on average, ensuring that car users have to walk, on average, 3 minutes longer from parking places than now, reducing public transport fares by 9d., and reducing waiting times (for public transport travel) by 3 minutes.

These are quoted solely as examples: other possible combinations of changes can be explored by the same method.

The analysis does not indicate in what general direction Leeds should seek to bring about the required changes in z-values. There are likely to be substantial side effects which need close examination. A more fundamental point is that one class of measures, those which provide disincentives to the use of cars, is virtually costless in public
finance terms, and may indeed yield a net revenue. It could be argued, however, that there might be high social costs. However, those measures which involve increasing the attractiveness of public transport require substantial financial investment in new or improved public transport systems. The improvements may not, in strict accounting terms, be economically viable, but they would yield high social returns. To decide how much of the reduction in the relative advantage of private over public transport (the \( z \)-value) should be achieved by disincentives to car use and how much by public transport improvements requires an assessment of the "costs" of possible undesirable side effects of disincentives (e.g. pressures to decentralise city centre activities) and of the benefits, measured in the widest terms, of an improved public transport system (such as the reductions in travelling time for users of the system, car owners and non-car owners).

**Evaluating a Public Transport System**

We now suggest how the model might be used to give a tentative estimate of the diversion of car owners which a proposed public transport system might achieve. Suppose we are considering a rapid transit line following a radial route from a residential suburb into a central business district. Ignoring future residential development which might be attracted to such a line if built, we proceed to define a corridor as the catchment area of the line. Within the corridor and covering its whole area, a representative sample of car owners is obtained. On the basis of information given by them, and information about the performance characteristics of the proposed rapid transit system (travel times, frequencies, etc.), \( z \)-values can be calculated for each person in the sample. Since information is needed from the sample only about the car journey, and not about another alternative mode, it is likely that it could be obtained from existing transportation study data banks (or from the questionnaires, if the data on store is not sufficiently detailed) provided the sample taken in the corridor was in fact large enough. From each \( z \)-value, individual probabilities of choice of car can be calculated; the mean value of these probabilities gives the best estimate of the proportion of car owners who would use their cars, and the proportion who would be diverted to the new line.

This method\(^*\) has the advantage over the use of -- say -- diversion curves on zonal data that individual characteristics such as walking times or costs are not used in aggregated form for inter-zonal predictions; they are used in disaggregated form up to the point where individual probabilities of choice are calculated, and only these probabilities are aggregated.

Any such improved public transport system will, of course, attract a large number of existing users of other public transport. Even the financial viability will ultimately depend on these; and judgments of the benefits accruing will take account of the reductions in travelling time experienced by all users of the system. We cannot attempt to predict people's choices between public transport modes, but we believe that a model of the same general nature as that used here to explain choices between private and public modes of transport will do this.

\(^*\)The method could also show, for instance, how many car owners had returned to using their cars after the building of a motorway into a city centre, such as the M4 from Slough and Maidenhead into the West End of London.
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We should emphasise that the method of prediction described above has not been attempted anywhere, and we have little idea of the operational problems that might be encountered in trying to apply it; for instance, we have not calculated what the minimum sample size would need to be. There may also be important conceptual problems: for instance, a radically new transport system might well have substantial redistributive effects on work trips originating in the corridor, and might invalidate the predictions based on modal choices alone. The secondary effects, arising from changes in congestion and travel times on the parallel road, might also upset the predictions. But we hope the method might at least indicate orders of magnitude of diversion of car owners.

APPENDIX A. THE MODEL

It is assumed that the decisions to travel, to travel for a specific purpose, and where to travel, have all been made. Two modes are available for travelling, and a model is required to describe the choice made between these two modes. The model still holds if more than two modes are available, because in general a choice will always resolve into a choice between two alternatives. Let us start, however, with 2 modes.

We assume a number of "dimensions" of travel (1 to k) each of which gives rise to some disutility (such as travel time, walking time, cost, etc.). Suppose that the measure of dimension p (1 to k) for mode i (1 to h) for person j (1 to n) is dpij. Further, let the importance of this dimension, or its "contribution to disutility", be λp,ij. Then the disutility of the pth dimension of mode i for person j is λp,ij dp,ij, and the disutility of travel by that mode, for that person, is Dij = \sum_p λp,ij dp,ij. This is the simplest linear form. One might argue that the contribution of a dimension to disutility is not linear – for instance, we may not mind waiting 3 minutes for a bus, but we may mind more than three times as much if we have to wait 9 minutes. One might argue that the effect of one dimension depends on the value of another dimension – i.e. that λp,ij = f(dp,ij) – for instance, the overall travel time may become more important if we have had to wait with aching feet for a train. One might argue that adding up different contributions to disutility does not represent how people feel about a given mode of travel. All these are valid points; but let us try this simple model first. There are good precedents for expressing total disutility as above.

We then postulate that a traveller will choose that mode i for which Dij < Dij, Dij, . . . . . . Dij (not including Dij), i.e. the mode with minimum disutility. If some change in circumstances changed the values of one or more dimensions of his disutility, and this caused an alternative mode to assume a lower value of D than his preferred mode, we should expect him to change his mode.

However, since one can reduce the choice to two modes it may be preferable, instead of comparing absolute disutilities, to define a relative disutility of one mode compared to the other. It may be that people tend to perceive characteristics of the modes relatively to one another, rather than separately and absolutely. For instance, absolute journey times of – say – 40 minutes by train and 35 minutes by car may not in themselves be important, if the traveller is willing to sacrifice a time of that
order of magnitude to accomplish the journey. But the relative measure may be important – the fact that the train takes 5 minutes longer, or 5 minutes longer in 35 minutes. If the latter possibility is allowed, one cannot define \( R_j \), the relative disutility for the \( j^{th} \) person, as \( R_j = D_{ij} - D_{2j} \).

A more general form is needed: \( R_j = \sum \lambda_{bij} f_p (d_{p1j}, d_{p2j}) \) where \( f_p \) is a more general way of expressing the relativeness between dimensions expressed for each mode. In the original form, the choice of minimum \( D \) implied that relativeness was measured by the difference between \( d's \); in the more general form, this may be the ratio of \( d's \), or the logarithm of the ratio, or something else. In the original form, if \( R_j \) were negative, \( D_{ij} < D_{2j} \), and mode 1 would be chosen; vice versa if \( R_j \) were positive.

In the more general form, if \( R_j \) were low mode 1 would be chosen, and if \( R_j \) were high mode 2 would be chosen. If \( x_{p1j} = f_p (d_{p1j}, d_{p2j}) \), then \( R_j = \sum \lambda_{bij} x_{p1j} \). Empirical work will examine which way of expressing "relativeness", i.e. which form of \( f_p \), comes nearest to explaining behaviour. A more fundamental task will be to develop a method for finding the weighting factors \( \lambda \), and for predicting what people will do when we know their \( R_j 's. \) Those dimensions which turn out to be unimportant – i.e. to make an insignificant contribution to relative disutility – will yield statistically insignificant values of \( \lambda \).

In order to find values of \( \lambda \), one must reduce the \( kn \) values of \( \lambda_{bij} \). One approximation is to assume the weighting factor for each dimension to be the same over the whole population; alternatively, different sub-categories of the population may be given different values of \( \lambda \) but a common \( \lambda \) within each sub-category for each dimension. In expounding the method of analysis, the first approximation is used; thus \( R_j = \sum \lambda_{bij} x_{p1j} \) is the general relative disutility function.

If people in choosing between two modes behaved exactly according to the model, and if all relevant dimensions were characterised, there should be a set of weighting coefficients, \( \lambda 's \), such that all users of mode 1 have relative disutilities (of mode 1 with respect to mode 2) below some "threshold" and all users of mode 2 have relative disutilities above this threshold. If there were any sets at all, however, there would in general be more than one set of weighting coefficients that separated the \( R_j 's \) for mode 1 users and mode 2 users. That set which separated the two populations best could be said to be the best explanatory set. Predicting modal split would consist of calculating the relative disutility for a particular person, given information about the relevant dimensions, and examining whether this relative disutility fell above or below the modal split threshold.

This method is useful as a starting point, but no more. Its assumptions and requirements are wide-ranging. It assumes, firstly, that people behave rationally, i.e. according to the model; secondly, that the same weighting coefficients apply to everyone; thirdly, that all relevant dimensions are taken into account; fourthly, that there is access to the entire population of users of both modes. Further, nothing has yet been said about the meaning of "separation" or about how to find the optimal set of weighting coefficients. In fact, it is extremely unlikely that in any real case a set of weighting coefficients could be found which would separate the two populations completely; it is very likely that there would always be some misclassification.
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of people – that is, users of mode 1 with a value of $R_j$ above the threshold, and users of mode 2 with a value of $R_j$ below the threshold. The criterion of separation could then be to minimise the number of misclassifications. This is not the only possible criterion, but it is one which is suggested by Beesley’s work on Ministry of Transport employees. Another might be to minimise the number of misclassifications, each one weighted by its distance – or the square of its distance – from the threshold.

The Two-Variable Case

Beesley’s work was not aimed at explaining modal choice but at finding the rate of trade-off between money and time which would best explain modal choice in terms of these two variables. Although he did not express it as a disutility function in the way described here, his task could be expressed as finding those weighting coefficients which give rise to least misclassification: the ratio of these weighting factors then gives the required trade-off, i.e. the value of time. He did not compare users of one mode with users of the other, but compared, within one mode, those people who chose a time saving at extra cost with those who chose a cost saving but incurred extra time. Those in the latter group were then transposed into the same quadrant and the task was to find the line through the origin which best separated the two groups of points. This transposition made the problem of choosing the line exactly the same as if the characteristics had been observed within two groups, one using each mode, the latter group enjoying values of time difference and cost difference as given by the transposed points in Beesley’s example. The best threshold line can be found by simple algebra, or almost by trial and error on the graph itself, and the gradient of the line gives the ratio between the weighting coefficients. Suppose the best threshold line is of the form

$$y = -mx$$

where $x$ is the time saving (a negative number), $y$ is the extra cost, and $m$ is a positive number. Points representing people’s values of time and cost difference can be taken as $(x_i, y_i)$, and their distances from the line are given by

$$d_i = \frac{m x_i + y_i}{1 + m^2} = \frac{m}{1 + m^2} x_i + \frac{1}{1 + m^2} y_i$$

This is identical to the expression of relative disutility, where $\frac{m}{1 + m^2}$ and $\frac{1}{1 + m^2}$ are the weighting coefficients, $d = 0$ gives the threshold, and $d$ itself is a measure of the excess disutility of one mode over the other.

In Beesley’s exercise, it is necessary to assume that this separation line passes through the origin in order to transpose one set of points into the same quadrant as the other. But when we are comparing users of two modes and plotting them in the same quadrant, it would not seem necessary to assume that the threshold line passes through the origin – i.e. that $d = 0$ at the threshold. This would be equivalent to assuming that the marginal value of time is equal to the average value of time – which is not necessarily true. It would seem better to develop a method without this assumption and to see what the results yield. A marginal value of time so obtained might be more reliable, since it is not constrained to be equal to the average value of time; it is, however, required to be constant in this linear formulation.

We need to examine the criterion for choosing the "best" line. In Beesley’s exercise

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the criterion chosen was the simplest – that of minimising the number of misclassified points. If the two populations differ in size, the number of points in one group is weighted to compensate for this, and misclassification is minimised on that basis. One criticism of this method is its extreme sensitivity to the position of points near the threshold; Beesley took pains to point out that the results from his analysis could be taken as little more than a guide.

But a much more fundamental criticism can be made of this criterion. The points used for the analysis are obtained from a sample drawn from a population of commuters. Any results obtained will apply to the population only within certain limits of confidence. Now the confidence of the estimates of value of time will depend on how sensitive these estimates are to possible variations between one sample and another. We have already said that these estimates are very sensitive to sample points near the threshold. An alternative method which would be much less sensitive to sample-to-sample variations involves using the sample in a less direct way, by characterising the population from which it was drawn and then using the resulting population attributes to achieve minimum misclassification. In particular, this could mean that the sample would provide best estimates of the means, variances and covariances of the factors (in this case time and cost differences) for the population, and a more sophisticated model, using a normality assumption, would then be used to derive values of the coefficients for minimum misclassification – by implication for the whole population.

The only conditions under which it could be argued that this latter method would provide poor estimates would be that the distributions of the factors in each population (i.e. for each mode) were grossly different from normal. If this were so, however, then doubt would be cast equally on the validity – as estimates for the whole population – of results obtained from the simple, pseudo-graphical misclassification procedure. There is already in this simple procedure an implicit assumption about the distribution of the factors in the population from which the sample is drawn. One final point is that the procedure has been used only for two variables: it would seem that an increase in the number of variables could make it extremely unwieldy. Clearly, a more streamlined method is needed.

To use the sample to provide best estimates of means, variances and covariances for the populations using each mode, and then to seek minimum misclassification with respect to a $k$-dimensional region (where there are $k$ explanatory factors) is exactly what linear discriminant analysis does.

**The Discriminant Analysis Approach**

We have thus derived, from first principles, a discriminant analysis solution: from basic notions of disutility and choice advancing to the simple misclassification criterion used by Beesley, and subsequently to a form using total population characteristics, in a way that is both behaviourally and intuitively valid. The relative disutility function becomes what is known as the discriminator or discriminant function.

It can be shown that the method of solution can be derived from two different standpoints: that of minimising misclassification with respect to some presumed threshold or region boundary, and that of not using the threshold concept at all, but of seeking instead to find conditions in which the separation between the two populations, as measured by the square of the difference between their means, is greatest
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in relation to the variance within each population. It can be seen intuitively that this will in fact achieve minimum overlapping of relative disutility values. In the second approach, the ratio of "variance between" the populations to variance within each population is maximised by using the weighting coefficients as variables, and consequently optimal values of the weighting coefficients can be found directly.

The first approach is due to Neyman and Pearson,16 and the second, which will be expounded here, is due to Fisher.17 In both, necessary conditions are that the distributions of the factors are multivariate normal, and that the two populations are homoscedastic with respect to the factors — i.e. that the covariance matrices of factors are the same for each population. This requirement will be discussed later. For the purpose of developing the model these conditions will be assumed to hold.

Discriminant analysis has been used in many fields – particularly biology and botany, where there are special problems of classification. It was in this context that Fisher developed his approach. The discriminant function, or relative disutility function, takes the form

\[ z_{ij} = \lambda_1 x_{1ij} + \lambda_2 x_{2ij} + \ldots + \lambda_k x_{kij} \]

or

\[ z_{ij} = \sum_{p=1}^{k} \lambda_p x_{ pij} \]

where \( z_{ij} \), \( x_{ pij} \), and \( \lambda_p \) are the relative disutility, factor value and weighting coefficient respectively for the \( p \)th factor (\( p = 1 \) to \( k \)) of the \( j \)th person (\( j = 1 \) to \( n_i \)) in the \( i \)th mode in the total population (\( i = 1, 2 \)). From this, the square of the distance between the means of the populations is

\[ (\bar{z}_1 - \bar{z}_2)^2 = \left[ \sum_{p=1}^{k} \lambda_p (\bar{x}_{p1} - \bar{x}_{p2}) \right]^2 \]

and the variance within the population is

\[ \sum_{p=1}^{k} \sum_{q=1}^{k} \lambda_p \lambda_q C_{pq} \]

assuming homoscedasticity, where \( C_{pq} \) is the common covariance matrix.* The task is to maximise

\[ G = \frac{\left[ \sum_{p=1}^{k} \lambda_p (\bar{x}_{p1} - \bar{x}_{p2}) \right]^2}{\sum_{p=1}^{k} \sum_{q=1}^{k} \lambda_p \lambda_q C_{pq}} \]

with respect to \( \lambda_p \), \( p = 1 \) to \( k \). Differentiating with respect to \( \lambda_p \) and assigning \( \frac{\partial G}{\partial \lambda_p} = 0 \), we have

\[ \sum_{q=1}^{k} \lambda_q C_{pq} = \frac{\sum_{p=1}^{k} \sum_{q=1}^{k} \lambda_p \lambda_q C_{pq}}{\sum_{p=1}^{k} \lambda_p (\bar{x}_{p1} - \bar{x}_{p2})} (\bar{x}_{p1} - \bar{x}_{p2}) \]

Multiplying through by \( C_{pq} \), the inverted matrix of \( C_{pq} \), and summing over \( p = 1 \) to \( k \) gives

\[ \lambda_q \propto \sum_{p=1}^{k} C_{pq} (\bar{x}_{p1} - \bar{x}_{p2}) \]

*It can be shown that for discriminant coefficients to be independent of sample size, this must be the common covariance matrix, not the sums of squares and products matrix.
Since the disutility function has no absolute value, we can choose any convenient constant. Here we shall say that

\[ \lambda_q = \sum_{p} C_{pq} (\bar{x}_{p1} - \bar{x}_{p2}) \]  

(1)

The discriminant, or disutility, function is now of the form

\[ z_{ij} = \sum_{q} \sum_{p} C_{pq} (\bar{x}_{p1} - \bar{x}_{p2}) \sigma_{qij} \]  

(2)

In our problem we do not know the population means and covariances, so values of means and covariances calculated from the sample are used as maximum likelihood estimates. We then have

\[ \lambda_q = \sum_{p} \hat{C}_{pq} (\hat{x}_{p1} - \hat{x}_{p2}) \]

and similarly for \( z \).

An important aspect of the discriminant analysis described here is the validity and significance of its results. Kendall suggests three ways in which one can ask whether a discriminator is "significant".

First, there may be a real difference between the populations but they may be so close together that a discriminator is not very effective. This difference is measured by the errors of misclassification, which, though minimal, may still be large. Or there may be a real difference between the populations but the sample may not be large enough to produce a very reliable discriminator; this is really a matter of setting confidence limits to the function or to its coefficients. Or it may be that the parent populations are identical and that a discriminant function is illusory.

When we are dealing with choice of mode, there is no doubt that there are two different populations of people – one travelling by one mode, the other by the other mode – so the third case (of an insignificant discriminator) could never apply. If the first case applies, it seems that one interpretation of the populations being "so close together" is that the factors used to characterise the relative disutility of one mode to the other do not include the important ones; another is that the assumptions of consistency and rationality implicit in the model are so tenuous that no set of factors can achieve an effective discriminator. If either interpretation is right, significance can be measured in terms of the probability of the observed discrimination occurring at random; for this the variance ratio (see below) is an appropriate indicator. Since the variance ratio involves measures of both the "distance" between the populations and the numbers in the sample, it can also indicate whether the sample is large enough to provide a reliable discriminator.

If the abilities of different factor sets to explain modal choice are being investigated, and each factor set provides a "significant" discrimination in terms of the variance ratio, then the distance between the populations is a direct measure of how good each factor set is, in terms of discrimination, for a given sample size. A more generalised measure is the multiple correlation coefficient which can compare different sample sizes and different relative proportions in the two populations using different modes. The multiple correlation coefficient is itself something of an absolute measure of discrimination, since it indicates how much of the variation in the sample is explained wholly by the discriminant analysis.

The extent to which individual factors are significant in the discriminant function
can be ascertained from a comparison of their coefficients with the standard errors of these coefficients. Standard errors have no direct interpretation in terms of the discriminant function itself, but it is possible to derive these standard errors with reference to the equivalent multiple regression problem.

Kendall\textsuperscript{19} indeed describes how the whole task of assigning values to the coefficients in the discriminant function may be tackled by a multiple regression approach. In this, a dummy dependent variate $y$ is introduced, taking values $\frac{n_2}{n_1 + n_2}$ and $-\frac{n_1}{n_1 + n_2}$ for users of mode 1 and of mode 2 respectively, where $n_1$ and $n_2$ are the numbers of users of mode 1 and mode 2. With the two matrices of observations $x_{pji}, x_{p2j}$, put together into one matrix $x_{pj}^T (j = 1$ to $n_1 + n_2)$, this new matrix is regressed on the dependent variate vector $y_j$ in the normal way. Kendall shows (op. cit. p. 345) how the resulting vector of regression coefficients, $\mu_p$, is proportional to the discriminant coefficients, $\lambda_p$, thus:

$$\mu_p = \lambda_p \frac{n_1 n_2}{n_1 + n_2} \left( 1 - \frac{\sum q \mu_q (\bar{x}_{q1} - \bar{x}_{q2})}{q} \right)$$

However, to make the discriminant coefficients independent of sample size, the derivation above inverts the covariance matrix, not the sums of squares and products matrix. Thus the equivalence between regression coefficients and discriminant coefficients as derived above is:

$$\mu_p = \lambda_p \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 2)} \left( 1 - \frac{1}{q} \sum \mu_q (\bar{x}_{q1} - \bar{x}_{q2}) \right)$$

or

$$\mu_p = \lambda_p \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 2)} \cdot \frac{1}{1 + \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 2)} \cdot \sum q \lambda_q (\bar{x}_{q1} - \bar{x}_{q2})}$$

i.e.

$$\mu_p = \lambda_p \cdot \frac{K}{1 + Ka}$$

where $K = \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 2)}$, and $a = 2d. = \bar{z}_1 - \bar{z}_2 = \sum p \lambda_p (\bar{x}_{p1} - \bar{x}_{p2})$.

For researchers who have ready access to multiple regression computer programs the regression approach will be much more convenient than discriminant analysis, particularly where comprehensive analysis of variance using a stepwise process is available. But if we are to use the regression function as shown below in the expression for the estimation of probabilities (this estimation derives directly from the form of the discriminant function), it will be necessary to convert the regression coefficients into “discriminant coefficients” by a factor derived from the above expression of proportionality, i.e.

$$\lambda_p = \frac{(n_1 + n_2)(n_1 + n_2 - 2)}{n_1 n_2} \cdot \mu_p \left( 1 - \frac{\sum q \mu_q (\bar{x}_{q1} - \bar{x}_{q2})}{q} \right)$$
Reverting to the analysis of variance for the regression function, it can be shown that the total sum of squares

\[ SST = \frac{n_1n_2}{n_1 + n_2} \]

and the sum of squares due to the regression function

\[ SSD = \frac{n_1n_2}{n_1 + n_2} \sum_p \mu_p (\bar{x}_{p1} - \bar{x}_{p2}) = \frac{n_1n_2}{n_1 + n_2} \cdot \frac{Ka}{1 + Ka} \]

The correlation coefficient is clearly given by \( R^2 = \frac{Ka}{1 + Ka} \)

The error (or residual) sum of squares is

\[ SSE = \frac{n_1n_2}{n_1 + n_2} \left( 1 - \frac{Ka}{1 + Ka} \right) \]

Residual variance is \( SSE \) divided by degrees of freedom, i.e.,

\[ S^2 = \frac{n_1n_2}{n_1 + n_2} \left( 1 - \frac{Ka}{1 + Ka} \right) / (n_1 + n_2 - k - 1) \]

Variance due to the discriminant function is \( SSD/k \), i.e.

\[ S_1^2 = \frac{n_1n_2}{n_1 + n_2} \cdot \frac{Ka}{1 + Ka} / k \]

and the variance ratio is \( F = \frac{s_1^2}{S^2} \)

Thus the standard errors of the regression coefficients are

\[ s.e. (\mu_p) = \sqrt{S^2 C^{pp}} \]

Standard errors of the discriminant coefficients are given by

\[ s.e. (\lambda_p) = (1 + Ka) / K \cdot \sqrt{S^2 C^{pp}} \]

It is worth emphasising that \( C^{pp} \) is an element of the inverted matrix (of sums of squares and products) which would be used in the regression analysis, not the inverted covariance matrix used in the discriminant analysis. The difference is that the regression matrix is calculated over the whole matrix of observations \( x_{pj} \), while the discriminant covariance matrix is the result of “pooling” the separate covariance matrices for each population. If a discriminant program is being used, a simple conversion can be used to obviate the need to calculate a regression type matrix from scratch:

\[ C_{pq} = (n_1 + n_2 - 2) \cdot C_{pq} + \frac{n_1n_2}{(n_1 + n_2)} d p d q \]

where \( d_p = \bar{x}_{p1} - \bar{x}_{p2} \), etc.

Most of the above indicators depend on the normality and homoscedasticity assumptions, and it is appropriate to develop ways of testing the sample for this. Homoscedasticity can be said to obtain if corresponding elements of the covariance matrices of the two populations lie within a factor or two of each other, with no
more than one in twenty departing from this. An indication of normality can be obtained from the shape of the frequency distributions of each factor in the two populations and the shape of the final z-value distributions. A further measure of normality at the area of overlap is whether or not the best-fitting exponential function for predicting modal choice probabilities is significantly different from the theoretical form based on the normality assumption. Finally, the actual percentage misclassification can be compared with the theoretical misclassification.

It will be noticed in the analysis of data that comparisons are made between the results obtained by different sets of factors, and the criterion often used to choose between them is the correlation coefficient, $R$. In this context we clearly require to know whether differences between values of $R$ are significant or not. This can be tested by using Fisher's transformation

$$z = \frac{1}{2} (\log_e(1 + R) - \log_e(1 - R))$$

and the property that $z$ is normally distributed with standard deviation $1/\sqrt{n - 3}$. But it is not clear whether this applies in our particular case, and an alternative approach has been adopted. This involved deriving an empirical estimate of the variance of $R$, using the "jack-knife" technique. For this, discriminant analysis was run ten times for one particular set of factors on only 90 per cent of the data, leaving out a different 10 per cent each time. If $R_i$ is the correlation coefficient for each run, then the variance of $R$ is given by

$$\hat{V}(R) = \frac{n}{n - 1} \sum\limits_{i=1}^{n} (R_i - \bar{R})^2 \quad \text{where} \quad \bar{R} = \frac{1}{n} \sum\limits_{i=1}^{n} R_i$$

where $n$ is the number of runs on the above basis, which is 10 here. The results of applying this are discussed in a footnote to the text.

**Modal Prediction**

The purpose of discriminant analysis in botany and biology is usually to classify; the purpose here would be to classify according to mode, that is, to predict modal choice. Now in the natural sciences discrimination can often be achieved with a very small number of (or even no) misclassifications; a deterministic classification is then entirely satisfactory. The greater the amount of misclassification at the optimum, the less appropriate is a deterministic classification procedure. (It will be remembered that 20 to 25 per cent misclassification is typical here.) It would be unrealistic to expect a model such as this to be very accurate in predicting any one person's modal decision under certain conditions, because, as has been mentioned already, some sweeping assumptions regarding consistency and rationality are implicit in the model. For planning purposes we are not interested in each individual's behaviour on its own, but in the behaviour of people in aggregate, whether aggregated by zone or in other ways. Fortunately, in predicting aggregate behaviour, we can assume that the net effect of eccentric, idiosyncratic factors is small, because, being random, they will tend to cancel. But prediction of choice of mode on a binary, deterministic, "above or below threshold" basis is in a sense attempting to predict each individual's behaviour, since the threshold is discontinuous. A much more satisfactory method is to assign a probability that one mode is chosen rather than the other, where this probability takes a value which depends on the value of the relative disutility function. Prediction would thus involve calculating the $z$-value for a commuter and assigning
to him a probability of choosing one mode on the basis of this value, rather than assigning him definitely to one mode or the other according to which side of the threshold value his z-value fell.

A theoretical expression for probability of choice can be derived as follows. Suppose we represent graphically the frequency distributions of values of \( z \), relative disutility of travel by mode 1, for users of both modes. The best estimate of the probability of a commuter with a given \( z \)-value—say \( z' \)—choosing mode 2 is the ratio of the ordinate of the mode 2 distribution at \( z' \) to the sum of the ordinates of the mode 2 and mode 1 distributions at \( z' \). If these frequency distributions are represented as \( F_1(z) \) and \( F_2(z) \), and \( p_2(z) \) is the probability of choice of mode 2 at \( z \), we have

\[
p_2(z') = \frac{F_2(z')}{F_1(z') + F_2(z')} = \frac{1}{1 + F_1(z')/F_2(z')}
\]

Under present assumptions, \( F_1(z) \) and \( F_2(z) \) are both normal distributions with the same variance. Let \( z_1 = z_2 = 2d \), and let \( Z = z + t \), where \( t \) is chosen so that \( Z_1 = -Z_2 \), i.e. the origin is shifted so that \( Z = 0 \) is midway between the means of the two distributions. Let the variance of each distribution be \( \sigma^2 \). Then we have

\[
F_1(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(Z+d)^2}{\sigma^2}}
\]

and

\[
F_2(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(Z-d)^2}{\sigma^2}}
\]

Thus

\[
p_2(z) = \frac{1}{1 + e^{-\frac{(Z+d)^2 - (Z-d)^2}{\sigma^2}}} = \frac{1}{1 + e^{-2dZ/\sigma^2}}
\]

Now it can be shown that when the constant term in equation (1) above is set to unity, then \( G \) in Fisher’s maximisation criterion, used to derive equations (1) and (2), takes the value of the distance between the means, \( 2d \), and this distance between the means is equal in value to the variance. That is, \( 2d = \sigma^2 \).

The probability expression becomes

\[
P_2(z) = \frac{1}{1 + e^{-\frac{Z}{\sigma^2}}}
\]

or

\[
P_2(z) = \frac{e^{(z+t)}}{1 + e^{(z+t)}}
\]

More generally, if the numbers of people using the two modes are different, there is an \textit{a priori} probability, not equal to one half, that one mode will be chosen more than the other. If \( n_1 \) and \( n_2 \) are the numbers using mode 1 and mode 2 respectively,
it can be seen from a diagrammatic representation that the ordinate of \( F_2(z) \) will be inflated relative to \( F_1(z) \) by a factor \( n_2/n_1 \). That is, \( F_2(z) \) can be taken as

\[
\frac{1}{2} e^{-\frac{1}{2}(Z-d)/\sigma^2}
\]

and \( F_1(z) \) as before. Thus

\[
p_2(z) = \frac{1}{1 + \frac{n_1}{n_2} e^{-\frac{2dz}{\sigma^2}}}
\]

\[
p_2(z) = \frac{1}{1 + \frac{n_1}{n_2} e^{(z+c)}}
\]

or

\[
\frac{n_2/n_1}{1 + n_2/n_1 e^{(z+c)}}
\]

This relationship is an S-shaped “logistic” curve.

In the use for prediction purposes of results based on analysis of a sample, there are two possible approaches. Firstly, the value of \( t \) can be found – it is the distance from the origin of the point midway between the means of the two distributions (the constant term) and probabilities predicted direct from the above expression. Another approach, which uses the sample data more directly, is to gather, from the actual frequency distributions of \( z \) yielded by the sample data, values of \( F_2(z)/(F_1(z) + F_2(z)) \) for equal sub-ranges of \( z \)-values covering the area of overlap of the distributions. These values are then used as probability estimates, and together with the corresponding \( z \)-values are fitted to a generalised version of the probability function:

\[
p_2(z) = \frac{F_2(z)}{(F_1(z) + F_2(z))} = \frac{e^{(z+c)}}{1 + e^{(z+c)}}
\]

This can be done by reducing the expression to a linear equation in \( z \) and \( \log \left( \frac{p}{1-p} \right) \) as follows:

\[
\frac{p}{1-p} = \frac{e^{(z+c)}}{1 + e^{(z+c)}}
\]

That is

\[
\frac{p}{1-p} = e^{(z+c)}
\]

\[
\log \left( \frac{p}{1-p} \right) = k(z + c)
\]

and a linear regression of \( \log \left( \frac{p}{1-p} \right) \) on \( z \) can be carried out. Note especially that the closeness of \( k \) to 1 and of \( c \) to \( t \) in such a regression will indicate the closeness of the distributions to normality for the \( z \)-range considered, and is one indication of the general validity of the normality assumptions inherent in the analysis.

The following describes the application of this to the problem of “calibrating” a predictor from the car-bus results described in the text.

Now calculations are easier if distributions are “standardised”, that is, if for instance
the car users' distribution is reduced by \( n_1/n_2 \), or in this case by 166/376. The expression to fit is then

\[
\log(p/(1-p)) = k(z + c)
\]

where \( p \) is the (standardised) proportion of car users in the total number of commuters with \( z \)-values lying in intervals 0.5 wide. The following values were calculated from the two frequency distributions of \( z \)-values for the sample, using Set 3 at 2d. a mile.

<table>
<thead>
<tr>
<th>( p )</th>
<th>mean ( z ) for the interval</th>
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</thead>
<tbody>
<tr>
<td>0.157</td>
<td>0</td>
</tr>
<tr>
<td>0.160</td>
<td>0</td>
</tr>
<tr>
<td>0.220</td>
<td>0</td>
</tr>
<tr>
<td>0.465</td>
<td>0</td>
</tr>
<tr>
<td>0.435</td>
<td>0.5</td>
</tr>
<tr>
<td>0.692</td>
<td>1.0</td>
</tr>
<tr>
<td>0.765</td>
<td>1.5</td>
</tr>
<tr>
<td>1.000</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Linear regression gave the following result, with correlation coefficient of 0.93:

\[
\log(p/(1-p)) = 1.04 (z - 0.431)
\]

That is,

\[
p = \frac{e^{1.04(z-0.431)}}{1 + e^{1.04(z-0.431)}}
\]

or, in non-standardised form,

\[
p(z) = \frac{2.26 e^{1.04(z-0.431)}}{1 + 2.26 e^{1.04(z-0.431)}}
\]

since 376/166 = 2.26.

Now the point halfway between the mean of the car users' \( z \)-values and the mean of the bus users' \( z \)-values works out at 0.461. Hence the probability expression fitted to the sample is very close to the expression based on a normally variate population, which is

\[
p(z) = \frac{2.26 e^{z-0.461}}{1 + 2.26 e^{z-0.461}}
\]

Thus the sample is close to normality for the range of \( z \)-values at the overlap for the two frequency distributions; this lends weight to the whole analysis. A further test is to see how well the process of summing the probability of choice of car for each person in the sample, on the basis of the fitted expression above, actually reproduces the overall car-using proportion. The actual car-using proportion is \( n_2/(n_1 + n_2) = 376/542 = 69.4 \) per cent. Calculating \( p(z) \) for each person in the sample, summing for the sample, and dividing by 542 yields a car-using proportion of 69.1 per cent, which confirms again that the sample distribution conforms closely to normality.

**APPENDIX B. HOMOSCEDASTICITY AND MULTICOLLINEARITY**

This appendix examines the covariance and correlation matrices of Set 3 at 2d. a car-mile, to test for homoscedasticity and to investigate the possibility of multi-
collinearity between variables. The following table gives the ratio of covariances between the car-using population and the bus-using population for the four significant factors; for homoscedasticity, all except one or two of these ratios should lie between 0.5 and 2.0. There is a particular problem with the use-of-car-for-work factor: this takes only one out of two values in each case, and in the bus-using population no person uses his car for work occasionally; so values of infinity result.

<table>
<thead>
<tr>
<th>Overall travel time difference</th>
<th>1.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess travel time difference</td>
<td>1.47</td>
</tr>
<tr>
<td>Cost difference</td>
<td>-0.656</td>
</tr>
<tr>
<td>Use of car for work</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Excluding the “use of car for work” factor, 4 out of 6 ratios lie within the required range. It is clear, then, that conditions of homoscedasticity are not strictly met. Unfortunately, while the literature maintains that these conditions should be met, there is nothing to indicate how accurate or inaccurate are results obtained under conditions such as those prevailing here.

Examination of the two correlation matrices of seven factors, one for car users, the other for bus users, reveals that out of 42 intercorrelations only 9 are significant at the 1 per cent level, and of these only two are greater than 0.30. These two are the correlations between overall travel time and excess travel time for each population (0.489 and 0.477 respectively). This is to be expected, since one is a component of the other. To test whether there was any association over and above this intrinsic connection, a separate correlation analysis was carried out on excess travel time and net in-vehicle time by each mode for each population. The correlations between excess travel time and overall travel time less excess travel time were as follows:

- Car users: for bus travel 0.075
  for car travel -0.068
- Bus users: for bus travel 0.150
  for car travel -0.189 significant at 5 per cent level

These correlations are nowhere near large enough to upset the use of the discriminant function for prediction purposes, provided that changes in excess travel time are also included as changes in overall travel time.

REFERENCES


University of Leeds

Mr. Quarmby is at present secondment to the Ministry of Transport, London.