Decomposition of Travel Related Expenditure Elasticities into Choice and Generation Components

Correction

John H E Taplin and Brett Smith*

1. Introduction and Correction
A recent paper in this Journal (Taplin, 1997) dealt with expenditure choice elasticities for a group of travel and accommodation items in terms of their ordinary demand elasticities. Because the dependent variables are expenditures, not quantities, the matrix \( G \) of expenditure choice elasticities should be expressed as:

\[
G = (I - W) F
\]

where:

\[
F = E + I
\]

\( I \) being the identity matrix, \( W \) a matrix formed by \( n \) repetitions of the row vector of expenditure shares \( w_j \) within the group, \( F \) the matrix of expenditure elasticities, and \( E \) the matrix of ordinary demand elasticities. The cross-price expenditure elasticities are identical to the corresponding ordinary elasticities, but each own-price expenditure elasticity takes a price change into account at two levels. The ordinary quantity demand response is substantially offset by the price change increasing or decreasing the expenditure on each item purchased. Thus, an own-price expenditure elasticity differs by 1.0 from the corresponding ordinary elasticity (see Henderson and Quandt, 1971, p.27).

2. Generation Elasticities
The elements of the expenditure choice matrix \( G \) account only for the substitution effects within the group, leaving the effects of the individual prices on group expenditure to be represented by the generation elasticities. These are the differences between the elasticities in matrices \( F \) and \( G \), so that the matrix of generation elasticities,

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1 The source of the previous error was the failure to note, in deriving \( G \) from \( E \), that if \( p_f = p_i \), then \( \partial p_f / \partial p_f = 1 \).

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\[ \Omega = (F-G) = WF. \]  

The elements in each column of \( \Omega \) are identical, so that an individual generation elasticity with respect to price \( j \),

\[ \omega_j = \sum_{k=1}^{n} w_k F_{kj}. \]  

This is the expenditure elasticity for the whole group with respect to the price of \( j \). Added to each expenditure choice elasticity in column \( j \) of \( G \), the generation elasticity \( \omega_j \) gives the corresponding expenditure elasticity in \( F \).

### 3. The Example: Holiday Travel and Hotel Expenditure

Table 1, using the example in Taplin (1997), shows expenditure choice elasticities derived from normal expenditure elasticities of demand for vacation travel and accommodation in Australia. The expenditure elasticities in the first part of the table differ from the ordinary elasticities in Taplin (1997) only in the diagonal elements, the own-price elasticities. These have been derived by applying equation (2), that is, by adding 1.0 to each corresponding ordinary quantity elasticity of demand. The revised choice and generation elasticities in the second part of Table 1 have been derived by applying equations (1) and (4).

### 4. Choice and Income-Compensated Elasticities

When decomposed in terms of the income-compensated elasticities (Appendix 1), the individual expenditure choice elasticity is:

\[ G_{ij} = \varepsilon^*_{ij} - \sum_{k=1}^{n} w_k \varepsilon^*_{kj} - v_j \left( \eta_{iY} - \sum_{k=1}^{n} w_k \eta_{kY} \right) + (\delta_{ij} - w_j), \]  

where \( \varepsilon^*_{ij} \) is the income-compensated elasticity with respect to \( p_j \), \( v_j \) is the expenditure proportion for the \( j \)th item in the entire consumer budget, \( \eta_{iY} \) is the \( i \)th income elasticity, and \( \delta_{ij} \) is the \( ij \)th element of the identity matrix \( I \). The generation elasticity in terms of income-compensated elasticities is:

\[ \omega_j = \sum_{k=1}^{n} w_k \varepsilon^*_{kj} - v_j \sum_{k=1}^{n} w_k \eta_{kY} + w_j. \]

### 4.1 Expenditure generation elasticities and separability

Estimation of choice and generation components under the assumption of weak separability is simplified by the use of a constant ratio between cross-price and income elasticities. The ratio is constant because of the assumed functional form of the consumer's utility, the marginal rate of substitution within the group being independent of purchases of items outside the group (Green, 1976). Thus,

\[ r = \varepsilon_{ic}/\eta_{iY}. \]
Decomposition of Travel Related Expenditure: Correction

Taplin and Smith

Table 1

Derivation of Expenditure Choice Elasticities from Expenditure Elasticities of Demand for Vacation Travel and Accommodation in Australia

<table>
<thead>
<tr>
<th></th>
<th>Overseas Air Fare</th>
<th>Overseas Hotel Price</th>
<th>Domestic Air Fare</th>
<th>Domestic Hotel Price</th>
<th>Car Operating Cost</th>
<th>All Other Prices</th>
<th>Income Elasticity</th>
<th>Budget Share %</th>
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<tbody>
<tr>
<td><strong>Normal Expenditure Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Vacation Air Trips: Overseas</td>
<td>-0.70*</td>
<td>-0.70</td>
<td>0.40</td>
<td>0.30</td>
<td>0.10</td>
<td>-0.50</td>
<td>2.10</td>
<td>1.101</td>
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<tr>
<td>Overseas Hotel Use</td>
<td>-1.28</td>
<td>0.20</td>
<td>0.10</td>
<td>0.20</td>
<td>0.10</td>
<td>-0.52</td>
<td>2.20</td>
<td>0.604</td>
</tr>
<tr>
<td>Vacation Air Trips: Domestic</td>
<td>1.76</td>
<td>0.25</td>
<td>-1.10*</td>
<td>-0.89</td>
<td>-0.15</td>
<td>-0.27</td>
<td>1.10</td>
<td>0.252</td>
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<tr>
<td>Domestic Hotel Use</td>
<td>0.54</td>
<td>0.20</td>
<td>-0.36</td>
<td>0.00*</td>
<td>-0.14</td>
<td>-0.24</td>
<td>1.00</td>
<td>0.623</td>
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<tr>
<td>Vacation Car Trips: Domestic</td>
<td>0.52</td>
<td>0.28</td>
<td>0.17</td>
<td>-0.39</td>
<td>-0.11*</td>
<td>-0.17</td>
<td>0.70</td>
<td>0.220</td>
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<tr>
<td>All Other</td>
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<td>0.0041</td>
<td>-0.0004</td>
<td>-0.0014</td>
<td>-0.0012</td>
<td>-0.011*</td>
<td>0.98</td>
<td>97.200</td>
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<tr>
<th></th>
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<th>Within Group Spending Share %</th>
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<tbody>
<tr>
<td>Vacation Air Trips: Overseas</td>
<td>-0.47</td>
<td>-0.56</td>
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<td>0.25</td>
<td>0.07</td>
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<td>0.15</td>
<td>0.07</td>
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<td>0.39</td>
<td>-1.11</td>
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<td>0.12</td>
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<td>9.0</td>
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<td>Domestic Hotel Use</td>
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<td>-0.37</td>
<td>-0.05</td>
<td>-0.17</td>
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<td>Vacation Car Trips: Domestic</td>
<td>0.75</td>
<td>0.42</td>
<td>0.16</td>
<td>-0.44</td>
<td>-0.14</td>
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<th>Expenditure Generation Elasticities</th>
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<td></td>
<td>-0.23</td>
<td>-0.14</td>
<td>0.01</td>
<td>0.05</td>
<td>0.03</td>
<td></td>
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</tbody>
</table>

* Each own-price expenditure elasticity has been derived from the corresponding ordinary own-price demand elasticity by adding 1.0.
The generation elasticity (6) now becomes (Appendix 2):
\[ \omega_j = -w_j \{ (r + \upsilon_c) \eta_{ij} + (1 - \upsilon_c) \eta_{i[GROUP]} - 1 \}, \]
where \( \upsilon_c \) is the expenditure proportion for all items other than those within the group. The expenditure choice elasticities can also be obtained by substituting into (1).

5. Conclusions
This note corrects an error in an earlier paper (Taplin 1997) which showed that a matrix of demand elasticities for a group of items that cannot be counted in the same physical units may be decomposed into choice and generation components. The result is corrected when expressed in terms of the matrix of expenditure elasticities, obtained by adding the identity matrix to the ordinary elasticity matrix. Expenditure elasticities take the price change into account at two levels. There is the ordinary quantity demand response and also the change in expenditure, for any given quantity, due to the fact that the items now cost more or less.

Appendix 1

Relationship between Choice and Income-Compensated Elasticities

The Hicks Slutsky decomposition:
\[ \varepsilon_{ij} = \varepsilon^*_{ij} - \upsilon_j \eta_{ij}, \]
where:
- \( \varepsilon^*_{ij} \) is the income-compensated elasticity of demand for \( i \) with respect to the price of \( j \);
- \( \upsilon_j \) is the share of \( j \) in total expenditure; and
- \( \eta_{ij} \) is the income elasticity of demand for \( i \).

The matrix of expenditure choice elasticities, \( G = (I - W) (E + I) = \)

\[
\begin{bmatrix}
(1 - w_1) & -w_2 & \cdots & -w_n \\
-w_1 & (1 - w_2) & \cdots & -w_n \\
\vdots & \vdots & \ddots & \vdots \\
-w_1 & -w_2 & \cdots & (1 - w_n)
\end{bmatrix}
\begin{bmatrix}
\varepsilon^*_{11} - \upsilon_1 \eta_{11} + 1 & \varepsilon^*_{12} - \upsilon_2 \eta_{12} & \cdots & \varepsilon^*_{1n} - \upsilon_n \eta_{1n} \\
\varepsilon^*_{21} - \upsilon_1 \eta_{21} & \varepsilon^*_{22} - \upsilon_2 \eta_{22} + 1 & \cdots & \varepsilon^*_{2n} - \upsilon_n \eta_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\varepsilon^*_{n1} - \upsilon_1 \eta_{n1} & \varepsilon^*_{n2} - \upsilon_2 \eta_{n2} & \cdots & \varepsilon^*_{nn} - \upsilon_n \eta_{nn} + 1
\end{bmatrix}
\]

Thus,
\[ G_{ij} = \varepsilon^*_{ij} - \sum_{k=1}^{n} w_k \varepsilon^*_{kj} - \upsilon_j \left( \eta_{ij} - \sum_{k=1}^{n} w_k \eta_{kj} \right) + (\delta_{ij} - w_j), \]
where \( \delta_{ij} \) is the \( ij \)th element of the identity matrix \( I \).
Appendix 2
The Generation Elasticity Based on a Constant Ratio under Separability

On the assumption of weak separability, the cross-price elasticities for items outside the group share a common ratio with their respective income elasticities,

\[ r = \varepsilon_{ij}/\eta_{ij}, \varepsilon_{ic} = r\eta_{ic}. \]

Homogeneity:

\[ \sum_j \varepsilon_{ij} + r\eta_{ij} + \eta_{ij} = 0, \]

\[ \sum_j \varepsilon^*_{ij} - \sum_j v_j \eta_{ij} + r\eta_{ij} + \eta_{ij} = 0, \]

\[ \sum_j \varepsilon^*_{ij} + (r + v_c)\eta_{ij} = 0. \]

Symmetry:

\[ \sum_k w_k \varepsilon^*_{kj} = \sum_k w_k \varepsilon^*_{jk} \]

\[ = w_j \sum_k \varepsilon^*_{jk}. \]

Hence, the generation elasticity expressed in terms of income compensated elasticities,

\[ \omega_j = \sum_k w_k \varepsilon^*_{kj} - v_j \sum_k w_k \eta_{kj} + w_j \]

\[ = w_j \sum_k \varepsilon^*_{kj} - v_j \sum_k w_k \eta_{kj} + w_j \]

\[ = -w_j [(r+v_c)\eta_{ij} - v_j \eta_{GROUPY} + w_j \]

\[ = -w_j [(r+v_c)\eta_{ij} + (1-v_c)\eta_{GROUPY} - 1]. \]

References

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