SUBSIDIES TO RELIEVE URBAN TRAFFIC CONGESTION*

By Roger Sherman

Traffic congestion is a familiar problem. One partial remedy which has been suggested from time to time is to subsidise buses, on the ground that bus passengers contribute less than car passengers to traffic congestion. Indeed, it is theoretically possible to identify conditions in which a bus subsidy will improve economic welfare. This paper states these conditions, which were derived elsewhere [12], and considers whether they are likely to be satisfied in real cities.

In brief, the theoretical argument runs along these lines. When a person drives his car into town, he faces only his own private cost of the trip, even though when there is congestion his own private cost will be lower than the social marginal cost of the trip.\(^1\) But there may be no feasible way to confront the independent decision maker with the costs associated with the delays he imposes on other drivers, costs which make up the difference between private and social cost. We don’t want to stop a car every 100 yards just to collect a toll, and more sophisticated devices for fee assessment appear to be unpopular politically.\(^2\) Taxes on inputs (e.g., a petrol tax) are too imprecise to serve as congestion charges. Since travel inputs need not be purchased as the same time as they are used, any inputs tax that effectively rations scarce road space during peak congestion periods must also be in effect at off-peak times when it is inappropriate.

A departure from ideal pricing in car travel thus seems inescapable; and, since the private cost of one travel mode will affect usage in another, it can then be appropriate to depart also from the normal ideal in pricing a substitute mode. More specifically, it can be better to price mass transit below its marginal social cost, simply because car transit is priced below its marginal social cost. This is not a claim that “two wrongs make a right”, for the result will still not be ideal (economists call it “second best”). But when consumers act on a price that does not reflect marginal social cost they are misled, and will fail to coordinate their activities; they will “misallocate” their incomes and, collectively, their resources. The misallocation is most severe between competing goods when only one of them is priced wrongly, because individuals will choose a wrong amount of the wrongly priced good relative to its competing goods. So, if there is no cure for one faulty price, a change in the same direction in the price of competing goods can reduce the misallocation.

The logic of this conclusion rests on the fact that elasticities are lower for broad categories of goods and services (e.g., travel) than for subcategories (e.g., bus travel).

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\(^1\) Private cost just equals average social cost, since each car can join in the traffic and experience average costs and delays. Marginal social cost includes, in addition to those average costs and delays borne by the marginal vehicle, the additional costs and delays imposed on all other vehicles by its presence. See [6], [15] or [16] for a thorough analysis.

\(^2\) The pros and cons of road pricing are set out in [6], [7] and [15].
So by shifting the impact of allocative effects to the broader level, where elasticities are lower, those effects can presumably be made smaller. In practice, so many long-run decisions rest on particular travel costs and opportunities that such general results may not hold. For example, subsidising buses might in particular circumstances create transitional location problems. We shall assume that this is not a problem in what follows, trusting that improvements in relative pricing among alternative modes will improve resource allocation. This second-best line of argument has special force in the case of public bus transit when car travel contributes to bus as well as car traffic congestion, and vice versa. If bus travel makes the smaller contribution to road congestion, persuading more persons to travel by bus can reduce the overall level of traffic congestion, as well as offsetting the wrong (low) price which invites excessive car travel.\(^3\)

We begin by stating conditions obtained from welfare maximising solutions to transit pricing. When satisfied, the conditions indicate that bus fares which lie below average cost are in order; so the conditions really are tests for the appropriateness of subsidies to buses. Then we examine these conditions in the light of what has been observed empirically for their crucial variables. Although precise answers are not available, we find that the conditions would often be satisfied; this suggests that bus subsidies may often be desirable to reduce traffic congestion. Implications for redistribution of income are also noted.

**CONDITIONS CALLING FOR SUBSIDY**

Conditions that call for a transit subsidy were obtained from a simple model in which private car and public bus travel used the same road space and each contributed to the congestion experienced by the other [12]. We shall summarise briefly these conditions which call for bus fares to be lower than average cost. The conditions are to hold at optimal solutions, and only with some risk of error can they be applied to circumstances as we find them in the real world. Nevertheless they provide a starting point for empirically identifying the appropriateness of subsidies. They were derived both with and without the existence of inputs taxes, and we shall discuss both cases, because together they raise a useful policy choice for some communities, even though the case in which inputs taxes already exist would clearly be the relevant one for most large cities. The derivation of the conditions also assumed that a car passenger mile contributed relatively more to the congestion of both car and bus modes than did a bus passenger mile, and that the relative marginal contribution was the same in both modes. That car passenger miles cause greater congestion seems reasonable from previous research, even when conservatively interpreted.\(^4\) And that each mode causes the same relative marginal contribution to both modes is plausible as long as passenger car units (p.c.u.’s) can serve reasonably as common-denominator determinants of congestion. Some such exchange rate between bus and car passenger miles is needed for measuring the extent of the congestion they cause.

\(^3\)We do not consider additional biases which favour car ownership and might be countered by altering the form of the consumer’s choice. For such arguments see [10] and [11].

\(^4\)A bus requires about three passenger car units ([9], pp. 200–201), and perhaps more, but carries well over three times as many passengers. We return to this point below.
(a) No Inputs Taxes

First, consider the case in which there are no inputs taxes. At off-peak times, when there is no congestion, highways will be rationed ideally, for no usage charge is then in order. But at peak times a solution with no inputs taxes will cause the greatest congestion, because there will be no pecuniary restraint on the use of highways. Thus, although no subsidy is necessary at off-peak times, it may be desirable to subsidise buses at peak periods, when car travellers will be basing their trip decisions on average rather than marginal social cost. With no inputs taxes, the welfare maximising condition calling for a bus fare below average cost, and hence for a subsidy, is: that the ratio of cost elasticities for bus relative to car passenger miles be less than the ratio of (compensated) demand elasticities of car and bus passenger miles with respect to the bus fare. We can express this condition more precisely if we put it in mathematical form.

First we must introduce some definitions. Let cost elasticity be represented by

$$\zeta_{jk} = \frac{d\xi_j}{d\xi_k} \cdot \frac{t_k}{g_j}, \quad j = a, b \text{ and } k = a, b$$

where $t_a$ and $t_b$ represent private car and bus passenger miles and $g_a$ and $g_b$ represent composite variables which indicate average input requirements. Since each mode affects congestion in both modes, both $g_a$ and $g_b$ are functions of both $t_a$ and $t_b$:

$$g_a = g_a(t_a, t_b)$$
$$g_b = g_b(t_a, t_b)$$

Our underlying assumption that each mode causes the same relative marginal congestion in both modes, but that car passenger miles always contribute more, can be represented as:

$$\frac{\partial g_a}{\partial t_b} \frac{\partial}{\partial t_a} < 1 \quad (1)$$

The relative contributions are equal here; and, since the contributions of car passenger miles in the denominators are assumed greater, the ratio of contributions from the two modes must be less than unity. Let demand elasticity be represented by

$$\eta_{jk} = K_{jk} \frac{P_k}{t_j}, \quad j = a, b \text{ and } k = a, b$$

where $P_a, P_b$ are the prices for car and bus passenger miles and $K_{jk}$ is the Slutsky effect on demand for the $j$th good of a compensated change in the price of the $k$th good. Using these terms we can restate the condition given above, for a bus subsidy to be warranted, as follows:

$$\frac{\zeta_{ab}}{\zeta_{aa}} = \frac{\zeta_{bb}}{\zeta_{ba}} < \frac{\eta_{ab}}{(-\eta_{bb})} \quad (2)$$

The equality of cost elasticity ratios is a consequence of the assumptions represented by (1).

Intuitively, condition (2) can be interpreted as follows. We assume that bus costs rise less than car costs for any given increase in travel (passenger miles) by either mode. As long as there is some substitution of bus for car travel whenever the bus fare is reduced (i.e., as long as $\eta_{ab} > 0$), a greater divergence between the responses of bus and car costs to increased travel (the response of bus costs being the smaller) will
tend to justify a subsidy to buses. And a greater tendency for bus travel to be substituted for car travel as the bus fare is reduced will also tend to justify a subsidy (for any given divergence in marginal bus and car costs). In the extreme case of perfect substitution between cars and buses a subsidy will always be justified (in that case (2) can be reduced to (1) by substituting in definitions and simplifying).

(a) Inputs Taxes

Now let us move on to the case in which inputs taxes are in use. The average cost of a passenger mile equals average inputs (e.g., \( g_a \)) multiplied by the price of the inputs, \( \pi \). Permitting inputs taxes is equivalent to permitting adjustments in both \( \pi \) and the bus fares. With this added policy instrument of an inputs tax, a perfectly optimal solution can be obtained at the peak.\(^5\) However, at this solution inputs will have a higher effective price, \( \pi' \), rather than \( \pi \), and the difference \( (\pi' - \pi) \) will represent the inputs tax imposed. Now at the peak the condition for a subsidy for buses to be warranted is

\[
\frac{\zeta_{ba}}{\zeta_{bc}} = \frac{\zeta_{bb}}{\zeta_{bs}} < \frac{\pi' g_b t_b}{\pi' g_a t_a}
\]

(3)

Condition (3) can be given the following intuitive interpretation. The left hand cost elasticity ratios are the same as in condition (2), and once again they indicate that, as our assumed increase in car costs is larger than that in bus costs, a subsidy will tend to be justified. But on the right hand side of condition (3) we now have the ratio of total expenditures on bus travel divided by total expenditures on car travel. So (for any given relative responsiveness in costs) a greater expenditure on bus relative to car travel will tend to justify a subsidy. Now of course there are tax proceeds that can help to pay the subsidy. And the subsidy will be more reasonable as bus travel expenditures are relatively greater, because bus travellers then bear a greater burden of the inputs taxes.

At off-peak times there is no congestion, and since more travel then will not raise average costs we have

\[
\frac{\partial g_a}{\partial t_a} = \frac{\partial g_a}{\partial t_b} = \frac{\partial g_b}{\partial t_a} = \frac{\partial g_b}{\partial t_b} = 0
\]

But if an optimal peak solution resulted in an inputs tax, so that \( \pi' > \pi \), the effective inputs price, \( \pi' \), would still be in effect at off-peak times. A welfare maximising off-peak bus fare in the presence of \( \pi' \) will be below average cost if the following condition holds:

\[
\frac{\eta_{ab}}{(-\eta_{bb})} < \frac{\pi' g_{ub} b}{\pi' g_{ua} a}
\]

(4)

Condition (4) calls for the ratio of bus to car off-peak compensated demand elasticities to be less than the ratio of bus to car off-peak travel expenditures.

An intuitive explanation of (4) can be constructed easily now, because (4) involves terms that have appeared before in conditions (2) and (3). For a given bus-to-car total expenditure ratio, a greater tendency for bus travel to be substituted for car travel when the bus fare is reduced will now work against a subsidy. Indeed, if there

\(^5\)Marchand pointed out how the inputs tax combined with optimal public transit fares could achieve an optimal solution in the presence of congestion [3].
is no substitution for car travel at all when the off-peak bus fare is reduced (if \( \eta_{ab} = 0 \) off-peak) a subsidy will definitely be justified. For any other ratio of elasticities, a greater expenditure on bus relative to car travel will tend to justify a subsidy. Again there will be tax proceeds to help pay for the subsidy, and bus travellers also bear more of the inputs taxes as the total expenditure on bus travel is greater.

These conditions for bus subsidies were obtained from separate solutions to problems at peak and off-peak times; demand functions in the two periods were thus assumed independent. While choice of time of travel between peak and off-peak might be influenced by fares in each period, the problem of cross elasticities according to time of day would be much harder to solve. Even without considering that possibility, however, we encounter limitations in data available for empirical investigation of whether subsidy conditions (2), (3), and (4) could hold. They thus appear as sufficiently ambitious starting points for an empirical check on the reasonableness of subsidies.

ESTIMATES OF CRUCIAL VARIABLES

(a) No Inputs Taxes

Turning to an empirical examination of whether these welfare maximising conditions for subsidies are met, we shall consider first the case with no inputs taxes. The only way to achieve an ideal welfare maximising solution at off-peak, no-congestion times is to forego inputs taxes. And, judging purely by a comparison of the amount of travel and the extent of misallocation at peak and off-peak times, some communities may on balance do better without inputs taxes. This solution is not seen in practice, at least in pure form, primarily because revenue is raised by inputs taxes to help pay for the construction of highways.

If there were no inputs tax in the United Kingdom, it would probably be desirable to have peak hour bus fares subsidised. We shall note later that this may not be true for the United States, where the case may be of more interest because inputs taxes there are so low. Taking only one of the two equal left-hand terms in (2), and substituting definitions, we obtain:

\[
\frac{\partial g_b}{\partial t_b} \cdot \frac{t_b}{g_b} \cdot \frac{P_b}{t_a} > \frac{K_{ab}}{(-K_{bb})} \frac{P_b}{t_b}
\]

which reduces to

\[
\frac{\partial g_b}{\partial t_b} \cdot \frac{t_b}{g_b} < \frac{K_{ab}}{(-K_{bb})}
\]

(2')

The left-hand side of this expression represents relative contributions to congestion. We have assumed this ratio to be less than unity (see equation (1)), but we will now make a more precise estimate of it. If we assume that p.c.u.’s cause congestion, we simply have to convert from p.c.u.’s to passenger miles for each mode. Values will

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vary from location to location, but at peak times private cars have variously been estimated to carry about 1-5 passengers, while buses carry an average of at least 20 passengers. (This seems a conservative figure: cf. [5], p. 232.) With buses requiring 3 p.c.u.'s ([9], p. 200), this ratio would have a value of 0-225. A more conservative p.c.u. value of 5 will give the ratio a value of 0-375. This figure still seems likely to lie below the value of $K_{ab}/(-K_{bb})$. Beesley and Foster used a value of 0-75 for $K_{ab}/(-K_{bb})$ ([1], p. 74) in considering an increase in public transit fare, thinking that three-quarters of those giving up public transit would take to car travel. It seems likely that a similar value would serve as an estimate in the other direction, and in any case that at least half the increase in public transit usage would come from former car users. These estimates are summarised in Table I.

Table I
Summary of Estimates for Crucial Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Peak</th>
<th>Off-Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial g_{b} / \partial t_{b}}{\partial g_{a} / \partial t_{a}}$</td>
<td>0-2 to 0-4</td>
<td>0-2 to 0-4</td>
</tr>
<tr>
<td>$\frac{K_{ab}}{(-K_{bb})}$</td>
<td>0-75 or lower</td>
<td>0-75 or lower</td>
</tr>
<tr>
<td>$\frac{\pi'<em>{gb}}{\pi'</em>{ga}}$</td>
<td>0-5</td>
<td>1-0 or higher</td>
</tr>
</tbody>
</table>

(b) Inputs Taxes

Now consider the more relevant case in which inputs taxes exist. By substituting for definitions and simplifying, conditions (3) and (4) for peak and off-peak times can be reduced to these conditions, (3') and (4')

\[ \frac{\partial g_{b}}{\partial t_{b}} < \frac{\pi'_{gb}}{\pi'_{ga}} \]  
\[ \frac{\partial g_{b}}{\partial t_{a}} < \frac{\pi'_{gb}}{\pi'_{ga}} \]  

(3')

\[ \frac{K_{ab}}{(-K_{bb})} < \frac{\pi'_{gb}}{\pi'_{ga}} \]  

(4')

Notice that these conditions contain no $t_{a}$ or $t_{b}$ terms. The expenditure ratio on the right-hand side contains only averages. Although values of $t_{a}$ and $t_{b}$ will influence some of the terms in (3') and (4'), $t_{a}$ and $t_{b}$ values need not themselves play a crucial role in determining whether a subsidy should be offered to bus riders.

On the basis of the estimates of Meyer, Kain and Wohl [5] for U.S. cities, the average costs of private car and bus travel are about the same at low traffic (off-peak) densities, with bus costs possibly higher (this is the source of $\pi'_{gb}/\pi'_{ga}$ figures in Table I). Bus travel falls to about half the cost of car travel at high densities, however, not because of congestion as emphasised here, but because Meyer, Kain and Wohl
relied on historical costs as average costs, which would fall for buses at the peak because of more intense utilisation. Congestion might cause all costs to rise more than Meyer, Kain and Wohl estimated, but we cannot say that it would seriously alter their estimated relative costs, because the greater contribution of cars to congestion would also lower \( \pi'g_b/\pi'g_a \). In any case, if \( K_{ab}/(-K_{ab}) \) is again taken to be about 0.75,6 or lower, it seems likely that condition (4') would be satisfied at off-peak times, when bus and car average costs are more likely to be equal. A subsidy to off-peak bus riders would then be in order. The rationale for it here, of course, is to offset the inputs tax, which is undesirable at off-peak times but cannot be "turned off" once it is imposed.

Satisfaction of condition (3') at peak times is likely but cannot be assured for all circumstances. As already noted, it is improbable that \( (\partial g_b/\partial t_b)/(\partial g_b/\partial t_a) \) will rise above 0.5 at the peak, and a more typical estimate will lie in the range between 0.2 and 0.4. While \( \pi'g_b/\pi g \) might fall as low as this in some cities, it would seem more often to lie above 0.4, even when \( \pi g_a \) includes all attendant car travel costs, right down to tolls and parking. Marginal cost exceeds average cost relatively more in car than in bus travel. Thus some peak-hour subsidy to bus transit would seem to be desirable in many cities. This will not be true for all cities, of course, and conditions which either reduce the relative contribution made by cars to traffic congestion or raise the average cost of car relative to bus travel would also discourage the peak-time bus travel subsidy. Relative average costs of modes can vary a great deal from place to place, as Meyer, Kain and Wohl make clear [5]. For example, an even distribution of bus passengers over a route will improve utilisation and thus lower average cost. Moreover, the use of average cost estimates for off-peak times alone is especially treacherous, since average bus transit costs can vary according to whether a peak period is sufficiently large to absorb capacity costs and thus lower the off-peak costs. Where off-peak costs of bus travel can be lowered in this way, the satisfaction of (4') also is less likely. While the author is aware of no similar comparison estimates of car and bus transit costs in the U.K., he sees no reason to expect drastic differences in relative costs— with the same qualification that considerable differences might arise under specific circumstances.

The value of \( \eta_{ab}/(-\eta_{ab}) \) may be lower in the U.S. than in the U.K., although no reliably precise estimates are yet to hand. One recent study of Boston passengers estimated transit demand elasticities \( \eta_{ab} \) of roughly \(-0.2\) to \(-0.3\) [14], compared with the estimates for London presented by Beesley and Foster ([1], p. 78) of \(-0.3\) to \(-0.6\). What is more, the Boston study found little or no evidence of a

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6 At off-peak times, the absolute value of the own-price elasticity of bus travel demand tends to be higher than at the peak. See [1], p. 78. It seems just as plausible then, however, to use the 0.75 figure as the fraction diverted from cars.

7 These results are not entirely consistent with Warner [17], who would seem to anticipate greater elasticity as more travellers own automobiles (although he did not work with continuous elasticity measures). About 60 percent of public transit users owned cars in Chicago at the start of the 1960s ([5], p. 141), while 43 percent of public transit users were estimated to own cars in London in 1961 ([1], p. 73). This estimate is probably high—it is based on a 1954 survey which found 21 percent of public transit users owned cars. A greater fraction also commuted by car in Chicago than in London. Propensities may be very different across countries, of course, and Warner would probably not want his hypothesis extended beyond the U.S. For elasticity implications similar to those noted in the text, and an excellent concise review of recent studies, see [4].
positive cross elasticity of demand. This means that the left-hand side of (4) (before simplification into (4')) might be small, e.g., well below one. To be comparable in this form, however, the right-hand side of (4) includes miles travelled by the two modes in the bus-to-car total expenditure ratio, which would also be smaller for the U.S. because of greater car transit usage. Nevertheless, on the basis of fare adjustments alone (the cross elasticity with respect to travel time appears greater than that with respect to fare alone [2], [4], [14] pp. 49–50), it still seems likely that (4) (and hence (4')) would be satisfied in many U.S. cities. Condition (2) would probably not be satisfied for U.S. cities, however, so subsidies might be inappropriate if there were no (or very low) inputs tax. Although they exist in the U.S., inputs taxes still appear to be well below the level that would equate the private and social cost of car trips in cities at peak times [15].

When inputs taxes exist, they must be known before the level of any subsidy can really be interpreted. It is therefore worth noting that the optimal peak inputs tax will reflect the difference between private and social marginal cost. The inputs tax derived in [12] is the product of the marginal cost of inputs multiplied by \((\xi_{oa} + \xi_{ba} \frac{b_{gb}}{b_{gba}})\). The cost elasticity term, \(\xi_{oa}\), ordinarily reflects the difference between social and private cost [15] and the additional term, \(\xi_{ba} \frac{b_{gb}}{b_{gba}}\), reflects a similar cost difference imposed on buses, in our case of congestion interdependence, by an additional car passenger mile. The optimal subsidy per bus passenger mile will depend on whether the contribution of buses to congestion exceeds or falls short of this contribution by cars. Indeed, the subsidy per bus passenger mile, \(s\), as presented in [12], can be expressed in the form

\[
s = g_b \pi \left[ (\xi_{oa} + \frac{b_{gb}}{b_{gba}} \xi_{ba}) - (\xi_{bb} + \frac{b_{gb}}{b_{gba}} \xi_{ab}) \right]
\]

(5)

where the left-hand term in brackets represents the contribution of cars to congestion, which is the basis of the inputs tax \((\pi' - \pi)\), and the right-hand (negative) term is the contribution of buses. As long as the contribution by cars to congestion is greater, \(s\) will be positive, or a subsidy will be in order. Conditions (3) and (3') are simply additional ways of expressing the relationship in (5).

THE DISTRIBUTION OF INCOME

Both the off-peak solution with no inputs taxes and the peak solution with inputs taxes are Pareto optimal. At these solutions, each person faces private costs or prices which equal social costs. But at the peak with no inputs tax, and also at off-peak times with an inputs tax, we have second best solutions. These second best solutions require income redistribution to make them Pareto optimal solutions. When there are no inputs taxes, income should be redistributed away from those who spend more of a marginal unit of income on transit at the peak. When inputs taxes exist, income should be redistributed toward those who spend more of a marginal unit of income.

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8For comparison, the 0.75 assumption of Beesley and Foster for \(K_{ab}/(−K_{bb})\) will combine for London with annual (peak) passenger miles of about 500 million by car (derived from [13]) and 3,500 million by bus ([1], p. 86) to produce an elasticity ratio, \(\eta_{ab}/(−\eta_{bb})\), well above 1.
on off-peak travel. 9 Crude approximations which reflect these implications might be achieved by fees for licences, special licences being required for travelling at peak times.

When inputs taxes exist it can be shown [12] that there will be net revenues for the state, even after payment of optimal subsidies, so licence adjustments to relieve appropriate users are entirely possible. Under any additional taxation plan to pay for the cost of highways, the distribution of the tax burden can reflect the same redistributinal objectives.

CONCLUSION

Coordination of transit decisions in cities is very difficult; we cannot easily charge drivers for the use of roads precisely when and where they use them. But public transit fares have not yet been used to reduce to a minimum the misallocation consequences of this problem. Technology will soon open the way to a wider choice of transit fare structures. The best (most efficient) solutions will almost surely involve some variation in fares by time of day (as well as by miles travelled), and they may also call for more public subsidy. In many cases, subsidies to buses may be desirable simply to reduce traffic congestion.

A general statement on whether subsidies are warranted for urban bus services is not appropriate. On balance, however, what data exist seem to suggest that a subsidy to buses would be appropriate in London, to be financed out of taxes on transit inputs. The same may be true for large U.S. cities. Results of this analysis are less applicable there, however, because, although taxes exist, they are not at the level assumed, which would equate the private and social marginal costs of peak travel by private car. This analysis also assumed that fares could differ by time of day, although inputs taxes could not.

REFERENCES


9See [3] and [12] for derivation of these implications. The importance of including redistribution implications is set out clearly by Mohring [8].
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