AIRLINE OVERBOOKING

Some Further Solutions

By William Vickrey

In earlier issues of this Journal there has been discussion of a method of dealing with airline overbooking [1], [2], [3]. Under this method, in the event of more passengers with confirmed reservations showing up than can be accommodated on the plane, they would be asked to submit sealed bids indicating the minimum amount of money they would be willing to accept as compensation for changing their plans at that moment to the next best alternative, the passengers submitting the lowest bids being then bumped and compensated accordingly. Professor Simon correctly claims that this method would be clearly superior, at least on the average and for any given level of overbooking, to methods based on random selection, selection in terms of position in some queue, or indeed other methods not mentioned such as selection on the basis of ultimate destination and connections, or the offer of arbitrarily fixed compensation to those quickest to respond to the offer. There remain, however, imperfections that lead one to ask whether one cannot do better than this.

As Professor Falkson points out, paying the bumped passenger on the basis of his bid gives him an incentive to overstate his price, and the degree of this overstatement by passengers may vary, so that the person with the lowest bid who is therefore bumped may not in fact be the person who would have accepted the lowest compensation. Simon does not indicate whether, in the case where \( n \) passengers must be bumped, each will be paid his own bid or all will be paid the same amount equal to the \( n \)th lowest bid; but the incentive to overbid is present in either case, though far weaker where all will be paid the same. It is even possible that in isolated cases the arbitrary method would produce a result closer to Pareto-optimal than the bidding procedure. This defect can be remedied fairly simply, however, by asking passengers to bid on the understanding that if there are \( n \) passengers to be bumped the payment will be a uniform one equal to the \((n+1)\)th lowest bid. In this case the individual passenger will find that his best strategy is to bid the actual lowest amount he would be willing to accept. To bid lower would introduce a probability that he might be bumped on unfavourable terms. To bid higher would merely reduce his chances of being bumped on favourable terms, while not affecting the amount he will receive if he is in fact bumped. The position of the airline will not be impaired: under fairly reasonable assumptions, the level of the bids will tend to be decreased just enough to ensure that the \((n+1)\)th bid under this revised proposal would average no higher than the \( n \)th lowest bid under the original proposal ([4], [5], and later work summarised in [6], [7]). The procedure would be strategically equivalent to the clumsier open-auction procedure in which passengers bid \textit{viva voce}, making successively lower bids until no passenger is willing to make a lower one.

If we ask whether the airline would actually gain by introducing any such auction procedure, the answer is not so clear. Much depends on its susceptibility to the ill-
will engendered by uncompensated bumping. If an airline is in a strong monopoly position on a given route, or has relatively little repeat traffic and can contrive to avoid having its record become generally known to prospective passengers, it may consider that it is maximizing its net revenues by a policy of relatively heavy overbooking and frequent bumping. It might find that inaugurating a policy of paying for the bumps that it is now imposing on the public without payment would cost it money, and that maximizing net revenue under these circumstances would require less overbooking rather than more. A different answer might be given if a policy of compensating for bumps enabled the airline to charge a higher fare, which would affect its place in various tariff and cartel arrangements.

Even where competition is such that the airline has reason to be jealous of its image in the mind of the travelling public, compensation for bumping would have to be considered in conjunction with such matters as compensation for lost baggage, overnight arrangements for passengers missing connections, and other types of compensation for service below standard. The institution of compensation on the basis of bids for bumping might be expected to be at least as cheap a form of enhancing the airline’s image as any other, but this cannot be guaranteed from the analysis.

A SIMULATED SPECULATOR’S MARKET

Falkson [2] suggests that what is really wanted is some form of advance auction that would avoid the waste of effort involved in the bumped passengers having to make the trip to the airport and possibly missing opportunities to make the best of alternative arrangements. He also suggests somewhat vaguely a system of flight priority insurance, without working out in detail how this could be put into practice. If, however, one follows this line of thought and uses as a model a free speculator’s market in future reservations, the following scheme emerges as probably as good as any that can be devised.

Airlines would sell reservations on future flights at prices that reflect fairly closely what the outcome of a competitive futures market in those reservations would produce at any given moment. The price for a reservation on a given future flight would vary from time to time according to the relationship between the number of seats already sold for that particular flight and the number that would normally be expected to have been sold at that time before the flight. The price at any given time for a further reservation on that flight would be such as to generate an expectation that, if the price were kept constant at this level, the remaining seats would just be sold at flight time. The price would in fact fluctuate either up or down as further reservations were made at a faster or slower rate than that anticipated; but there would at any given time be no expectation that the price in the future would on average be either higher or lower by any substantial amount, though possibly a slightly rising price expectation might be maintained to encourage early reservations, especially as departure time drew near. Reservations would be regarded as final sales, not normally subject to recontracat, but passengers changing their plans would be entitled to turn in their reservations at the price ruling at the time of turn-in, less an appropriate handling charge to cover the associated costs and to discourage speculation by people with no intention of travelling.

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One would, of course, not expect to have a separate pricing schedule or algorithm for each flight independently. Flights or flight legs would be grouped in five or ten classes according to how far ahead passengers normally make reservations for them, using such criteria as national or international, short or long-haul, and the overall schedule frequency. Thus London–Glasgow would probably fall in the shortest lead category, London–Paris would fall in a somewhat longer lead category, London–New York longer still, and Lisbon–San Juan the longest. For each class of flights a table would be prepared showing, for each number of days or number of hours before departure time, the proportion of seats normally sold by that time on occasions when the price is equal to the standard price for that flight and traffic demand is such that the plane is just comfortably full by departure time. When sales for any flight were running ahead of this normal pattern, the tables would give premium percentages to be applied to the normal fare according to the proportions of sold and unsold seats and the time remaining to departure; and conversely, where sales for a flight were slower than the normal pattern, the table would give discounts to be applied. Thus, when an enquirer asks whether space is available, the reservations computer returns an answer giving the applicable fare for the requested flight as well as, possibly, the differing fares for some of the more likely alternatives. Obviously, such a scheme would have been difficult to implement before the advent of computerised reservation systems, but with them the difficulties, while considerable, seem surmountable.

It is an essential part of the scheme that the making of a firm reservation should be considered a final sale subject to resale if plans change, rather than the securing of an option; and it is this feature that is effective in dealing with the problems of the no-show and overbooking. In principle, no-shows would be entitled only to a return of the amount at which their ticket could be sold at the time of departure, though the severity of this sanction might be mitigated by putting a floor under the refund, either as a percentage of the amount paid or as a percentage of the standard fare, or possibly both. A floor would also have to be used as a limit to the discount at which reservations would be sold; otherwise a passenger, having originally booked at a higher fare, would be able to gain by turning in his original reservation for a refund and rebooking at the current lower fare. Marginal cost pricing principles would allow the fare to fall to very low levels when vacant seats remain on a given flight, the low fare representing then merely the cost of added fuel, meals, service, wear and tear and reduction of amenity to passengers resulting from the occupancy of another seat; but a floor substantially above this level may be desirable in practice to preserve at least some revenue for the airline. This is particularly necessary for airlines whose service includes some low density routes, where economies of scale are significant and the maintenance of a high schedule frequency is of substantial value to passengers.

The advantages of the scheme go far beyond merely dealing with the problem of overbooking; indeed, if that were all that would be accomplished it would hardly be interesting. The scheme should make it possible to achieve load factors of 0.85 up to 0.95 and possibly higher, while at the same time almost eliminating instances where a passenger urgently wanting to travel on a given flight finds that it is sold out. With this increase in load factor, average fares could be cut nearly in half. Passengers able to adjust their plans would be given an incentive to select the less heavily booked
flights, leaving more space for others on the more heavily booked flights. Those able to shift their plans on fairly short notice would be able to pick up bargains at a quarter or even less of the present fares. Those having urgent last-minute changes in their plans would be assured of having space available at some price almost always; under present practices space is frequently not available at any price, short of sub rosa inside dealings with the airline personnel.

Even with computerised reservation service in full operation, the system is not without its disadvantages, which may outweigh the advantages where the unit of sale is small or little inconvenience is caused to marginal potential customers if they are turned away at the last minute. This will apply to long-distance telephone calls or short-haul flights on routes with frequent service. But where the unit of sale is large, and customers incur trouble and expense in preparing to use the service, as in long-distance flights or flights on low-frequency routes, the benefits of the scheme should greatly outweigh the costs and disadvantages. At least for the fairly large class of passengers who travel non-stop from origin to destination on single-hop flights and who make their reservations at the point of origin, either one-way or round-trip, there is not much of a problem. The businessman who is insensitive to price can merely, as now, call up and ask for a reservation on the schedule of his choice. He will rarely need to concern himself about confirmation or about the price, since he can be very confident of being able to get the reservation and moderately confident that the impact of more flexible passengers on the reservations market will keep price differentials to a range within which they would not be likely to affect his decision. The price-sensitive shoppers and the bargain hunters can convey to the agent serving them the general nature of their preferences, upon which the agent can ascertain from the reservations centres of the various airlines concerned the current quotations for the more likely schedules and report them to the customer, who can then make his choice among the various schedules and rates. This is, indeed, a substantial increase in complexity over present practice in this relatively simple case, but the improved results would seem to make it well worth while.

Group Sales

An interesting question is what the practice should be when a reservation is requested for more than one seat at a time, as with groups travelling together. One is tempted to provide that the price quoted for the first passenger in the group should be extended to all passengers in the group, even though if the reservations were made separately but in rapid succession the price for the successive reservations would tend to rise, and to rise especially sharply if the reservations were being made close to departure time. However, in spite of the fact that this would be fairly closely analogous to the present practice of granting special low rates to groups of various kinds, it seems preferable for a number of reasons to price a group reservation as though it were a sequence of single reservations. This in general will mean that if a group wishes to travel together in the same plane, and is not large enough to warrant the use of a charter flight or to cause the airline to run an extra flight in part for the group, it will tend to pay more than if its members spread themselves over two or more flights. This result, while it may seem surprising at first blush, is entirely in accordance with the costs implicit in the situation. A large group making reservations on a single flight is substantially increasing the likelihood that individuals subse-
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sequently attempting to make reservations on that flight will find the price prohibitive and will thereby be diverted to flights which to them are less desirable. The larger the group, the greater the number of persons that will have to be diverted and the greater the average reduction for each person in the desirability of the service finally used. Similarly, the later the group reservations are made the smaller will be the number of potential subsequent reservations-makers from among whom the diverted passengers must be selected and the greater the loss in service desirability. The pricing of group reservations as if made singly thus produces prices that conform to costs, and produces appropriate incentives for distributing the group over a number of flights if this is feasible, or for making the group reservations at the earliest possible date so as to allow the necessary displacement of other passengers to take place at minimum cost to them.

Pricing on a one-by-one basis also obviates the need for measures to prevent the speculation which might occur if speculators were able to make spurious reservations for a large group based on the price of the first reservation, and then resell them at prices enhanced by the spurious demand thus created. While this could be obviated by placing special restrictions on resale of group tickets, there would remain a considerable incentive to overbook for groups.

Should Quotations be Firm Offers?

A closely related problem concerns the degree to which the prices quoted for the purpose of enabling the customer to make a selection are to be considered firm offers, even within a specified time limit. From the standpoint of good customer relations, it would seem desirable to hold these prices fixed for some reasonable time during which the customer could collect the various quotations and make up his mind. On the other hand, there would be the added administrative chore of keeping track of all these quotations, the persons to whom they were made, and the time limits applicable to them. Further, if the time period for which the quotation is to constitute a firm offer is at all substantial, this opens the possibility that an agent for a group, or a speculator, might ask for such quotations in a large number of different names simultaneously and independently, thus getting the same low price quotation for all, and then make the several purchases at these low prices, thus acquiring a substantial block of reservations at a price considerably lower than would have obtained if they had been purchased in a more normal way. This problem could be dealt with by treating each outstanding firm offer as a sale for the purpose of computing further price quotations during the period of its validity. Even this procedure would be vulnerable to manipulation, albeit one fairly costly to carry out, by maintaining a constant flurry of enquiries regarding a given flight that would maintain its price at a spuriously high level over a period of time, during which potential passengers would be diverted by the high price to other flights, and then allowing the outstanding quotations to expire and picking up the manipulator's own reservations at a price depressed by reason of the diversion of passengers which he had engineered. The costs of carrying out such a manipulation, however, would probably be so great relative to the possible gains that in most cases the danger could be disregarded.

Even so, the cost of maintaining even a fairly short list of firm quotations would be considerable. The simpler procedure would be to treat quotations as made for information and guidance only and not as firm offers; when the passenger had made
his choice on the basis of this tentative set of prices he would then make a firm request for a specific reservation, possibly specifying the maximum price he was willing to pay (either the price just quoted or one somewhat higher), above which he would prefer some other flight for which he had been quoted a price. Usually, of course, the quoted price would hold. In some instances, however, as when in the interim a large block of reservations had been made, the effective price at the time of the passenger's firm request would be substantially higher than the previous quotation. If this price were higher than the maximum indicated by the passenger the reservation would not be made, and he would be asked to make another selection. Ordinarily reductions in price below the quotation would amount to only a single step, resulting from the lapse of time during which no reservations were made; in rare instances large reductions could occur in the event of a large block of cancellations. Even so, it is possible that these price variations would generate sufficient unfavourable passenger reaction to require the more costly procedure of holding quotations as firm offers for an appropriate time interval, much as at present reservations are held subject to the subsequent purchase of a ticket within a reasonable time.

Through Bookings

A more difficult problem is that of passengers wishing to make arrangements for trips involving connecting flights and complicated itineraries. For single-flight trips involving more than one hop in the same plane, it would seem easy and appropriate to adopt the straightforward procedure of deriving a through fare by adding up the currently applicable fares for the separate legs and subtracting an amount representing the saving from reduced costs of passenger handling at the intermediate stop. Connecting flights, however, especially where different airlines are involved, are more troublesome.

There is no particular difficulty, even here, if the problem is merely one of quoting a fare for an itinerary already specified: simply adding the fares for the separate flights, possibly with an arbitrary concession representing check-in and ticketing costs, would again be satisfactory (though here the costs involved in interline accounting may cancel most of the savings in check-in costs over caring for entirely separate passengers for the different legs of the trip). The problem is to provide the prospective traveller with an appropriate selection of itineraries from which to choose. Under present circumstances, with no variations in fares, it is already often difficult and time-consuming, even with the aid of quick-reference time-tables, for a travel agent to find the itinerary best suited to the needs of his client. Most travel agents, indeed, will stop considerably short of a complete search unless prodded to the point of harassment by the client.1 If each potential itinerary has a different price attached, and if even these prices are not available in a tariff but must be obtained by enquiry, the complexity is multiplied. Where times of departure and of

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1The writer's own experience includes finding, after considerable search, that, rather than any of the connections listed in the Official Airline Guide, the best after-business connection from Washington to San Francisco is via an on-line connection at Atlanta; that to have dinner in Denver and meet a class the following morning in New York is not impossible after all, but requires going via Dallas; and that the best overnight connection from Boston to Tokyo is made at Chicago, not at New York or on the West coast.
arrival are the only matters of concern, it is possible to eliminate a large number of itineraries that are "dominated" by one of those selected for consideration,\textsuperscript{2} i.e., that depart earlier and arrive later than the dominating itinerary. Where price is a consideration, however, domination \textit{strictu sensu} would require in addition that the price of the dominated itinerary be as high or higher.

Perhaps when the next generation of computers comes along it will become possible to key in the potential passenger's needs in fairly general terms, including a schedule of the value placed by the passenger on free time at the origin and free time at the destination, and possibly even of time at various alternative stopover points, and to have the computer respond with an appropriate selection of itineraries that are close to optimal in terms of these specifications. For the present, however, the best that can be hoped for is for a travel agent to formulate, with the aid of such intuitions, experience, and reference materials as he can dispose of, a number of alternative itineraries which he hopes will include one best meeting his client's needs, submit these itineraries to the reservations system to obtain quotations, and report the results to his client for selection.

Where agents are located at a distance from the nearest centre where current reservation information is maintained, and especially where more than one airline is involved, this is a fairly expensive procedure both in agents' time and in communications charges. To be sure, the present system is not without its drawbacks, many of which would be eliminated under the proposed scheme: for instance, the multitude of rates based on arbitrary criteria as to routes, length of stay, number and composition of the group, and the like. But, even with allowance for the elimination of such complexities in current tariffs, the added complexity and cost is substantial.

One possible solution would be to provide for interline tickets to be sold at fixed fares according to simplified versions of the present tariffs, or possibly to apply the responsive pricing scheme only to the originating airline portion of the trip, and to make bookings for the subsequent connections at fixed fares. It would not be possible, in general, to close off the passenger's option to proceed one leg at a time, taking his chances on the price fluctuations for the successive legs of the itinerary, but being virtually assured of not being stranded by being unable to obtain space on the continuing flight. Disparities would then occur between those booking through and those booking one leg at a time. These would be particularly awkward where a trip can be made with either on-line or interline connections, or with either a single flight or connecting flights. But the disparities would not be any greater, on the whole, than those that arise now in the application of various reduced-rate tariffs.\textsuperscript{3}

\textsuperscript{2}Procedures have been developed by the writer for generating, within reasonable limits of computer time, a complete listing of undominated itineraries between the more important origin-destination pairs, and for providing data that would facilitate an exhaustive search by an agent in the case of the more exotic trips, provided that timing is the dominant consideration. To allow for possible fare differentials, especially if these fluctuate from day to day, makes the problem vastly more complex. (The routings cited in the previous footnote, incidentally, proved to be available at no extra fare.)

\textsuperscript{3}Existing tariffs are indeed already so complex that in planning any trip involving several flights it is often worth while to obtain independent quotations through several channels. Thus on a recent trip the writer was assured when purchasing the ticket that a certain possible alteration in the itinerary would remain available at no extra charge, but when booking the change he was required to pay an additional fare. When on returning home he asked about a possible refund of this overcharge, the eventual advice was that the fare ought to have been even higher than what had been paid, and that the traveller had better leave well enough alone!
POTENTIAL GAINS FROM EFFICIENT PRICING

Whatever solution is adopted for these ancillary problems, an efficient pricing technique for the bulk of the airline traffic would yield substantial returns, far outweighing any possible costs of the procedure. Indeed, rough calculations based on assumptions that seem not too far from the facts indicate that this potential gain might be of the order of 55 per cent of gross revenues earned in those operations to which the scheme would be applicable. Even if the assumptions on which this estimate is based are considered grossly wide of the mark, the benefits on any reasonable appraisal of the system amply justify at least experiments in the more favourable segments of the airline travel market.

To be sure, this potential 55 per cent gain is to be compared, not with the rather unsatisfactory status quo, but with what might be attained with a comparable degree of astuteness applied to attacking the problem of unsold seats along more conventional lines, involving discriminatory promotional fares of various kinds designed to attract off-peak traffic, combined with reduction in excess capacity where this can be accomplished without undue reduction in effective frequency. Here, however, there are fairly severe limits on what is practicable. Given complete freedom to adjust fares on a flight-by-flight and leg-by-leg basis, the airlines could in principle come fairly close to full utilisation of capacity with fares fixed for each schedule on the basis of projections made, say, three to six months in advance. This might well increase substantially the frequency with which potential passengers would be unable to find space at any price on the schedule that suited their needs; almost certainly there would be little or no improvement in this respect over present conditions. In practice the exigencies of tariff specification within a manageable scope of words usually result in substantial compromise, and it is often difficult to frame a promotional tariff in terms comprehensible to the customers which will fill otherwise empty seats without adding to peak loads or unduly impairing revenues from peak traffic. Opinion within the industry seems to be that load factors of 60 to 65 per cent would represent a highly satisfactory and possibly ideal level of achievement under present practices, and that load factors could not be pushed much if any above 70 per cent without serious deterioration in the availability of service. While such an improvement would involve a potential gain of 10 per cent or perhaps 20 per cent on current gross revenues in terms of social efficiency, the balance of gain attainable from going all the way to a “responsive” pricing system would seem to be well worth the effort, since the gain over the best that would otherwise be available could well amount to the equivalent of 25 per cent of gross revenues.

EFFICIENT PRICING AND COMPETITION

Indeed, the main real obstacle to such a scheme may be, not doubts about its impact on efficiency, but fears of its consequences on the competitive relations between airlines. At present the industry operates largely as a form of cartel, with rates kept at a uniform level by agreement, leaving only various forms of “non-price” competi-

4See Appendix.

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tion. Attempts have occasionally been made to limit even some of the aspects of the non-price competition, as in the case of the famous “sandwich war”. The present proposal would tend to bring the industry much closer to the competitive ideal. Even if an agreement were reached over the way in which the price would be made to depend on the state of reservations at a given time, it would be much more difficult to ensure that the various airlines adhered to such a convention, whether formally subscribed to or merely established as the conventional norm. Even if there were no deviation from the established rate function, competition in the introduction of new schedules would tend to drive rates and profits down to levels where only normal profits were earned; since introducing a new schedule would almost automatically fill it, albeit at somewhat reduced rates, less risk would be involved than when a new schedule can attract traffic only in terms of non-price parameters. Cartel action would then, to be effective, have to turn to regulating frequencies and possibly timings of flights. It may be hoped, however, that the effect would be to turn competition towards efficiency and reduce excessive competition in frills, unneeded schedule frequency, and the like.

**Regulation of Private Monopoly and the Use of Escrow Funds**

Routes served exclusively by nationalised airlines would cause no special difficulty, assuming that the management was reasonably well motivated to act in the public interest, or at least to maximise the level of service (within whatever financial restraints were imposed) rather than to maximise an accounting profit or surplus. Regulated private airlines with a route monopoly, however, are a different matter, since, even if the regulatory authority were to keep control over the rate formula, the airline by cutting back service would push up the average level of effective rates. This difficulty can be handled by establishing, in addition to the reactive rate charged to the customer according to the level of demand, a retention rate which would determine the amount that the airline would be entitled to retain for each revenue passenger carried. Any excess of the reactive rate over the retention rate would be paid into an escrow fund, and whenever the reactive rate was below the retention rate the airline would be entitled to withdraw the difference from the escrow fund. Failure of the airline to expand its operations to the point justified by demand would merely lead to an accumulation in the escrow fund, which the airline company could realise only by expanding operations so as to drive the reactive rates below the retention rate.

**Impacts on Unregulated Monopoly**

The problem of curbing the monopoly power of private companies is thus not insuperable. Even so, it is probably not entirely academic to ask what would be the effect of permitting an unconstrained monopolist to use such a pricing scheme rather than limiting him to the use of a uniform, non-discriminatory rate. Unfortunately not much can be said on this in general terms. One can, of course, assert with confidence that opening up a new option to the monopolist can hardly hurt him, and is likely to work substantially to his advantage. But whether the monopolist’s reaction would be such as to impose an additional burden on consumers greater or less than the gain to the monopolist, or even to confer a benefit on consumers, would depend on circumstances.
At one extreme, consider a case where the off-peak demand has a sharp downward kink, and where its duration relative to that of the peak is sufficient to make the location of this kink decisive for the price the monopolist will find most profitable if prices are uniform. Suppose too that the peak demand is nevertheless large enough relative to the cost of expanding capacity to induce the single-price monopolist to expand his capacity sufficiently to take care of the peak demand at the price determined by the kink in the off-peak demand. If the monopolist is allowed to discriminate in these circumstances he will keep the off-peak price the same, and, if the peak demand is not too elastic, raise the peak price and cut back on capacity – a reaction that will raise rates to peak users by more than the resulting gain to the monopolist, and thus constitute a move away from Pareto-optimality. At the other extreme, if we consider the same case except that peak demand is sufficiently elastic for marginal revenue from expanding peak sales to exceed the cost of expanding capacity, the monopolist will find it to his advantage to reduce the peak price below the off-peak price and to expand his capacity. In this somewhat paradoxical case both the peak users and the monopolist benefit, while the situation of the off-peak users remains unchanged. Other examples can doubtless be constructed, not involving such blatantly paradoxical results, in which all parties benefit by the greater freedom of action accorded to the monopolist. And there are also possibilities that the monopolist may gain substantially at the cost of a relatively minor loss to the customers.

On the whole, however, the likelihood of cases arising in which the gain to the monopolist is obtained only at the cost of excessively high burdens to the customers seems sufficient to indicate great caution in allowing reactive pricing to be used by unregulated monopolists. Where there is public ownership, however, or where regulation or competition can be effective in keeping prices down to levels yielding only normal profits, reactive prices provide an opportunity for substantial improvement of the efficiency of utilisation of the service offered by the airlines. This is especially true of the longer flights on routes with a volume of traffic sufficient to support a high frequency of service.

Low Frequency Routes

Routes where frequency is low present another problem. It is possible that the level of service that could be financed by reactive prices at levels that would bring about full utilisation of the service would involve either an unsatisfactory frequency of schedules or the use of low-capacity aircraft with a relatively high cost per seat-mile. Since this is essentially a case where there are substantial economies of scale, with overall marginal cost substantially below overall average cost, one appropriate remedy would be to subsidise the operation, either from general revenues or from excess revenues earned on routes of heavier density where the economies of scale are substantially smaller. If subvention is not possible, a second best solution will be to arrange the rates so that traffic in each time period is some uniform fraction of what it would have been if a subsidy had been available sufficient to defray the intra-marginal residue of cost and permit prices to be set equal to marginal cost [8]. A practical rule-of-thumb approximation to this would be to establish a level of service that appears to be optimal, given the constraints, and then place a floor under the rates, sufficiently above the marginal cost of having a passenger occupy an otherwise empty seat to permit the revenue constraint to be met. It is of course
possible, though unlikely, that following this prescription would result in so slight a variation of rates as to make the whole procedure hardly worth while. In any case the establishment of the optimum level of service would involve the exercise of considerable judgement in appraising the relevant elasticities and cross-elasticities of demand, and complete accuracy is not to be hoped for. But in appropriately selected situations reactive pricing can yield very high dividends.

APPENDIX

Estimating the gain from responsive pricing

Let

\[ Q_o = \text{total current available seat miles}, \]
\[ Q_1 = \text{total available seat miles under responsive pricing}, \]
\[ s = \text{incremental cost of accommodating passengers in existing space}, \]
\[ c = \text{cost per seat mile, exclusive of incremental passenger costs } s, \]
\[ P_o = \text{current revenue per passenger mile}, \]
\[ i = \text{index of flight leg classes, classified according to load factor}, \]
\[ f_i = \text{average load factor of flight legs in the } i\text{th class, currently}, \]
\[ w_i = \text{proportion of total available seat miles in the } i\text{th class}, \]
\[ P_i = \text{responsive price to be charged for the flight legs in the } i\text{th class}, \]
\[ x_i(p) = \text{demand for travel on flight legs in the } i\text{th class at price } p, \]
\[ \epsilon = \text{elasticity of demand, assumed uniform for all classes of flight legs,} \]
\[ F = \sum f_i w_i = \text{overall average load factor}. \]

Now if current revenues are just sufficient to cover current costs, we have

\[ \sum P_o w_i f_i Q_o = c Q_o + s \sum w_i f_i Q_o, \quad \text{or} \]
\[ (P_o - s) \sum w_i f_i = c. \quad (1) \]

The constant elasticity demand curve for the \( i \text{th class can be written} \)

\[ x_i(p) = w_i f_i Q_o \left[ \frac{P_o}{p} \right]^\epsilon, \quad (2) \]

and the new price \( p_i \) which will result in filling the available space in the \( i \text{th class of flight legs will be given by} \)

\[ x_i(P_i) = w_i Q_1 = w_i f_i Q_o \left[ \frac{P_o}{P_i} \right]^\epsilon, \quad (3) \]

it being assumed that the space offered in all classes of flight legs is reduced uniformly by the ratio \( Q_1/Q_o \). Solving (3) for \( P_i \) we get

\[ P_i = P_o \left[ \frac{Q_o f_i Q_1}{Q_1} \right]^\frac{1}{\epsilon}. \quad (4) \]

If revenues are to be sufficient to cover the cost under the new regime, the (uniform) contraction in the volume of space offered must be such as to satisfy

\[ \sum P_i w_i Q_1 = c Q_1 + s Q_1, \quad \text{or} \]
\[ \sum P_i \left[ \frac{Q_o f_i}{Q_1} \right]^\epsilon w_i = c + s. \quad (5) \]

Solving for \( Q_1 \), we get
\[ Q_1 = Q_o \left[ \frac{P_o}{\epsilon + s} \right]^I \left[ \Sigma f_i \frac{1}{w_i} \right]^I. \] (6)

The gain in consumers’ surplus resulting from lowering prices from the uniform \( P_o \) to the various levels \( P_i \) can be expressed as

\[ \Delta S = \frac{1}{1 - \epsilon} Q_o P_o \left( P_o - P_i \right) \left( \Sigma w_i f_i \right) \]

\[ = \frac{1}{1 - \epsilon} Q_o P_o \left( P_o^{1\epsilon} - P_i^{1\epsilon} \right). \] (7a)

If we put \( F = \Sigma w_i f_i \) for the overall average current load factor, and \( R = Q_o P_o F \) for total current revenues, we have, using (4),

\[ \Delta S = \frac{1}{1 - \epsilon} Q_o \left( F^{1\epsilon} - \left( \frac{P_o}{P_o} \right)^{1\epsilon} \right) \]

\[ = \frac{1}{1 - \epsilon} Q_o P_o \left( 1 - \left( \frac{Q_o f_i}{Q_i} \right)^{1\epsilon} \right) \]

\[ = \frac{1}{1 - \epsilon} Q_o P_o \left[ F - \frac{Q_o}{Q_i} \left( \frac{Q_o f_i}{Q_i} \right)^{1\epsilon} \right] \] (8b)

and using (5a) and (6), this becomes

\[ \Delta S = R \left( \frac{1}{1 - \epsilon} \left[ 1 - \left( \frac{\epsilon + s}{P_o} \right)^{1\epsilon} \left( \Sigma f_i^{1\epsilon} w_i \right)^{1\epsilon} \left( \frac{1}{F} \right) \right] \right) \] (9)

For the case where \( \epsilon = 1 \), the above expression is indeterminate, and the corresponding solution is

\[ \Delta S = R \left[ \log \frac{P_o}{\epsilon + s} + \frac{1}{F} (F \log F - \Sigma w_i f_i \log f_i) \right], \] (10)

where the logarithms are natural.

Evaluation of the above formula, using data supposedly representative of recent experience on the domestic routes of a U.S. carrier, suggests \( \Delta S = 0.55R \). This figure was obtained on the basis of assuming \( \epsilon = \frac{1}{4}, \) a figure somewhere near the lower end of the range of estimates produced by industry sources, and probably a very conservative figure. The average load factor for given flight numbers over a two-week period in October 1970 was distributed as follows: 1.4 per cent of the flight numbers averaged under 0.10 load factor, 7.1 per cent between 0.10 and 0.20, and for subsequent load factor classes, frequencies of 0.166, 0.226, 0.190, 0.144, 0.0106, 0.051, 0.031, and finally 0.001 for the flights averaging over 0.90 load factor for the fourteenth-day period. This yields an average load factor of \( F = 0.43 \), and a value of 0.45 for the expression \( (\Sigma f_i^{1\epsilon} w_i)^{1\epsilon} \), which is fairly insensitive to moderate changes in the dispersion of load factors about a given mean. Thus, even though data by individual flights would presumably have given a somewhat wider dispersion of load factors, this would not greatly affect the value of \( \Delta S \). Ticketing and passenger service expenditures are estimated at 18 per cent of the total; but, since much of this expense is determined in terms of peak traffic conditions, only about 10 per cent out
of this 18 per cent is estimated to constitute net incremental cost of carrying passengers in available seats, so we can put \( s/P_o = 0.10 \), and \( e/P_o = 0.9 \). \( F = 0.387 \), and thus obtain the above result.

If on the other hand all planes were loaded uniformly to 43 per cent of capacity, the two rightmost parentheses cancel, and we have \( \Delta S = 0.63R \). At the other extreme, if we start from a situation in which the 0.43 load factor results from 43 per cent of all flights being full and the rest empty, we get \( \Delta S = -0.135R \). This result reflects the fact that if the empty flights are filled by charging a zero price the value of the service obtained will be less than the cost of the extra service provided, while meeting these costs would entail raising fares on the full flights and cutting back on the number of flights provided.

A load factor of 0.43 may be considered unusually low, and is well below what would be aimed at even without responsive pricing. If we start from a load factor of 0.70 (perhaps the highest level that one could maintain with satisfactory service without responsive pricing) and assume that that 0.70 is the result of a distribution of load factors uniform over the range from 0.40 to 1.00 (which might be fairly representative of what could be expected under these conditions), we obtain \( \Delta S = 0.25R \).

From this benefit something might, to be sure, be subtracted on the ground that increased loading and reduced frequency may somewhat impair the value of the service, though this would seem to be adequately taken care of by assuming a low value of elasticity. Possibly one might on this account want to use a value for \( s/P_o \) greater than 0.10. On the other hand, no allowance is made in these estimates for cross-elasticity: the elasticity values are in terms of a uniform change in all prices, which presumably involves shifts between travelling and not travelling by air at all. In addition to these shifts there would be shifts of travel time from peak to off-peak times. This shifting will generate substantial additional benefits over and above those that are in principle included in the above formulas; but there is no easy way of evaluating these additional benefits, particularly in the absence of experience with responsive pricing there is little data available by which the appropriate cross-elasticities could be estimated. Thus the estimate of \( \Delta S = 0.25R \) can be considered a rock-bottom figure, and the actual gain from shifting to responsive pricing would almost certainly be substantially greater than this.

REFERENCES


*Columbia University*