A NETWORK ANALYSIS OF AIRPORT ACCESSIBILITY IN SOUTH HAMPSHIRE

By Harvey W. Armstrong

INTRODUCTION

Transport costs can play an important role in influencing the location and the economic viability of many forms of activity. For some activities (for example, retail trade) the influence of accessibility, through transport costs, on the site of the most beneficial location can be great; for others (for example, high value-added manufacturing) it is much smaller.

In reality the location of transport facilities is merely a special case of the location of activity in general, and the planning of transport merely a special case of land-use planning in general. The importance of accessibility in the location of transport facilities (as shown by the relative importance of time savings in cost-benefit studies of transport plans) simply reflects the special nature of the good produced (i.e., movement over space). Thus the quality of the transport good produced by, say, an airport depends crucially on access of consumers to that good (surface access), because of the intrinsic nature of the good (its perishability).

In the economic evaluation of the location of airports, excessive expense has restricted detailed analyses of the forces affecting location by Cost Benefit Analysis (C.B.A.) to a very small number of sites. This has meant that some form of "sieving" of potential sites, based on preliminary analyses of the major forces at work, must precede the C.B.A. "Sieving" has taken many forms, including "sieve maps" and (e.g., Roskill Commission) very simplified C.B.A. of a "long list" of sites, looking briefly at only the major locational forces.

There is an obvious need for improvement:

(i) in the techniques of analysis of the major locational forces such as surface accessibility;

(ii) in the aids to "sieving" of sites.

This analysis is intended as a contribution to these two areas. The Isochrone Map (utilising equal time, distance or cost contours) has been widely used, both in the preliminary analysis of surface access to airport sites and as an aid to "sieving". Isochrone mapping in this context suffers several disadvantages:

(a) Individual friction-of-distance contours of each potential airport site, like other network measures of accessibility [1], reflect only the quality of the private and public transport networks and miss the second major element in surface access, namely, the uneven spatial distribution of users. For instance, an isochrone map study of road access in England would probably show the most accessible site to be somewhere in the Midlands, whereas the concentration of potential air users in the south east would tend to shift the most accessible site to the south. This problem can be overcome to some extent by superimposing isochrone maps on to population maps and visually comparing the two ([2], page 118).
(b) Isochrone mapping is time consuming when a large number of potential sites are to be considered, which is the purpose in the first place of "sieving".

Preliminary C.B.A. of a "long list" of sites suffers the same disadvantages as other methods: there is a direct trade-off between the cost (in time and money) of the analysis on the one hand, and the quality of the preliminary C.B.A. on the other hand. This militates against a maximum number of sites being considered in the analysis and "sieving".

Accessibility studies are capable of considerable refinement when the road system is looked on as a binary system. The roads themselves form the links, while airport sites and city centres, as well as crossroads, form the other element, the nodes of the network. Drawn out in this form, the whole system is reduced to a linear graph and is then amenable to graph theoretic techniques using matrix algebra. The purpose of operations performed upon the graph is to build up a more accurate series of "accessibility surfaces" in which the links (sometimes called "edges" or "arcs") or the nodes ("vertices") can be weighted by considering not only road travel times but the importance of link and node in terms of their use by adjacent potential airport users. In this study, unlike some others, weighting was in fact applied to the links.

Data requirements for this analysis are rather severe, and data availability was a constraint upon the analysis. This was true not only for the detailed layout of future road nets, but more particularly for the future population estimates needed for the weighting. For this reason this study uses the 1966 data, which is available down to the level of enumeration districts for population and matches the detail available for the road network. Furthermore, the area covered was restricted to the South Hampshire region. The analysis itself was undertaken in association with the South Hampshire Airports Study [3]—hence the data sources used. But even on regional airports data was poor, and it was lacking especially on the large populations living in Bournemouth and East Dorset areas within the regional airports catchment area. This means that accessibility findings of this study refer only to access to the South Hampshire region as defined. This puts Hurn, one of the sites studied, at a disadvantage, since it is associated most closely with these western population centres. It would have been desirable to extend our analysis to cover planned networks and population distributions for 1980, one of the projection dates of the study, but data was not available in sufficient detail for this exercise.

METHODOLOGY AND LAYOUT

Some reduction in the total road network was required in order to produce a matrix that could be handled by the C.D.C. 6600 computer in the time available. Thus all minor roads were excluded, leaving the A and B roads as the network employed. This net was reduced to a graph as follows:

(i) *Nodes* were defined as all junctions on the A and B road system, all airport sites under review, and road entrances and exists to the study region (external cordon).

(ii) *Links* were defined as all A and B roads connecting nodes in the region.

The resulting graph has 156 nodes and formed the basis for succeeding operations. A map of the study area and graph is shown in Figure 1, page 302.
Stage 1

The preliminary analysis was made with a smaller graph, consisting of 89 nodes and weighted for road quality only. This analysis helped to prove the method and also provided a check on the operations performed on the bigger 156 node matrix. In fact the latter proved too large for all operations to be completed in the time available. The 89 node graph was formed as follows:

(i) Any isolated node pair less than 0.8 minutes apart was coalesced to form a single central node.

(ii) Closely spaced city nodes were reduced to a single node representing the central built-up core of the city. This reduces detailed accessibility in these areas with respect to the airports; but this is probably not so critical as might be imagined, since the time elements involved are relatively small, and also because airports are unlikely to be sited in city centres where the distortion will be greatest. For Portsmouth a second node represents the airport, while in Southampton the “city node” covers the dense set of junctions of the peninsula between the rivers and is supported by nodes on the outer city periphery. The resulting nodes are shown in Figures 2 and 4.

The graph was then used as a basis to construct an 89 by 89 matrix using the nodes as row and column headings. Each cell could then be used to store information with respect to the two nodes it referred to in the matrix. Two series of operations were carried out.

In the simpler case, no weighting was used on the links and only the presence or absence of a direct link between each pair of nodes plotted. The result is a “connectivity matrix” $C^1$, which is binary, i.e. one/zero in form. A powering procedure (see Appendix: Stage 1A) allows a shortest path matrix to be derived from the connectivity matrix $C^1$. Cells in such a shortest path matrix show the number of links between each pair of nodes $(ij)$ by the shortest possible route (shortest path) through the network. Summing the rows (or columns) of this matrix gives an individual measure for each node of accessibility by shortest paths through the network to all other nodes. The node values may be plotted on a map and accessibility contours interpolated from them, giving a final representation of the “accessibility surface”. This is shown in Figure 4, and it should be remembered that low values indicate high accessibility to other points in the network.

The second series of operations represents one step beyond the “shortest path” matrix discussed above. Here the basic connectivity matrix $C^1$ not only recorded direct links but gave such links a weighting depending upon the time taken to travel each link. The road speeds used for the time values were derived from Road Research Laboratory data and combined with link length to give the time estimate.

A powering procedure of the weighted connectivity matrix $C^1$ similar to that used for the shortest path procedure produces a final matrix, $T$ (see Appendix: Stage 1 B). Sums of the columns (or rows) of this give a measure of accessibility of each node to all other nodes in the network as before, but this time weighted by the travel time values used. Again the node values may be plotted and contours drawn to give an accessibility surface, as shown in Figure 2. Note that in this case higher values will indicate more accessible nodes. The use of contours assumes that there is a uniform gradient between nodes; this is likely unless minor roads not included in the analysis render the gradient uneven.
Stage 2

The second stage comprised the analysis of the full 156 node graph. In addition to the finer mesh of nodes, an attempt was made to weight the links even further by including not only road times but also population location on the network.

To make the population location element amenable to analysis, “catchment areas” for each link were first established (Figure 1). All points within a link “catchment area” are closer to that link (in time) than to any other, using all roads as channels of movement. Watersheds will therefore be points equidistant from two links, and not, as is more usual, from two nodes. This assumes that people will travel directly to and from the links (A and B roads), utilising minor roads only as feeders to the A and B roads.

Within the link catchment area it was now necessary to establish the population of “air users”, or rather potential air users, in order to obtain the weighting for the link. Basic population data for this must be detailed, and preferably available to the level of enumeration districts (E.D.s). For this reason the 1966 sample census was used, since no later estimates as fine as this were available. To obtain the “air user potential” would require knowledge of the socio-economic factors affecting the propensity to fly in the population of each E.D. The closest “mesh” available in the census covers the larger urban and rural district units, and even here household income is excluded.

The Roskill Commission [4] findings on the propensity to fly on business trips were utilised, together with occupation group data from the 1966 census, to give estimates of potential business air users at the urban and rural district level. Scaling down to E.D. level was estimated from existing population distributions within the cells. For potential non-business air users the Family Expenditure Survey of 1966 provided figures which were used to derive the percentage of the population in each income group, and this was then applied to the E.D. data. Similarly, the Roskill Commission data on non-business propensities to fly were used for this sector of the total. Summation of business and non-business data gave a final “total air user population” for each E.D. The assumption behind these calculations is that such elements as family size and family incomes for the Hampshire area are similar to those for the “Rest of South East” estimates of the Family Expenditure Survey. Again, in the non-business sector, no account is taken of age, sex or family influences.

Thus weightings were obtained for the E.D.s, and the figures were then allocated to the link catchments. Thus for each link a travel time and an air user population value were now available. However, there may still be an uneven distribution of population within a catchment area. To overcome such lopsided internal distributions, each link was considered as two directed links (i.e., i to j and j to i), which means that a two-way road is treated as two roads, one going in each direction, and weighting for movement in each direction can be achieved.

For example—in Figure 1, the area 17–39 has the population concentrated around node 17 on the edge of Southampton. A hypothetical example is shown in Figure 5. By considering the link 10–11 as two directed links (10–11, 11–10) and weighting for movement in each direction, some compensation is achieved. Thus, let the “population potential” be:

\[ PP = \frac{P}{D} \]
where \( P \) is the population in each enumeration district assumed to act through the enumeration district centroid to the node concerned, and \( D \) is the time taken by minor road from the enumeration district centroid to that node.

Hence for node 10 (direction 11–10):

\[
PP = \frac{2000}{2} + \frac{100}{5} + \frac{10}{5} + \frac{10}{10} = 1041
\]

and for node 11 (direction 10–11):

\[
PP = \frac{10}{2} + \frac{100}{10} + \frac{100}{10} + \frac{2000}{20} = 122.
\]

Thus we have modified population weightings for each link, and the graph no longer gives a symmetrical connection matrix \( C^1 \)—it is a directed graph.

Finally, the total travel time lengths of each link, as used in Stage 1, and the population weighting are combined to give one index per link:

\[
A = \frac{PP}{t}
\]

where \( A \) is accessibility index, and \( PP \) and \( t \) are the “population potential” and time value respectively. The higher the value of \( A \) the greater the accessibility on the link, e.g., short links in densely populated areas will have high values. \( A \) values were again scaled to give a range 0–1, where 0 represents the “poorest” link with respect to the weighting, and 1 the most accessible link.

For studies where the “catchment areas” of the individual links are small relative to the area under consideration, the calculation of population potential \( (PP) \) for each link would be unnecessary, since the variation in simple air user population \( (P') \) among catchment areas would be sufficient to allow for the uneven distribution of population in the area. The weighting index for each link would then be:

\[
A = \frac{P'}{t}
\]

where \( P' \) = air user population located in each catchment area

\( t \) = time taken to traverse the link.

This formula for the link weighting to go into the connectivity matrix \( C^1 \) is bringing together the two elements of our definition of accessibility—the population being served \( (P') \) and the length and quality of the road network serving them \( (t) \). Here they enter the weighting equation with equal importance. Thus a long link serving a large population will have a similar weighting to a short link serving a small population. It is conceivable that in principle any number of weighting systems are possible. Thus if one considered for some reason that \( P' \) (or \( t \)) should be given an increased weight it would be possible to adjust the equation accordingly. The effects of such an adjustment upon the final accessibility measures are difficult to foresee without first running the technique through. Arbitrarily raising values of \( P' \) would probably raise the accessibility values of nodes close to, or within, population centres. Raising \( t \) would probably increase accessibility values of nodes at the road network centre in comparison with the results presented in Figure 3.

Powering the weighted connectivity matrix \( C^1 \) to its solution time and summing matrices again yields accessibility values for nodes to the whole region—this time reflecting road length and population location (see Appendix: Stage 2). The contour
plot of the resulting surface is shown in Figure 3—column sums being used.

It should be noted that, unlike the binary matrix showing the “shortest path” surface in Figure 4, Figures 2 and 3 indicate high accessibility where high values are indicated on the contours.

Results of the Accessibility Analysis

This section gives the results of the analysis made upon both the 89-node and 156-node graphs.

The 89 graph accessibility surface

Contour spacing on Figure 2 is at intervals of 200 million units and it can be seen that the range is relatively large, running from 1.86 million for Node 1 to 2.160 million for Node 48. A sharp peak in accessibility occurs around nodes 48 and 50 on the eastern outskirts of Southampton. This peak is, as might be expected, fairly close to the geographical centre of the road network studied, although with a significant western skew.

The roads in the Southampton–Romsey areas, on the north side of the city itself, produce a zone of high surface accessibility along the A27 and A35 and extending from Southampton towards Romsey along the A3057 and A27. Congestion, which enters the study via the road speed estimates, is almost certainly the background to the low values in Southampton itself and in the western approaches through Totton. Accessibility declines eastward from the peak at nodes 48–50 along the A27 from Southampton towards Portsmouth, but rises in the less congested areas north of the island on the line running through Wickham and Havant—again the congestion factor in Portsmouth is reflected here.

The value of this analysis is that it throws up sites for which more detailed (and perhaps even cost/benefit) studies may be worth while on the basis of surface access. This particular 89 node graph gives only a partial view, however, as there are no population weightings (see later section on accessibility on the 156 surface).

The analysis can also be used to present various other pieces of useful information in airport planning. It can be used to rank potential airport sites with respect to network accessibility. For example, in Table 1, Eastleigh is seen as the most accessible of four airport nodes to the South Hampshire planning region.

<table>
<thead>
<tr>
<th>Airport Node</th>
<th>Accessibility Index (Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33 Eastleigh</td>
<td>305.0</td>
</tr>
<tr>
<td>62 Lee</td>
<td>98.4</td>
</tr>
<tr>
<td>76 Portsmouth</td>
<td>39.6</td>
</tr>
<tr>
<td>89 Hurn</td>
<td>3.8</td>
</tr>
</tbody>
</table>

From individual cells in the \( T \) matrix other useful data can be extracted to show:

(a) which airport site would be best located for links with other airports in a multiple airport system (Tables 2, 3 and 4);

(b) how single airport or multiple airport combinations stand with regard to access to particular areas in the region. The most interesting point here is the accessibility of different sites to city centres (Table 5).
TABLE 2

Airport Site Accessibility

<table>
<thead>
<tr>
<th>Node</th>
<th>Access to other three Airports (000 units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>Eastleigh 648-4</td>
</tr>
<tr>
<td>62</td>
<td>Lee 766-0</td>
</tr>
<tr>
<td>76</td>
<td>Portsmouth 307-4</td>
</tr>
<tr>
<td>89</td>
<td>Hurn 55-4</td>
</tr>
</tbody>
</table>

Lee is the central site in a 4-airport system.

TABLE 3

Inter-Site Access between Two Airports

<table>
<thead>
<tr>
<th>Node Combination</th>
<th>Access Sum (000 units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastleigh + Lee</td>
<td>526-0</td>
</tr>
<tr>
<td>Eastleigh + Hurn</td>
<td>55-0</td>
</tr>
<tr>
<td>Eastleigh + Portsmouth</td>
<td>67-4</td>
</tr>
<tr>
<td>Portsmouth + Hurn</td>
<td>0-014</td>
</tr>
<tr>
<td>Portsmouth + Lee</td>
<td>240-0</td>
</tr>
<tr>
<td>Lee + Hurn</td>
<td>0-333</td>
</tr>
</tbody>
</table>

Eastleigh + Lee is the best combination.

TABLE 4

Three-Airport Groups

<table>
<thead>
<tr>
<th>Node Combination</th>
<th>Access Sum (000 units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portsmouth + Eastleigh + Lee</td>
<td>1,666-8</td>
</tr>
<tr>
<td>Hurn + Lee + Portsmouth</td>
<td>480-7</td>
</tr>
<tr>
<td>Eastleigh + Lee + Hurn</td>
<td>1,162-7</td>
</tr>
</tbody>
</table>

Portsmouth + Lee + Eastleigh is the best group, closely followed by Eastleigh + Lee + Hurn.

TABLE 5

Accessibility to City Centres

<table>
<thead>
<tr>
<th>Airport</th>
<th>Accessibility Index (million units)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Southampton (Node 15)</td>
</tr>
<tr>
<td>33 Eastleigh</td>
<td>8290-0</td>
</tr>
<tr>
<td>62 Lee</td>
<td>468-0</td>
</tr>
<tr>
<td>76 Portsmouth</td>
<td>7-86</td>
</tr>
<tr>
<td>89 Hurn</td>
<td>120-0</td>
</tr>
</tbody>
</table>

Each city airport (Southampton and Portsmouth) is best situated for its local city centre. Lee is perhaps the best situated with regard to both city centres.

The Shortest Path Matrix

It is of some interest to consider the simple binary matrix that results at this point, before going on to look at the 156 matrix. In this shortest path (see Stage 1) case redundant paths (i.e. non-direct paths) through the graph between nodes are omitted. A shortest path final summed matrix $T$ was derived for the 89 node graph,
but only with the initial connectivity matrix \( C \) being treated as a binary matrix (presence of a link = 1; absence = 0). The results were plotted in Figure 4 as a surface again. Here low values show high accessibility. Comparison of Figure 4 with the 89 graph surface weighted for road times (Figure 2) shows that the strong peak of accessibility of the latter to the east of Southampton is replaced by a less intensive zone in the Eastleigh area; but otherwise the pattern is similar. The problem of how important redundant or non-shortest paths between nodes are is quite a difficult one in this type of analysis.

For comparison, airport rankings using the shortest-path matrix are shown in Table 6. The results are broadly similar to the weighted 89-graph, except that Hurn now ranks slightly higher than Portsmouth in value.

### TABLE 6

<table>
<thead>
<tr>
<th>Node</th>
<th>Column/Row Sums</th>
</tr>
</thead>
<tbody>
<tr>
<td>33 Eastleigh</td>
<td>487</td>
</tr>
<tr>
<td>62 Lee</td>
<td>642</td>
</tr>
<tr>
<td>76 Portsmouth</td>
<td>780</td>
</tr>
<tr>
<td>89 Hurn</td>
<td>765</td>
</tr>
</tbody>
</table>

### Accessibility on the 156 Surface

Figure 3 sets out the accessibility surface for the 156 node weighted graph. Table 7 gives the ranking of the four airport sites in terms of accessibility on the road network to the existing air user population.

### TABLE 7

<table>
<thead>
<tr>
<th></th>
<th>Column Total (Terminal Point)</th>
<th>Row Total (Origin Point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>53  Eastleigh</td>
<td>0.197</td>
<td>0.016</td>
</tr>
<tr>
<td>94  Lee</td>
<td>0.038</td>
<td>0.000000884</td>
</tr>
<tr>
<td>143 Portsmouth</td>
<td>0.00000332</td>
<td>0.089</td>
</tr>
<tr>
<td>142 Hurn</td>
<td>0.00000106</td>
<td>0.00000117</td>
</tr>
</tbody>
</table>

(Strictly, four noughts should be added after the decimal point in each case.)

Figure 3 indicates that inclusion of an air-user population weighting, together with a more complex graph, has altered the pattern considerably from that obtained with the 89-node graph. The two population centres of Portsmouth and Southampton, at either extreme of the region, seem to have split the 89-node single peak (see Figures 2 and 3) into twin peaks at eastern and western peripheries of the region. The peaks lie in the semi-rural fringes of the two cities. The troughs lie in the cities of Portsmouth and Southampton, more particularly in the heavily built-up zones. This results from two attributes in the accessibility equation:

(i) Urban congestion enters via road speed assumptions; this puts the most accessible points on the fringes and to the north of the two cities.

(ii) Air user population tends to be in higher income population groups, whose residential areas lie on the city peripheries rather than at city centres.

The central trough along B3035 (between nodes 92 and 96 on Figure 1) contrasts starkly with the higher values here on the 89-node graph. This area has low population, and so, in spite of being central to the road graph, is reduced in importance in
the 156-graph. From Table 7 it is apparent that Eastleigh is again the key terminal point of the four, but is replaced by Portsmouth as the key origin. This is quite unexpected, and the only explanation that suggests itself seems to be that, under the wider assumptions of the analysis in this graph, Portsmouth Airport becomes easily accessible to the large population of Portsea Island, and this overshadows the road quality rating. Note too that the population weighting brings Hurn ahead of Lee. The overall peak of the surface is around Romsey, to the north west of Southampton.

Similar extractions can also be made for airport-to-city-centre and airport-airport accessibility, but there are some problems, as Hurn was not reached completely by the \( C^{15} \) matrix. Also there is something of a problem in interpretation, as these values from the 156 matrix are not just road access, but also include population weightings. Airport-to-city-centre examples are shown in Table 8.

<table>
<thead>
<tr>
<th>Airport Node</th>
<th>Southampton City Centre</th>
<th>Portsmouth City Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>53 Eastleigh</td>
<td>*354 ( \times 10^{-15} )</td>
<td>Not reached</td>
</tr>
<tr>
<td>94 Lee</td>
<td>180 ( \times 10^{-16} )</td>
<td>260 ( \times 10^{-17} )</td>
</tr>
<tr>
<td>143 Portsmouth</td>
<td>Not reached</td>
<td>228 ( \times 10^{-13} )*</td>
</tr>
<tr>
<td>142 Hurn</td>
<td>177 ( \times 10^{-19} )</td>
<td>Not reached</td>
</tr>
</tbody>
</table>

**Table 8**

Airport to City Centre

<table>
<thead>
<tr>
<th>Airport Node</th>
<th>Southampton City Centre</th>
<th>Portsmouth City Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>53 Eastleigh</td>
<td>*223 ( \times 10^{-14} )</td>
<td>Not reached</td>
</tr>
<tr>
<td>94 Lee</td>
<td>392 ( \times 10^{-16} )</td>
<td>177 ( \times 10^{-17} )</td>
</tr>
<tr>
<td>143 Portsmouth</td>
<td>Not reached</td>
<td>195 ( \times 10^{-13} )</td>
</tr>
<tr>
<td>142 Hurn</td>
<td>245 ( \times 10^{-18} )</td>
<td>Not reached</td>
</tr>
</tbody>
</table>

*Maximum values.*

As expected, Eastleigh and Portsmouth are most accessible of all to their "own" city centres, with Lee again a good all-round site for both areas and Hurn behind all the others.

The Report [3] of the South Hampshire Airports Study recommended Hurn as the regional airport. The apparent contradiction of this study does not refute this finding. Firstly, this study merely looks at surface access, which is only one of many factors in airport location. Secondly, the 156 surface shows a site near Romsey as a peak access site. An extension of the study area to the Bournemouth–Poole and Dorset areas, had it been possible, would certainly have "pulled" the high access peak further west towards Hurn airport site.

It is hoped that this type of analysis will be useful as a "sieve" technique, since surface access plays such a predominant role as a location factor in cost benefit analysis.

An important conceptual problem is that of handling redundant paths in the analysis or determining their importance, and the main practical problems are those of obtaining spatially ordered data. But, in spite of difficulties, comparison of Figures 2 and 3 shows very clearly that the absence of population weighting in accessibility studies (Figure 2) can cause an assessment of the most accessible site.
FIGURE 5
HYPOTHETICAL ZONE

KEY
- ENUMERATION DISTRICT CENTROID
- (100) POTENTIAL AIR USERS
- ----- E.D. BOUNDARY
-  ➔ ROAD LINK
almost completely the reverse of a more genuine study weighted by air user location (Figure 3). This throws severe doubts on the use of isochrone maps as a “sieve” technique, and on previous accessibility studies using graph theory, which have not only failed to take account of uneven air user distribution, but have also tended to use binary graphs and matrices and so missed even road quality and length weightings.

Thanks must be extended to Dr. K. R. Sealy for invaluable help, to Laurie Baker for the indispensable computer programs and to many sources associated with the South Hampshire Airports Study [3] for data.

APPENDIX

Stage 1

(A) The Shortest Path Matrix: 89 Node Graph

The 89-node shortest path matrix is derived from the simple binary “connectivity matrix”, $C^1$, by a powering procedure. The binary connectivity matrix $C^1$ is powered and two-step connections between nodes are introduced in the matrix $C^2$. A shortest path matrix may then be derived, $D^2$, in which not only single step, but also two-step shortest paths may be indicated by filling in any new cell entries appearing in $C^2$ as two-step entries in $D^2$ (i.e. no connection = 0, one-step = 1, two-step = 2). The powering process of $C^1$ is continued, appropriate shortest paths being extracted at each stage, until all cells in the $D$ matrix are filled. The number of powerings of $C^1$ required to fill all the cells is called the solution time of the matrix or the diameter of the graph. The diameter then measures the maximum number of links in the shortest paths between each pair of nodes [1]. In this case, the diameter = 18 = $D^{18}$; i.e., the longest shortest path in the network has 18 links in it. This final matrix, $D^{18}$, records all the shortest paths between nodes in the network. Summing the rows (or columns) of this final matrix of shortest paths gives the total number of links (via shortest paths) necessary to connect each node with all others in the network. This can be regarded as a measure of accessibility of each node to the whole network.

Since the matrix is symmetrical, the sum of the matrix rows equals the sum of the matrix columns, and accessibility of each node to the whole network is the same whether nodes are taken as originating (row) or terminating (column) points in the network.

(B) Weighted Matrix: 89 Node Graph

The values in the $C^1$ matrix were not binary in this case, but weighted values reflecting time required to travel each link. To make this information amenable to computation the time estimates were scaled relative to zero, using the index:

$$A'_y = 1.00 - \frac{A_y}{A_{y,max} + 1.00}$$

where

- $A'_y$ = time estimates scaled relative to zero,
- $A_y$ = access time of node $i$ from node $j$,
- $A_{y,max}$ = time taken to traverse the most inaccessible (i.e. longest) link in the graph.
Hence longer links would approach zero, and shorter link routes would appear in the range 0 to 1, the shortest of all approaching 1.00.

The 89 node graph was again taken as undirected, i.e. all links can be traversed in either direction, and the matrix \( C^1 \) would therefore again be symmetrical. The introduction of time weighting replaced the binary form by a value matrix \([5]\); and, since powering of this matrix assigns a greater value to direct than to indirect connections, an automatic time decay function is built into it.

The matrix was then powered up to its diameter as before, producing a final matrix \( C^{18} \). The summation of the matrices \( C^1 \) to \( C^{18} \) gives the final matrix \( T \), and, since it is symmetrical, row and column totals are equal. Elements in \( T \) include:

(a) Number of direct links between node pairs \((i,j)\),
(b) Number of path sequences between \( i \) and \( j \),
(c) Time taken to traverse (a) and (b).

Stage 2

*Weighted Matrix: 156 Node Graph*

As with the 89 node graph, a preliminary connection matrix \( C^1 \) was constructed and then powered. In this instance, however, for technical reasons it was not possible to continue the operation until the solution time was reached, and the powering was stopped when the \( C^{12} \) matrix was reached.

In this case, since \( C^1 \) was not symmetrical:

(a) Column sums give access of each node to every other node as a *terminal point*;
(b) Row sums give access of each node to all others as an *origin point*.

Figure 3 gives the column (terminal) sums of the matrix \( C^{15} \), and it would be equally possible to plot row (origins) sums if required. As already noted, the size of the matrix was too great for the powering process to be completed in the time available, and the results refer to the \( C^{15} \) matrix and not to \( T \), the sum of all levels. Experience with powered matrices of this order shows a tendency for patterns to stabilise after 5 to 10 iterations, and the fact that the solution time was not reached is less serious than might appear.

REFERENCES


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