DERIVED DEMAND FOR FREIGHT TRANSPORT AND INTER-MODAL COMPETITION IN CANADA

By Tae Hoon Oum*

INTRODUCTION

Many public decisions concerning taxes, subsidies and price regulation, as well as carriers' decisions on optimal pricing, require precise knowledge on demand for freight transport. Most previous studies on this subject have attempted to estimate the parameters of demand functions on the basis of various ad hoc models¹: for example, Morton [21], Antle and Haynes [2], Kullman [18], Arthur D. Little Inc. [3], Turner [31], Tcheriev et al. [29], and Hariton et al. [16]. The ad hoc demand models perform reasonably well in forecasting demand, but they have two shortcomings for investigating price responsiveness and possibilities of cross-modal competition: (i) The set of variables to include and the functional form to use for the estimation of demand model are arbitrary, yet the parameter estimates are likely to be sensitive to them. (ii) In general, neither the structures of shippers' preferences that the ad hoc demand models assume to approximate, nor the properties of approximation, are known.

On the other hand, some studies have estimated Cobb–Douglas demand models, which may be considered as derived demand models. However, the Cobb–Douglas demand model is inappropriate for multi-modal demand studies, because it places a severe restriction on the parameters of inter-modal competition.² To our knowledge, no empirical study has investigated systematically the price responsiveness and competition between various modes of freight transport on the basis of a derived demand

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²The term "ad hoc model" refers to all demand models that are specified arbitrarily by researchers without referring explicitly to shippers' production and distribution technology.

²The Cobb–Douglas demand model for factors of production is consistent only to a Cobb–Douglas production function which is dual to a Cobb–Douglas cost function. Thus the Cobb–Douglas demand model restricts the elasticity of substitution between every pair of inputs to unity.
model allowing for free variation of parameters of intermodal competition.

The purposes of this paper are to formulate a demand model for intercity freight transport as an intermediate input to the production and distribution sectors of the economy (hereafter called "economy") and to estimate the price elasticities and the elasticities of substitution between three modes of freight transport: railway, highway, and waterway carriers (excluding ocean shipping). The lag structure in shippers' response is also investigated, since it is quite possible that in the short run shippers may not respond fully to price changes because of institutional and technological rigidities or because of inertia in their behaviour.

The plan for this paper is as follows: A basic model of modal cost shares (revenue shares from the carriers' point of view) is derived in section I. In section II, alternative forms of share models are specified and nested hypotheses for testing share models are designed. The details on data construction and the sources of data are given in section III. Section IV discusses methods of estimation and results of the testing of hypotheses. In section V the empirical results are presented. Concluding remarks are given in section VI.

I. MODEL FORMULATION

In this study, the freight transport services are considered to be a subset of input factors for firms engaging in production and/or distribution of goods and services. It is assumed that there exists in the Canadian production and distribution sectors a twice continuously differentiable production function relating the gross output \( Y \) to the services of inputs: capital \( K \), labour \( L \), and freight transport \( T \). It is further assumed that: (i) the production function is linearly homogeneous (constant returns to scale), increasing and quasi-concave in the inputs; and (ii) the freight transport services \( T \) are separable from the other inputs in the production function. (Appendix B tests the separability hypothesis. The result shows that the separability is satisfied in practice.) Then the production function can be written as follows:

\[
Y = f [K, L, \overline{T}(T)]
\]

where \( T \): the economy's level of output,
\( T = R, H, W \)
\( R \): freight carried by railway carriers,
\( H \): freight carried by highway carriers,
\( W \): freight carried by waterway carriers, and \( \overline{T}(T) \) is linearly homogeneous in \( T \).

The following analysis is based upon the duality between production and cost functions, a result established originally by Shephard [27] and Samuelson [25] and refined by Uzawa [32], McFadden [19, 20], Shephard [28], and Diewert [11, 12].

\[3\] By specifying the transport cost function of the economy in a "flexible" functional form, the free variation of substitution parameters is ensured. Pipelines and air freight carriers are excluded from the analysis in this study because no significant possibilities of competition are expected to hold between any of these and the above three modes.
The duality theory implies that, if producers minimise input costs, the cost function satisfying certain regularity conditions\(^4\) contains sufficient information to describe completely the production technology. Thus, rather than specifying a functional form for the production and then solving the constrained cost minimisation problem, we can directly specify a cost function as in (2).

\[
C(Y, P_K, P_L, P_T)
\]

where \(P_K\): rental price of capital,
\(P_L\): price of labour,
\(P_T\): prices of freight transport services,
and \(C\) is linearly homogeneous in \(Y\) and satisfies the regularity conditions (footnote 4).

The homothetic separability equivalence theorem (Blackorby et al [8]) says that, if the production function is increasing and satisfies conditions for continuity, monotonicity and quasi-concavity, the homothetic separability of \(T\) from \((K, L)\) in the production function is equivalent to the separability of \(P_T\) from both \((P_K, P_L)\) and \(Y\) in the cost function \((C)\).

This allows us to construct a sectoral unit cost function\(^5\) \((C^T)\) for the freight transport input as in (3a) that is independent of both the level of gross output \((Y)\) and the prices of inputs other than transport services \((P_K, P_L)\).

\[
C^T(P_1, P_2, P_3)
\]

where \(P_1\): price of freight service by railway carriers,
\(P_2\): price of freight service by highway carriers,
and \(P_3\): price of freight service by waterway carriers.

This sectoral unit cost function \((C^T)\) still preserves the regularity properties of the aggregate cost function \((C)\).

For the purpose of estimation we need to postulate a specific functional form for the sectoral unit cost function \((C^T)\). For the study of intermodal competition, the functional form should allow for free variation of Allen partial elasticities of substitution (APES) and be sufficiently flexible to provide a valid second order approximation to an arbitrary twice differentiable cost function. Any one of the following forms may be used for this purpose:

1. mean of order \(r\) \((r = 1, 2, \ldots, n)\) cost functions (Hardy et al. [15]);
2. generalised Leontief cost function (Dievert [11]);
3. translog cost function (Christensen et al. [9, 10]);
4. square root quadratic (SRQ) cost function.

The translog function is chosen arbitrarily in this study.

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\(^4\)The regularity conditions that are required to determine uniquely the corresponding production function are that the cost function be increasing, linearly homogeneous and quasi-concave in the input prices.

\(^5\)Blackorby et al. [8] discuss in detail the homothetic separability equivalence theorem and the procedure for deriving the sectoral unit cost function.
Before specifying the sectoral unit cost function in the translog form, it is necessary to examine one other important point: that is, the effect of temporal change in technology may not be Hicks-neutral in its factor augmentation. To identify the cost function correctly, therefore, the neutrality of factor augmentation should be hypothesised and tested properly. To represent the cost function with non-neutral factor augmentation, the following form of factor augmenting price, $P_{it}^*$, is employed in place of the corresponding observed price, $P_{it}$.

$$P_{it}^* = P_{it} \cdot e^{t_i T_t}, \quad i = 1, 2, 3,$$

where $t_i$ is the parameter of factor augmenting price function for the $i$th mode ($i = 1$, railway mode; $i = 2$, highway mode, $i = 3$, waterway mode). $T_t$ is the trend variable to present the state of technology in year $t$.

For our three-mode sectoral unit cost model, the translog function without assumptions of neutrality of factor augmentation and Hicks–Samuelson symmetry may be specified as:

$$\ln C^T(P_{1t}^*, P_{2t}^*, P_{3t}^*) = \ln a_0 + \sum_{i=1}^{3} a_i \ln P_{it}^* + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} \ln P_{it}^* \ln P_{jt}^*$$

$$= \ln a_0 + \sum_{i=1}^{3} a_i (\ln P_{it} + t_i T_t) + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} (\ln P_{it} + t_i T_t) (\ln P_{jt} + t_j T_t)$$

where $P_{it}$ is the freight rate of $i$th mode in year $t$,

$T_t$ is a trend variable to represent the state of technology in year $t$,

$a_0$ is a scale factor of cost, and

$a_i$'s, $t_i$'s and $b_{ij}$'s are the parameters of the cost function.

The linear homogeneity property of our sectoral unit cost function then imposes the following restrictions on the parameters of (3b):

$$\sum_{i=1}^{3} a_i = 1, \quad \sum_{j=1}^{3} b_{ij} = 0 \quad \text{for all } i = 1, 2, 3 \quad (4)$$

and

$$\sum_{i=1}^{3} b_{ij} = 0 \quad \text{for all } j = 1, 2, 3.$$

If the output level is taken as predetermined, the firms produce the given output at minimum cost, hiring inputs at their market prices. As a result, the cost minimising input demand functions may be formed as follows.

First we logarithmically differentiate equation (3b),

$$\frac{\partial \ln C^T_t}{\partial \ln P_{it}} = \frac{\partial C^T_t}{\partial P_{it}} \cdot \frac{P_{it}}{C^T_t} = a_i + \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) \ln P_{jt} + \left[ \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) t_j T_t \right] i = 1, 2, 3.$$
and then using Hotelling–Shephard Lemma, we obtain the input demand functions as:

\[
X_{it} = \frac{\partial C^T_i}{\partial P_{it}} = \left[a_i + \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) \ln P_{jt} + \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) l_{ij} T_i \right] \cdot \frac{C^T_i}{P_{it}}
\]

(5)

Then the cost minimising expenditure share functions can be written as:

\[
S_{it} = a_i + \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) \ln P_{jt} + c_i T_i \quad i = 1, 2, 3 \quad t = 1, 2, \ldots, T.
\]

(6)

where \(c_i = \frac{1}{2} \sum_{j=1}^{3} (b_{ij} + b_{ji}) l_{ij}, \quad \sum_{i=1}^{3} c_i = 0 \) due to (4),

and \(S_{it}\) is the expenditure (revenue) share of \(i\)th mode.

Since we are estimating \(\frac{1}{2}(b_{ij} + b_{ji})\) and not \(b_{ij}\) directly, it is not possible to test the hypothesis that \(b_{ij} = b_{ji}\). In other words, the model with Hicks–Samuelson symmetry imposed cannot be distinguished by observation from the model with the symmetry not imposed.\(^6\) Therefore, the symmetry restrictions, \(b_{ij} = b_{ji}\), are to be imposed in all models that are to be estimated in this paper. This implies that the matrices of elasticities of substitution, \([\sigma_{ij}]\), are symmetric and the input demand functions are integrable over the domain of definition (Samuelson [26]).

The share functions (6) with symmetry condition imposed are in a convenient form to estimate by any multivariate regression techniques.

For translog cost function, Berndt and Wood [7] have derived the Allen partial elasticities of substitution (APES) between inputs \(i\) and \(j\), \(\sigma_{ij}\), and the partial price elasticities of input demand, \(E_{ij}\), as:

\[
\sigma_{ij} = \frac{b_{ij} - S_i}{S_i} + 1 \quad i = 1, 2, 3
\]

(7)

\[
\sigma_{ij} = \frac{b_{ij}}{S_i} + 1 \quad i, j = 1, 2, 3, \quad i \neq j
\]

\[
E_{ij} = \left(\frac{\partial X_i}{\partial P_{j/iisoquant}}\right) \cdot \frac{P_j}{X_i} = \frac{b_{ij} + S_i \cdot S_j}{S_i} = S_j \cdot \sigma_{ij} \quad i, j = 1, 2, 3.
\]

(8)

Equation (8) is in fact the price elasticities along an isoquant, and thus does not include the effect of change in the demand for the final product in response to the changes in freight rates.

However, Allen [1] has derived the price elasticities of Marshallian (ordinary) demand,\(^7\) \(F_{ij}\), that includes both the substitution effect and the effect of changes in demand for final products, as:

\(^6\)I am indebted to the anonymous referees for pointing out this analytical fact with a detailed derivation. The point was wrongly treated in my original work.

\(^7\)In this paper, the term “Marshallian (ordinary) demand”, borrowed from consumption theory, refers to the input demand when the level of shippers’ output is allowed to vary in response to changes in freight rates; it is therefore distinguished from the input demand along an isoquant.
\[ F_{ij} = \frac{d \ln X_i}{d \ln P_j} = S_j (\sigma_{ij} + \eta), \quad i, j = 1, 2, 3 \]  

where \( \eta = \left( \frac{d Y}{dp} P \right) \) is the price elasticity of demand for the shippers' products (potential cargoes).

II. STOCHASTIC SPECIFICATIONS AND ALTERNATIVE SHARE MODELS

In practice, there are errors in the adjustment to the cost minimising modal shares. Therefore, an empirical implementation requires that our share models be imbedded in a stochastic framework. Adding a disturbance term to each equation in (6) and imposing the linear homogeneity conditions (4), we obtain:

\[ S_{it} = a_i + \sum_{j=1}^{2} b_{ij} \ln \left( \frac{P_{jt}}{P_{3j}} \right) + \epsilon_i T_t + \epsilon_{it} \]  

for \( i = 1, 2, 3, \quad t = 3, 4, \ldots, T. \)

where \( a_3 = 1 - a_1 - a_2, \quad c_3 = -c_1 - c_2, \quad \) and \( b_{3j} = -b_{1j} - b_{2j}, \)

\( j = 1, 2, 3. \)

This share model will be referred to as the basic model in order to distinguish it from other models.

Since the three shares always sum to unity, the sum of the disturbances across the three equations in (10) is zero at each observation \( t. \) This implies that the disturbance covariance matrix is singular and non-diagonal, and consequently the likelihood function is undefined. This singularity problem can be circumvented by defining a density function for disturbances of any two equations and by obtaining the corresponding likelihood function.

Since the maximum likelihood estimates are invariant to the equation deleted, we arbitrarily drop the disturbance from the \( S_3, \) equation and specify that the disturbance column vector \( \epsilon_i^0 = [\epsilon_{i1}, \epsilon_{i2}] \) is normally independently identically distributed (NIID) with a mean vector of zeroes and a nonsingular covariance matrix, \( \Omega^0. \)

Two additional points need to be considered: serial correlation in disturbances and potential lag in shippers' response to price changes. Because of potential cyclical variations in the modal shares, our time series data are likely to reveal serial correlations. The lagged behaviour in shippers' adjustment may be caused by a variety of reasons, such as contract obligations to a certain mode, imperfect information, capital committed already to a certain mode, and inertia. To obtain the consistent parameter estimates of the long-run equilibrium demand function, this dynamic nature of the lagged adjustment has to be taken into account in modelling our share function. These two considerations provide us with alternative specifications of

\footnote{In order to base the testing of hypotheses on equal numbers of observations, all models hypothesised in this section start from year \( t = 3. \)}
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Share models: autoregressive model, partial adjustment model, and a combination of the two.

Following the Nadiri–Rosen [22] tradition, the partial adjustment model for our share functions is defined as:

\[ S^*_t = a + B \cdot \ln P_t + c \cdot T_t + U^*_t \]

\[ S_t - S_{t-1} = \Gamma \cdot (S^*_t - S^*_{t-1}) \]

where \( S^*_t \) is a 3 \times 1 vector of long-run equilibrium shares in year \( t \),
\( S_t \) is a 3 \times 1 vector of observed shares in year \( t \),
\( P_t \) is a 3 \times 1 vector of prices in year \( t \),
\( \Gamma \) is a 3 \times 3 matrix of partial adjustment coefficients,
\( a \) is a 3 \times 1 vector of intercepts,
\( B \) is a 3 \times 3 matrix of coefficients, \( b_{U_t}'s \),
\( c \) is a 3 \times 1 vector of trend coefficients \( c_{U_t}'s \),
\( T_t \) is the trend variable of year \( t \),
\( U^*_t \) is a 3 \times 1 vector of disturbances.

A simple manipulation gives the system (12) in which the vector of shares is expressed as a function of an observable price vector and a one-period lagged vector of shares:

\[ S_t = \Gamma \cdot a + \Gamma \cdot B \cdot \ln P_t + \Gamma \cdot c \cdot T_t + (I - \Gamma) \cdot S_{t-1} + \Gamma \cdot U^*_t \]

\[ t = 3, 4, \ldots, T. \]

Now, the disturbance vector, \( U^*_t \), of the partial adjustment model (11) is assumed to follow a first order autoregressive scheme as in (13).

\[ U^*_t = R \cdot U^*_{t-1} + \epsilon^*_t \]

\[ t = 3, 4, \ldots, T \]

where \( R \) is a 3 \times 3 matrix of autoregressive coefficients,
\( U^*_t \) is a 3 \times 1 vector of autoregressive disturbances,
and \( \epsilon^*_t = [\epsilon^*_1, \epsilon^*_2, \epsilon^*_3] \sim \text{NIID}(0, \Omega^*) \).

After substitution of equation (13) into equation (12), a rearrangement gives the partial adjustment model with autoregressive disturbances as:

\[ S_t = (I - R) \cdot \Gamma \cdot a + \Gamma \cdot B \cdot \ln P_t - R \cdot \Gamma \cdot B \cdot \ln P_{t-1} + \Gamma \cdot c T_t \]

\[ - R \cdot \Gamma \cdot c T_{t-1} + (I - \Gamma + R) \cdot S_{t-1} + R \cdot (\Gamma - I) \cdot S_{t-2} + \epsilon_t \]

where \( \epsilon_t = \Gamma \cdot \epsilon^*_t = \Gamma \cdot (U^*_t - R \cdot U^*_{t-1}) \).

The adding-up property of the three shares imposes restrictions on the parameters of both the adjustment and the autoregressive processes: the column sums of adjustment matrix \( \Gamma \) must be identical if equation (11b) is to hold, and similarly the column sums of matrix \( R \) of autoregressive coefficients must be identical if the linear homogeneity conditions (4) and equation (13) are to hold. Moreover, when the restrictions on the equality of column sums are not imposed, the ML estimates of parameters and the likelihood ratio (\( \lambda \)) statistics are no longer invariant to the equation deleted. Furthermore, Berndt and Savin [6] and Berndt et al [5] have shown that the singu-
larity of contemporaneous covariance matrix causes under-identification of the parameter of the autoregressive process and of the adjustment matrix, except for the cases of restricted $R$ and $\Gamma$ matrices in which each row of $R$ and $\Gamma$ has at least one zero element.

In order to maintain reasonable degrees of freedom and also to avoid the under-identification of the parameters, our empirical analysis is confined to the case of diagonal $R$ and $\Gamma$ matrices. Again all the diagonal elements of $R$ (or $\Gamma$) should have an identical value because of the equality of column sums. This implies that there will be a single autocorrelation coefficient ($\rho$) and a single partial adjustment coefficient ($\gamma$) for all the three share functions in equation (14).

After imposition of the diagonality of matrices $R$ and $\Gamma$ and the linear homogeneity conditions (4), the partial adjustment model with autoregressive disturbances in equation (14) becomes:

$$S_{it} = \gamma[a_i(1 - \rho) + \sum_{j=1}^{2} b_{ij} \ln(P_{jt}/P_{3t}) - \rho \ln(P_{j,t-1}/P_{3,t-1})]$$

$$+ \epsilon_i(T_t - \rho T_{t-1}) + (1 - \gamma)S_{i,t-1} + \rho(\gamma - 1)S_{i,t-2} + e_{it} \quad (15)$$

$i = 1, 2, 3. \quad t = 3, 4, \ldots, T.$

where $a_3 = 1 - a_1 - a_2$, $\epsilon_3 = -\epsilon_1 - \epsilon_2$, and $b_{3j} = -b_{1j} - b_{2j}$.

By setting $\gamma = 1$ in equation (15), the autoregressive model with instantaneous adjustment is obtained as:

$$S_{it} = a_i(1 - \rho) + \sum_{j=1}^{2} b_{ij} \ln(P_{jt}/P_{3t}) - \rho \ln(P_{j,t-1}/P_{3,t-1})$$

$$+ \epsilon_i(T_t - \rho T_{t-1}) + \rho S_{i,t-1} + e_{it}, \quad (16)$$

$i = 1, 2, 3. \quad t = 3, 4, \ldots, T.$

Similarly, by imposing $\rho = 0$ in equation (15), we obtain the partial adjustment model as:

$$S_{it} = \gamma[a_i + \sum_{j=1}^{2} b_{ij} \ln(P_{jt}/P_{3t}) + \epsilon_i T_t] + (1 - \gamma)S_{i,t-1} + e_{it} \quad (17)$$

$i = 1, 2, 3. \quad t = 3, 4, \ldots, T.$

As in the basic share model (10), we arbitrarily drop the $S_{3t}$ equation from models (15), (16) and (17), and specify

$$\epsilon^0_t = [\epsilon_{1t}, \epsilon_{2t}] \sim \text{NIID}(0, \Omega^0)$$

where $\Omega^0$ is now a non-singular contemporaneous covariance matrix.

Having specified the four alternative share models, (10), (15), (16) and (17), we design a set of nested hypotheses as in Figure 1 in order to determine an appropriate model to use for our purpose. Each arrow in Figure 1 links a pair of null and alternative hypotheses which are to be tested. The neutrality of factor augmentation is also tested for every alternative share model, independently of these nested hypotheses.
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Model (15)
\((\gamma \neq 1, \rho \neq 0)\)

Model (17)
\((\gamma \neq 1, \rho = 0)\)

Model (16)
\((\gamma = 1, \rho \neq 0)\)

Model (10)
\((\gamma = 1, \rho = 0)\)

Model (10): basic share model
Model (16): autoregressive share model
Model (17): partial adjustment share model
Model (15): partial adjustment model with autocorrelation

Figure 1: Nested Hypotheses

III. DATA CONSTRUCTION AND SOURCES

The data required for the estimation of the four alternative share models, and thus for the estimation of our translog cost function in the Canadian economy 1945–1974, are the price indexes and revenue shares of the three modes of transport.

The revenue shares were calculated from data available in various Statistics Canada publications (Appendix A). Although revenues per ton-mile could be calculated from data available in Statistics Canada publications (except for waterway), a serious problem arises in constructing the appropriate price indexes for the three modes of freight transport. This is because the average revenues per ton-mile are largely dependent on the average length of haul and the average volume per shipment, both of which are likely to vary over time. Although ideally the price index for each mode should be constructed through a two-dimensional Divisia aggregation of the price indices for separate mileage and volume brackets, it is impossible to do this because necessary data are lacking. Instead, the average revenues per ton-mile for railway, highway and airway carriers are transformed so as to reflect only the variation in average length of haul from year to year. The findings of other studies, which relate the revenue per ton-mile to the length of haul and other factors influencing revenue, are utilised to construct the formulae for the transformation (Appendix A).

Since the ton-mile data for waterway carriers are not published, we use the price index calculated on the basis of grain transport rates from Thunder Bay to the east coast ports through the Great Lakes, Welland Canal and St. Lawrence Seaway. This is believed to be a good indicator for the level of inland waterway freight rates in Canada.

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Since the effect of changes in average volume per shipment on the average revenue per ton-mile was not taken into account in the transformation of data, we are implicitly assuming that the change in average volume occurs in a similar pattern for all the three modes. If not, it could introduce a bias in their relative price levels.

Table 1 tabulates the revenue shares and the transformed price indexes of the three modes, deflated by the GNP deflator, for the period 1945–1974.

We notice that over the period 1945–74 the revenue share of trucking services increased from 6 per cent to 47 per cent, that of railways decreased from 71 per cent to 44 per cent, and that of waterways decreased from 23 per cent to 9 per cent. Part 2 tabulates the transformed price indexes deflated by the GNP deflator for the same period, and shows that, relative to the GNP deflator, the price index for trucking services increased by about 47 per cent, whereas those for both railways and waterways decreased by approximately 45 per cent.

IV. ESTIMATION AND TESTING OF HYPOTHESES

In most demand models based on highly aggregate data like ours, the prices may not be considered exogeneous to the model, and consequently the possible dependence between the right-hand-side variables and disturbances poses a complication in the estimation process. From the nature of a regulated industry, however, carriers enjoy only limited freedom in pricing their services. Therefore, in our case, it seems not unreasonable to assume that the prices are exogeneous to the share models.

The maximum likelihood method of estimation ensures that the parameter estimates are invariant to the equation deleted for all the share models specified in section II. Since the basic share model (10) involves a general linear constraint [Goldberger, 13] under the symmetry restriction, the constrained IZEF (Iterative Zellner Efficient) method is used for its estimation. Oberhofer and Kmenta [23] have shown that if the constrained IZEF converges in finite steps, it provides parameter estimates numerically equivalent to the constrained ML estimates obtained by determinant minimisation. For the estimation of the other three share models (15), (16) and (17), which are nonlinear in parameters, an iterative nonlinear least square systems estimator [Berndt et al., 4] is used. This is essentially a combination of the IZEF procedure with the Gauss–Newton method of non-linear least squares, and is equivalent to the ML estimation. The convergence was achieved for every alternative share model. The asymptotic likelihood ratio (λ) test is used for testing all the hypotheses about neutrality of factor augmentation and the form of share model. Theil [30] has shown that, asymptotically, $-21n\lambda$ has a chi-square distribution, the degree of freedom of which is equal to the number of independent restrictions imposed in the null hypothesis. Table 2 tabulates the observed chi-square statistic $(-21n\lambda)$ and the degree of freedom for each pair of null and alternative hypotheses.

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9 The convergence criterion used is that (i) the largest change in parameter estimates from one iteration to another should be no greater than 0.1 per cent, and (ii) the largest absolute deviation of the elements of transformed residual covariance matrix from the identity matrix should be no greater than 0.001.
## Table 1

Revenue Shares and Price Indexes of the Three Modes of Intercity Freight Transport in Canada, 1945–74

<table>
<thead>
<tr>
<th>Year</th>
<th>Modal Revenue Shares</th>
<th>Price Indexes*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Railway</td>
<td>Highway</td>
</tr>
<tr>
<td>1945</td>
<td>0.7091</td>
<td>0.0606</td>
</tr>
<tr>
<td>1946</td>
<td>0.6925</td>
<td>0.0639</td>
</tr>
<tr>
<td>1947</td>
<td>0.7269</td>
<td>0.0742</td>
</tr>
<tr>
<td>1948</td>
<td>0.7285</td>
<td>0.0841</td>
</tr>
<tr>
<td>1949</td>
<td>0.7369</td>
<td>0.0955</td>
</tr>
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<td>1950</td>
<td>0.5726</td>
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</tr>
<tr>
<td>1951</td>
<td>0.7330</td>
<td>0.1049</td>
</tr>
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<td>1952</td>
<td>0.7002</td>
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</tr>
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<td>1957</td>
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</tr>
<tr>
<td>1958</td>
<td>0.6234</td>
<td>0.2204</td>
</tr>
<tr>
<td>1959</td>
<td>0.6140</td>
<td>0.2446</td>
</tr>
<tr>
<td>1960</td>
<td>0.5960</td>
<td>0.2497</td>
</tr>
<tr>
<td>1961</td>
<td>0.5912</td>
<td>0.2588</td>
</tr>
<tr>
<td>1962</td>
<td>0.5904</td>
<td>0.2655</td>
</tr>
<tr>
<td>1963</td>
<td>0.5756</td>
<td>0.2829</td>
</tr>
<tr>
<td>1964</td>
<td>0.5845</td>
<td>0.2785</td>
</tr>
<tr>
<td>1965</td>
<td>0.5583</td>
<td>0.3082</td>
</tr>
<tr>
<td>1966</td>
<td>0.5605</td>
<td>0.3103</td>
</tr>
<tr>
<td>1967</td>
<td>0.5210</td>
<td>0.3597</td>
</tr>
<tr>
<td>1968</td>
<td>0.4981</td>
<td>0.3778</td>
</tr>
<tr>
<td>1969</td>
<td>0.4974</td>
<td>0.3845</td>
</tr>
<tr>
<td>1970</td>
<td>0.4325</td>
<td>0.4331</td>
</tr>
<tr>
<td>1971</td>
<td>0.4341</td>
<td>0.4630</td>
</tr>
<tr>
<td>1972</td>
<td>0.4295</td>
<td>0.4747</td>
</tr>
<tr>
<td>1973</td>
<td>0.4215</td>
<td>0.4796</td>
</tr>
<tr>
<td>1974</td>
<td>0.4422</td>
<td>0.4711</td>
</tr>
</tbody>
</table>

*The price indexes are deflated to show the changes in real freight rates. Of course, the parameter estimates of our share models are invariant to the deflation.

Part 1 of table 2 shows that, at 5 per cent level of significance, the neutrality of factor augmentation is rejected in favour of non-neutrality in all the share models (10), (15), (16) and (17). This implies that changes in model shares result not only from changes in freight rates but also from technological changes in the economy.

In testing the nested hypotheses about the form of share model, therefore, the non-neutrality of factor augmentation was assumed for every share model. Part 2 of table 2 shows the following:
Table 2
Asymptotic Likelihood Ratio Test Statistics

<table>
<thead>
<tr>
<th>Null Hypothesis (H₀)</th>
<th>Alternative Hypothesis (H₁)</th>
<th>Degrees of Freedom</th>
<th>( \chi^2 )-statistics</th>
<th>Test Favour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1: Tests about neutrality of factor augmentation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic model (10):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 10a</td>
<td>Model 10b</td>
<td>2</td>
<td>21.82**</td>
<td>Model 10b</td>
</tr>
<tr>
<td>Autoregressive model (16):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 16a</td>
<td>Model 16b</td>
<td>2</td>
<td>11.86**</td>
<td>Model 16b</td>
</tr>
<tr>
<td>Partial adjustment model (17):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 17a</td>
<td>Model 17b</td>
<td>2</td>
<td>7.15*</td>
<td>Model 17b</td>
</tr>
<tr>
<td>Partial adjustment model with autocorrelation (15):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 15a</td>
<td>Model 15b</td>
<td>2</td>
<td>6.55*</td>
<td>Model 15b</td>
</tr>
<tr>
<td>Part 2: Nested hypotheses testing about share models:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 10b</td>
<td>Model 15b</td>
<td>2</td>
<td>16.62**</td>
<td>Model 15b</td>
</tr>
<tr>
<td>Model 16b</td>
<td>Model 15b</td>
<td>1</td>
<td>0.11</td>
<td>Model 16b</td>
</tr>
<tr>
<td>Model 10b</td>
<td>Model 16b</td>
<td>1</td>
<td>16.50**</td>
<td>Model 16b</td>
</tr>
<tr>
<td>Model 17b</td>
<td>Model 15b</td>
<td>1</td>
<td>2.36</td>
<td>Model 17b</td>
</tr>
<tr>
<td>Model 10b</td>
<td>Model 17b</td>
<td>1</td>
<td>13.75**</td>
<td>Model 17b</td>
</tr>
</tbody>
</table>

Critical Chi-square Values:
\[ \chi^2:1(0.01) = 6.64 \]
\[ \chi^2:2(0.01) = 9.21 \]
\[ \chi^2:1(0.05) = 3.84 \]
\[ \chi^2:2(0.05) = 5.99 \]
\[ \chi^2:1(0.10) = 2.71 \]
\[ \chi^2:2(0.10) = 4.61 \]

**Reject \( H_0 \) at 1 per cent level of significance.
*Reject \( H_0 \) at 5 per cent level of significance.
Model with subscript a indicates neutrality imposed, and b indicates neutrality unimposed.

1. The basic share model (10) has to be rejected, no matter what the alternative hypothesis is.
2. The autoregressive model (16) cannot be rejected even at 1 per cent level of significance.
3. The partial adjustment model (17) also cannot be rejected at 1 per cent level of significance.

Although a fundamental difficulty arises in determining which of the two models is the better, the autoregressive model (16) is preferred \textit{a posteriori} on the Bayesian criterion [Press, 24], because it attained a higher value of the likelihood function than the partial adjustment model (17), and it is used in the subsequent analysis.\(^{10}\)

However, before we analyse the details of the autoregressive share model (16), it

\(^{10}\)The observed values of logarithm of likelihood functions are:
Model 16c (Autoregression Model): 82.0127
Model 17c (Partial Adjustment Model): 90.6383.
may be interesting to examine the lag structure of shippers' response to the changes in freight rates, using the partial adjustment model (17). The estimate of partial adjustment coefficient ($\gamma$) is 0.60, which implies that only 60 per cent of deviation of the observed shares from the optimal shares at the beginning of a period is adjusted during the year. Assuming a geometric lag, the parameters of lag distribution can be computed as shown in Griliches [14]:

\[
\text{Mean lag} = \frac{1 - \gamma}{\gamma} = 0.66 \text{ year}
\]

\[
\text{Variance} = \frac{1 - \gamma}{\gamma^2} = 1.11 \text{ year}
\]

This tells us that the shippers' response has exhibited a geometric lag distribution with a mean of 0.66 year and a variance of about 1.1 year.

Table 3 presents the ML parameter estimates of the translog transport sectoral cost function for the Canadian economy, 1948–1974, estimated on the basis of the autoregressive model (16). The conventional $R^2$-values (computed as one minus the ratio of the residual sum to the total sum of squares in each equation) are 0.9870, 0.9904 and 0.9322 for share equations of railway, highway and waterway carriers, respectively.

A cost function is well-behaved if it is quasi-concave and linearly homogeneous in input prices if its input demand functions are strictly positive. Since the translog cost function does not satisfy these regularity conditions globally, it is necessary to check local properties at each data point. The linear homogeneity conditions were originally imposed in our share model. The positivity of the input demand functions
is satisfied because the fitted shares of the three modes for each of the 27 years are all positive. The local concavity of the estimated translog cost function was checked by computing the eigen values of the matrices of elasticities of substitution $[\sigma_{ij}]$. The local concavity was not satisfied at the data points for the years before 1958 but was satisfied at the data points thereafter. Consequently, the following results of analysis are tentative and possibly subject to future revision.

V. EMPIRICAL RESULTS AND IMPLICATIONS

To measure the possibilities of substitution and price-responsiveness of demand, we compute the estimated Allen partial elasticities of substitution ($\sigma_{ij}$) and partial price elasticities ($E_{ij}$) as formulated in (7) and (8), respectively. These computations are all based on the autoregressive share model (16). Because the estimated $\sigma_{ij}$ and $E_{ij}$ exhibit rather stable trends over the period 1948–1974, we present in Tables 4 and 5 the estimated $\sigma_{ij}$ and $E_{ij}$, respectively, for only six selected years. The signs of the $\sigma_{ij}$ and $E_{ij}$ estimates agree generally with our expectation. One exception is that $\sigma_{22}$ and $E_{22}$ are positive for the early years of the study period. Nevertheless, several important conclusions emerge from Tables 4 and 5:

(i) The demand for railway freight services is only slightly responsive to the change in railway freight rate; but the own-price elasticity has been increasing in absolute value over time. The ML estimate of $E_{11}$ for 1974 is $-0.291$, while for 1950 it is only $-0.093$. The demand for trucking services after the year 1958 has shown a slight sensitivity to its own freight rate. As in the case of railways, this sensitivity has also been increasing over time. The ML estimate of $E_{22}$ for 1974 is $-0.155$, while for 1960 it is only $-0.045$. The demand for waterway services has been quite sensitive to its own freight rate and remains steady over the entire study period. The ML estimates of $E_{33}$ for 1974 and 1950 are $-0.744$ and $-0.738$, respectively.

(ii) Railway and highway carriers exhibited a complementary relationship till 1955, and thereafter an increasingly competitive relationship. The ML estimates $\sigma_{12}$, $E_{12}$ and $E_{21}$ for 1974 are $0.302$, $0.144$, and $0.127$, respectively, whereas for 1950 they are $-1.181$, $-0.102$ and $-0.878$. This conforms with our expectation that in the years immediately following World War II the highway carriers complemented the railway carriers rather than competing against them, but that the rapid technological advance in highway transport in the late 1950s and the continuous improvement in highway systems enabled the highway carriers to compete against the railways.

(iii) A highly competitive relationship has been found between the railway and waterway carriers throughout the entire study period. This relationship has been

---

11Common practice is to eliminate the data that do not satisfy the concavity condition and re-estimate the cost functions from the remaining data. We preferred not to do so because: (i) usage of only 17 data points (1958–74) may not justify the asymptotic tests employed throughout this study; (ii) the lack of concavity may reflect the fact that the majority of trucking users in the early years had no alternative modes to choose from. (Note, from Table 4, that the violation of concavity is mainly caused by the positivity of $\sigma_{22}$).

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### Table 4

**ML Estimates of Allen Partial Elasticities of Substitution, Canadian Freight Transport 1950–74**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_{11}</td>
<td>-0.126</td>
<td>-0.211</td>
<td>-0.313</td>
<td>-0.414</td>
<td>-0.605</td>
<td>-0.693</td>
</tr>
<tr>
<td>σ_{21}</td>
<td>12.846</td>
<td>1.174</td>
<td>-0.183</td>
<td>-0.410</td>
<td>-0.374</td>
<td>-0.323</td>
</tr>
<tr>
<td>σ_{31}</td>
<td>-4.350</td>
<td>-4.936</td>
<td>-5.294</td>
<td>-5.802</td>
<td>-6.862</td>
<td>-7.360</td>
</tr>
<tr>
<td>σ_{12}</td>
<td>-1.181</td>
<td>-0.215</td>
<td>0.074</td>
<td>0.202</td>
<td>0.292</td>
<td>0.302</td>
</tr>
<tr>
<td>σ_{13}</td>
<td>1.153</td>
<td>1.188</td>
<td>1.224</td>
<td>1.271</td>
<td>1.387</td>
<td>1.455</td>
</tr>
<tr>
<td>σ_{23}</td>
<td>-1.380</td>
<td>-0.358</td>
<td>0.004</td>
<td>0.151</td>
<td>0.258</td>
<td>0.277</td>
</tr>
</tbody>
</table>

### Table 5

**ML Estimates of Partial Price Elasticities of Demand, Canadian Freight Transport 1950–74**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{11}</td>
<td>-0.099</td>
<td>-0.143</td>
<td>-0.191</td>
<td>-0.228</td>
<td>-0.277</td>
<td>-0.291</td>
</tr>
<tr>
<td>E_{12}</td>
<td>-0.102</td>
<td>-0.037</td>
<td>0.018</td>
<td>0.064</td>
<td>0.126</td>
<td>0.144</td>
</tr>
<tr>
<td>E_{13}</td>
<td>0.196</td>
<td>0.178</td>
<td>0.173</td>
<td>0.164</td>
<td>0.151</td>
<td>0.147</td>
</tr>
<tr>
<td>E_{21}</td>
<td>-0.078</td>
<td>-0.146</td>
<td>0.045</td>
<td>0.111</td>
<td>0.134</td>
<td>0.127</td>
</tr>
<tr>
<td>E_{22}</td>
<td>1.112</td>
<td>0.200</td>
<td>-0.043</td>
<td>-0.131</td>
<td>-0.162</td>
<td>-0.155</td>
</tr>
<tr>
<td>E_{23}</td>
<td>-0.234</td>
<td>-0.054</td>
<td>0.0</td>
<td>0.019</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>E_{31}</td>
<td>0.858</td>
<td>0.806</td>
<td>0.747</td>
<td>0.702</td>
<td>0.636</td>
<td>0.611</td>
</tr>
<tr>
<td>E_{32}</td>
<td>-0.120</td>
<td>-0.061</td>
<td>0.001</td>
<td>0.048</td>
<td>0.112</td>
<td>0.133</td>
</tr>
<tr>
<td>E_{33}</td>
<td>-0.738</td>
<td>-0.745</td>
<td>-0.748</td>
<td>-0.750</td>
<td>-0.748</td>
<td>-0.744</td>
</tr>
</tbody>
</table>

slowly increasing over time. The ML estimates of σ_{13}, E_{13} and E_{31} for 1974 are 1.455, 0.147 and 0.611, respectively, and for 1950 they are 1.153, 0.196 and 0.858.

(iv) Highway and waterway carriers showed a complementary relationship up to the 1950s, and since then an increasing competitive relationship. This may be caused by the relative increase in the share of trucking services in the intercity freight market. The ML estimates of σ_{13}, E_{23} and E_{12} for 1974 are 0.277, 0.028 and 0.133, respectively, whereas for 1950 they are -1.380, -0.234 and -0.120.

A general deduction from these results is that, as the revenue share of trucking services increases relative to the other modes, the demand for both railway and waterway services has become increasingly sensitive to the rates charged for trucking services. The price elasticities estimated by various ad hoc models may be considered as the approximation to the price elasticities of ordinary demand (F_{ui}), which include both the substitution effect and the effect of change in the level of shippers’ output in response to the changes in freight rates.

These price elasticities of ordinary demand (F_{ui}) could be computed by using
VI. CONCLUDING REMARKS

The demand for the three modes of freight transport has been formulated and estimated as the intermediate inputs to the production and distribution sectors of the economy. The resulting demand model has allowed free variation of substitution parameters.

One of the main problems which impede this kind of research is the availability of adequate data on freight rates. Although they were adjusted so as to reflect variations in the average length of haul, the data used in this study are by no means ideal. Nevertheless, the results show that the estimated share model explains extremely well the variations in the shares of the three modes, and the estimated parameters of the demand functions conform generally with our expectation.

The study finds that there is significant competition in varying degrees between the three modes of freight transport, and that the cross-price elasticities estimated by using the ad hoc demand models are likely to understate this. The study also finds that there is a lag structure in the shippers' response to changes in freight rates.

The findings in section V are suggestive, but they clearly require further verification. The improvements in the price and quantity data permitting two-dimensional (length of haul and volume per shipment) Divisia aggregation of price indexes should prove useful in refining the results of this study. The application of this kind of derived demand model at a disaggregate level should be useful for analysing multimodal demands for freight transport, especially inter-modal price competition.

APPENDIX A

Sources of Data and Transformation of Price Indexes

(I) Sources of Data:
A. The total freight revenue, revenue ton-miles and average length of haul for the railways were collected from Dominion Bureau of Statistics (DBS) 52–207 (1945–69), Statistics Canada 52–207 (1970–74), Railway Transport, Part I (Comparative Summary Statistics).

B. The total freight revenue, revenue ton-miles and total revenue tons for the intercity for-hire and contract trucking were collected from the following sources,

\[\eta = -1, \text{ are } -0.772 \ (F_{t1}), -0.334 \ (F_{t2}), 0.046 \ (F_{t3}), -0.293 \ (F_{s2}), -0.633 \ (F_{s3}), -0.073 \ (F_{s3}), 0.191 \ (F_{s4}), -0.345 \ (F_{s4}), -0.412 \ (F_{s4}) \]

Note that these values are smaller than the corresponding values of \(\eta\) for 1974 in Table 5, and especially that all the cross-price elasticities of ordinary demand except \(F_{t3}\) and \(F_{s3}\) are negative, indicating complementary rather than competitive relationships.

12For example, the price elasticities of ordinary demand for 1974, computed under the assumption that \(\eta = -1\), are

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and the average length of hauls was computed accordingly:

DBS 53–205 (1956–60), Motor Carriers: Freight,
   Part I, class 1 and 2. Part II, class 3 and 4.
Special Report of the DBS Daily Bulletin, April 1969,

Note: The private intercity highway freight carriers are not included in our analysis.

C. The total freight revenues of the inland and coastal shipping industry were collected from DBS 54–205 (1945–69) and Statistics Canada 54–205 (1070–74): Water Transportation. The rate index for grain transport from Thunder Bay to the Canadian East Coast ports was used as an approximation to price index for water transport. The grain transport rate index is available in Canadian Grain Exports, Canadian Grain Commission (various years).

D. Data on labour and capital inputs used in Appendix B to test the separability of transport services were compiled from the following sources: Statistics Canada 72–002 (Employment, Earnings, and Hours), Statistics Canada 13–543 (Fixed Capital Flows and Stocks, Manufacturing, Canada), Table 28 of the revised National Accounts. Rental price for the capital goods was computed under the assumption that the rental bill equals value added minus the wage bill. Of course, the labour and capital employed in the transport sector were excluded in compiling these labour and capital input and price data.

(II) Transformation of Price Indexes

The findings of other studies which relate the revenue per ton to the length of haul and other factors were employed to transform the average revenue per ton-mile to reflect the changes in length of haul from year to year.

A. Railway: The transformation formula which we adapted from Heaver and Oum [17] is

\[ R_t^* = R_t \cdot \left( \frac{D_0}{D_t} \right)^{0.65 - 1} = R_t \cdot \left( \frac{D_t}{D_0} \right)^{0.35} \]

where \( R_t^* \): the revenue per ton-mile after transformation for the year \( t \).
\( R_t \): the observed revenue per ton-mile for the year \( t \).
\( D_0 \): average length of haul for the base year 1963.
\( D_t \): average length of haul for the year \( t \).

B. Highway: The data presented in Morton [21] were used to estimate a Cobb–Douglas rate function that relates the average truck-load rate of each mileage-weight block to the mean values of the lower and upper limits of the mileage and weight for the block. From this study, the following transformation formula was derived:

\[ R_t^* = R_t \left( \frac{D_t}{D_0} \right)^{0.15} \]

where all the variables are defined as before.
APPENDIX B

Separability of Transport Services

In the context of the differentiable production function in equation (1), the separability of transport services from other inputs is satisfied if and only if the following condition holds:

\[ \frac{\partial}{\partial X} \left( \frac{f_i(\cdot)}{f_j(\cdot)} \right) = 0 \quad \text{for all } i, j = R, H, W \quad \text{and for all } X = K, L \]

where \( f_i(\cdot) \) and \( f_j(\cdot) \) are the first partial derivatives of the production function with respect to freight services of the \( i \)th and \( j \)th modes, respectively.

This implies that rate of technical substitution between any pair of modes is invariant to the level of capital or labour employed in the economy. Since it is not clear whether or not this condition is met in reality, the separability should be hypothesised and tested empirically.

The proper test is to compare the five-input \((R, H, W, K, L)\) translog cost functions with and without separability imposed. This procedure requires one to estimate four expenditure share equations at the same time. A preliminary attempt has shown that use of the proper test would be impractical because:

(i) For some models, the parameter estimates failed to converge to stationary values even after 100 iterations.

(ii) In those models where convergence was achieved, many parameter estimates had wrong signs which made the share equations meaningless.

(iii) Because of the smallness of expenditure shares of freight services relative to those of capital and labour inputs, the share equations of freight modes had relatively low \(R^2\) values. (Note that systems estimator was used to estimate all the models.)

Therefore, an ad hoc approach is to be used to test the separability. The three-input basic share model in equation (10) is regarded as the separable basic model and the following as the non-separable basic model:

\[ S_{it} = a_i + \sum_{j=1}^{5} b_{ij} \ln \frac{P_{jt}}{P_{3t}} + c_i T_i \quad i = 1, 2, 3 \tag{A1} \]

where \[ \sum_{j=1}^{5} b_{ij} = 0 \quad \text{for all } i = 1, 2, 3, \]

\[ b_{ij} = b_{ji} \quad \text{for all } i, j = 1, 2, 3, \]

\[ \sum_{i=1}^{3} b_{ij} = 0 \quad \text{for all } j = 4, 5, \]

\( P_{4t} \) : rental price of capital stocks in year \( t \),

\( P_{5t} \) : price of labour in year \( t \),
DERIVED DEMAND FOR FREIGHT TRANSPORT

and others are defined as in equation (3b).

Note that the equation (A1) is obtained by merely adding

\[ \ln \left( \frac{P_{4s}}{P_{3r}} \right) \quad \text{and} \quad \ln \left( \frac{P_{5s}}{P_{3r}} \right) \]

as additional independent variables to the model (10).

The non-separable versions of other share models hypothesised in section II are formed by applying the same procedures as those used to arrive at (A1).

As shown below, in all cases, it was impossible to reject the separable versions at 5 per cent level of significance.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \chi^2 )-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic model (10)</td>
<td>6.541</td>
</tr>
<tr>
<td>Autoregressive model (16)</td>
<td>3.847</td>
</tr>
<tr>
<td>Partial adjustment model (17)</td>
<td>4.151</td>
</tr>
<tr>
<td>Partial adjustment model with autocorrelation (15)</td>
<td>3.212</td>
</tr>
</tbody>
</table>

Remarks: \( H_0 \): separable model  
\( H_1 \): non-separable model  
degrees of freedom: 4  
\( \chi^2 \): 4 (\( \alpha = 0.05 \)) critical value = 9.488

Although the testing procedure adopted here is far from being robust, the values of observed \( \chi^2 \)-statistics relative to the critical value suggest that the freight transport services may be treated as separable from other inputs in the economy. This result may be used in justifying the usual practice of not including rental price of capital and price of labour as the arguments of transport demand functions.

REFERENCES


