THE POSSIBILITY OF PROFITABLE BUS SERVICE

By Philip A. Viton*

This paper investigates the question where and under what conditions express bus services, as for example the observed New York City bus operations, can succeed, in the sense of being profitable. The approach taken is to model the bus operator as the sole franchisee between a residential area and a central business district, and to assume that his only competition in the provision of transit services comes from the automobile. The bus operator can vary the level of service he offers, and is assumed to do so in such a way as to maximise profits. Thus, the model enables us to answer two related questions: first, under what conditions will profitable service be possible? And secondly, what will be the profit-maximising level of service?

Zupan and Pushkarev (1977) have investigated Where Express Buses Work. Their concern is with the minimum residential densities that may result in sufficient passenger volume to keep average bus operating cost at a given level. For example, if one considers a residential area of two square miles, located ten miles from a CBD of 50 million square feet, and it is desired to run five express buses in a two-hour peak between the residential area and the CBD at a speed of 35 mph while keeping costs at $0.10 per passenger mile, then Zupan and Pushkarev find (p. 36) that one must place the service in a residential area exceeding 2.75 dwelling units per square foot. Their model holds constant a single measure of service quality—the number of buses per hour on a route; thus they cannot account for the observed fact that existing bus services do make profit in areas of widely differing residential densities by varying the level of service offered.

A more useful way of approaching the problem appears to be to study explicitly the profit function of the bus operator. This approach has the advantage of being able to account for profitable or unprofitable service in the presence of varying levels of cost and of varying demographic characteristics and income distribution of the population. Since the specification of a profit function involves the demand for a particular level of bus service, this study necessarily reminds us that the bus service is generally

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1 "The express bus system consisted in 1973 of some 31 approved routes serving all five boroughs and carrying about 260,000 passengers per week. Since then the number of approved operating routes has increased to . . . 35" (Pignataro and Falcochio (1976), p. I.1).
competing with (at least) the private car, and that the demand for bus service will depend to some extent on the prices and "service quality" offered by the competing mode. Further, since they examine only CBD size and residential density, Zupan and Pushkarev must make some implicit assumptions about the relative attractiveness of CBD over alternative destinations. However, since we often have explicit knowledge of the number of trips originating in the residential area and ending in the CBD, it would seem that more information could be gained by explicitly discussing the number of persons with the assumed travel patterns.

The structure of this paper is as follows. In section 1, I sketch a theory of mode choice which has been stated by previous writers, and show an empirical estimate of a choice function for San Francisco Bay Area commuters. Section 2 discusses an abstract market for transit services, and sets up cost functions for such services, depending on just those service characteristics revealed as important to the mode choice decision by the model of section 1. The parameters of the cost function are estimated, again on the basis of Bay Area operating costs. Finally, in section 3, I discuss the feasibility of profitable service, and its characteristics.

1. COMMUTER MODE CHOICE

Consider a population of consumers, each facing a binary choice of transport mode. The consumers maximise utility, and the utility of each mode \( j \) to the \( i \)th consumer is taken to be a linear function of the perceived characteristics of the mode \( (x_j) \) and the attributes of the consumer \( (a_i) \), plus a stochastic term reflecting unobserved (or unobservable) characteristics of mode or individual. Introduction of the stochastic element means that we can only analyse the probability that a given choice is made; each distribution of the stochastic term yields a set of mode-choice probabilities. For computational reasons, surveyed by Domencich and McFadden (1975), the stochastic terms for different individuals are often assumed to have independent Weibull distributions. Under this assumption, the binary choice probability of choosing the auto is given by the well-known conditional logit form. Denote by \( a \) and \( b \) the auto and bus modes. Then the probability of selecting the auto, \( P_i^a \), is

\[
P_i^a = \frac{\exp \left[ \beta'(Z_i^a - Z_i^b) \right]}{1 + \exp \left[ \beta'(Z_i^a - Z_i^b) \right]} \tag{1.1}
\]

where \( Z_i^a = (a_i, x_i) \), \( Z_i^b = (a_i, x_i) \), and \( \beta \) is an unknown weighting vector, estimable, for example, by maximum likelihood methods.\(^2\)

McFadden and his associates have recently completed a detailed study of the mode choices of Bay Area commuters.\(^3\) They surveyed the mode choices of commuters and proceeded to estimate choice probabilities of the form (1.1). In general, two sorts of models were estimated. The first class incorporated detailed information about the individual travellers: for example, annual income and drivers per household. The second class of models, the so-called "naive models," recognise that the detailed

\(^2\) See, for example, Domencich and McFadden (1975).

\(^3\) For a summary, see McFadden, Talvite et al. (1977).
knowledge about commuters supposed by models of the first class is often lacking. The "naïve" models rely for their explanatory power mainly on the service characteristics of the various modes. Table 1 shows one naïve model, estimated for Bay Area commuter (auto versus bus) decisions; the Appendix gives an example of a more detailed model estimated on the same sample. Table 2 compares the explanatory power of the two models.

From Table 2 it may be seen that, while the naïve model does result in some loss of explanatory power, this loss is not substantial. The likelihood ratio index is analogous to the more familiar $R^2$ statistic of regression analysis, except that, as McFadden and Talvitie (1977) point out, "values tend to be considerably lower than those of the $R^2$ index . . . values of 0.2 to 0.4 for [the likelihood ratio] represent an excellent fit" (p. 35, note 1). On this criterion, both models do about equally well. A more practical criterion is their success in prediction. The naïve model correctly predicted 80% of the choices of the sample population; the detailed model was correct 84% of the time. We may therefore conclude that use of a naïve model will not introduce serious error into the results.
2. THE SUPPLY OF TRANSPORT SERVICES

In this section I study the competition between the auto and express bus modes. After describing the nature of automobile travel, I assume that the bus company has a monopoly of express bus supply. I then show that the price and service quality which will maximise profits for the bus operator are obtained by maximising his unconstrained profit function. Finally, I turn to a description of the market within which the competition is assumed to take place, and give empirical estimates of the supply-side determinants of bus profit.

2.1. Competition with the private automobile

I assume, first, that each individual car owner is a price-taker: that is, that no action by him will alter the prices he faces if he uses his car. The results on consumer mode choice reviewed in section 1 make this a reasonable assumption: the concept of auto cost used by Train as an independent variable included only the out-of-pocket auto costs of gasoline, oil, maintenance and parking (see Reid (1977), p. 42). Unlike, for example, insurance costs, which are to some extent dependent on the driving patterns of the individual, the prices of these out-of-pocket items may reasonably be considered given in the usual sense of economic theory.

My second assumption is that there is some given attainable speed on the roads, and that this speed is independent of the volume of traffic. This is of course an unrealistic assumption: see for example Morlok (1978) and Keeler and Small (1977) for discussions of the actual dependence. The following considerations, however, lend plausibility to the assumption. The first has to do with the market within which the provision of transport takes place. As will be explained in the next section, two kinds of roads are involved, arterials/city streets and limited access highways. We model the bus operator as providing express service on the highways, with stops only on the arterials. Thus the speeds for both modes on the highways will be the same; and since, from equation (1.1), mode choice depends only on the differences between the characteristics, speed on the highway is irrelevant. On the arterials and city streets, note that the attainable speeds depend as much on the geometric layout of the roads (the number and nature of crossings and the provision of traffic lights) and on the frequency of stops—which are assumed to be given—as on the number of vehicles. In general, bus speeds will be less than auto speeds; but as a first approximation we may consider them fixed and independent of traffic volume.

2.2. Intermodal competition

Under the assumptions of the last section, no action taken by an individual consumer will be perceived as altering the prices or service characteristics he or she would face if selecting the automobile. I now assume that the bus company can perceive the mode-choice probability functions. Thus, it knows the downward-sloping demand curve for its product. We study the profit-maximising actions of the bus company.

Suppose that there is some fixed number of commuters $Q$ on a given bus route in a given period, and that there are $G$ types of individuals indexed by $g = 1, \ldots, G$. 

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Denote by $d_g$ the proportion of the total commuters in group $g$. If the characteristics of the bus and auto modes are, respectively, $x^b$ and $x^a$, then we may write down, from equation (1.1), the probabilities $P^a_g$ and $P^b_g$ that an individual of type $g$ will select the auto or the bus.

For a given set of modal characteristics $x = (x^a, x^b)$ the total demand for auto travel will be

$$D^a(x) = Q \sum_{g=1}^{G} d_g P^a_g(x, a_g)$$

(2.1)

where $a_g$ is a vector of socio-economic characteristic of group $g$. Then the residual demand for bus service at level $x^b$ is

$$D^b(x) = Q - D^a(x)$$

(2.2)

The bus operator is assumed to maximise profits. From the vector $x^b$ of bus service characteristics, one, say $x^b_k$, will be the fare. If the cost of providing service at level $x^b$ is $C(x^b)$, then the profit function of the bus company is

$$\pi^b(x) = x^b_k D^b(x) - C(x^b)$$

(2.3)

In general, this will depend explicitly on the demand for auto travel. But it can be seen that in this case, because there are only two modes and the mode-choice probabilities depend on both bus and auto characteristics, it follows that there is no explicit dependence on auto demand. We have

$$\pi^b(x) = x^b_k [Q - D^a(x)] - C(x^b)$$

$$= x^b_k [Q - Q \sum d_g P^a_g(x, a_g)] - C(x^b)$$

(2.4)

But for each $g$,

$$P^a_g = 1 - P^b_g; \quad \text{and} \quad \sum d_g = 1$$

so

$$\pi^b(x) = x^b_k Q \sum d_g P^b_g - C(x^b)$$

(2.5)

Therefore in this case, profit-maximisation by the monopolist involves a maximand dependent only on the bus mode-choice probabilities. One has the usual necessary conditions for a maximum:

$$\frac{\partial \pi^b(x)}{\partial x^b_k} : \quad Q \sum d_g P^b_g + x^b_k Q \sum d_g \frac{\partial P^b_g}{\partial x^b_k} = 0$$

(2.6)

and, in setting service levels, for each $x^b_k$ different from $x^b_k$

$$\frac{\partial \pi^b(x)}{\partial x^b_k} : \quad Q \sum d_g \frac{\partial P^b_g}{\partial x^b_j} - \frac{\partial C(x^b)}{\partial x^b_k} = 0$$

(2.7)

That is, the profit-maximising fare level is found by equating the incremental decrease in total bus revenues (since the sign of $\frac{\partial P^b_g}{\partial x^b_k}$ will normally be negative) to total bus demand, and the level of service by setting the marginal revenue from improved

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*It is assumed that the sufficient conditions always hold.
service levels to the marginal cost of providing them. Thus, in order to determine the profit-maximising fare and choice of service levels, it suffices to consider the maximisation of the unrestricted profit function given in (2.5).

2.3 The market

The market within which service is provided is now described in more detail. The observed profitable express buses in the New York City area generally collect their passengers in a residential area and provide express (non-stop) service on a limited-access highway. The market structure assumed here retains this essential layout.5

The market to be considered is shown schematically in Figure 1. A circular residential area of radius \( r \) miles is connected by an expressway or major arterial to a central business district. I assume, for mathematical convenience, that all roads in the residential area are laid out along the radii of the circle. The two types of trips with which we shall be concerned take place in this market as follows. For auto trips, the driver first drives along the residential streets to the expressway entrance at A. He then proceeds along the expressway to C, a distance of \( L + XL/2 \) miles. The point C is the expressway exit; the driver then proceeds along city streets a distance \((XL + 1)/2 \) miles to a parking lot near the work location.

The bus trip is somewhat more complicated, since we must allow for the bus trip being in general more circuitous than auto journeys. The present setting supposes it to be set out thus: proceeding along the residential streets the bus collects its passengers. Instead of getting on the expressway at A, however, the bus proceeds a further \( XL/2 \) miles on residential streets before joining the expressway at B. From this point on, the trip is the same as the auto trip, except that the expressway portion is only \( L \) miles. Thus, the disadvantage of the bus trip is not that it covers a greater distance than the car, but rather that it takes longer to cover the same distance. The parameter \( XL \) is referred to as a “circuity factor” in what follows, though, as the above discussion indicates, it is actually a proxy for circuity. The assumption that the circuity in the residential area equals the circuity in the relevant part of the downtown distribution area is made only to economise on parameters; clearly, it is easily modified. The additional one unit of distance at the CBD end of the trip is included to allow analysis of the case in which circuity is zero. Without this additional distance, zero circuity would be equivalent to assuming that the expressway delivers the commuter straight to the parking lot.6

From the demand model of Table 2 we see that there are three measures of transit service which are important to consumers: the fare, in-vehicle time, and excess time. From the assumptions on congestion noted previously, in-vehicle time is given, for

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5 This structure was first proposed by Pozdena (1976). See also Viton (1978) and Keeler, Small, Viton et al. (1975).

6 Since both costs and demand estimation are taken from a study of Bay Area transport, it may be well to note that this spatial pattern is represented in the Bay Area. Indeed, the express route 31 run by Alameda-Contra Costa Transit is set out along just these lines. The residential area is Richmond. There is a freeway entrance at Richmond, but the 31 proceeds instead down San Pablo Avenue and does not join the freeway (I-80) until Carlson Boulevard. It leaves the freeway at 27th Street, and proceeds down Telegraph Avenue through the business district of Oakland, ending at Broadway and 4th Streets. Thus the service pattern of route 31 is almost exactly described by Figure 1.
fixed speeds along the roads. Excess time is the sum of access time to get to the nearest stop and time spent waiting for a bus. I now show that these measures of service quality are determined by the bus operators' selection of three variables—the number of routes covering the residential area; the bus headway, or number of buses per hour on a given route; and the fare.

Suppose that the residential area is covered by \( R \) equally spaced routes, laid along the radii of the circle as shown in Figure 1. Assume that travellers in the residential area are uniformly distributed along the radii, and that the distribution of the various groups of commuters is similarly random. (If different portions of the residential area are inhabited exclusively by one or more groups, each portion may be analysed separately in the manner explained below.) The average commuter lives at a distance \( r/2 \) miles from the point \( A \). If he walks at 3 mph, the time taken to get from a given house to the nearest stop on one of the \( R \) routes provided is

\[
\text{Walk} = \frac{\pi r}{6R}
\]  

(2.8)
hours. Similarly, suppose that arrivals at this stop are Poisson distributed, and that the bus company runs $1/\psi$ buses per hour over the whole system. Then the headway (the inter-bus interval) observed on each route is $\psi R$ hours. It can be shown that, if arrivals are Poisson distributed, the rule-of-thumb “wait equals half the headway” correctly describes the average wait. That is,

$$\text{Wait} = \frac{\psi R}{2}. \quad (2.9)$$

Thus, a model of operator performance which includes as variables the bus headway, the number of routes provided, and the fare will suffice to determine the demand for bus services in the mode-choice model of section 1.

2.4. Bus costs

The final step before we can write down the complete profit-maximisation problem facing the bus operator is to specify the cost function $C(x^b)$ of providing the service level $x^b = (R, \psi, f)$ where $f$ is the fare charged. Assume that the technology of service is of the Leontief (fixed factor proportions) type, with two inputs, vehicle hours and vehicle miles. Then it can be shown (Varian (1978)) that the associated cost function is linear in the prices of the two inputs. If $C_H$ is the cost per hour of providing service and $C_M$ the cost per mile, then, if $S$ is the (given) expressway speed and SPD the residential/city street speed and if the unit of output is the round trip, the cost per round trip is

$$C^* = 2\left(C_H L + \frac{r + XL + 1}{SPD} + C_M (L + r + XL + 1)\right) \quad (2.10)$$

and the cost function $C(x^b)$ is

$$C(x^b) = \frac{C^*}{\psi} \quad (2.11)$$

The actual values of $C_H$ and $C_M$ used in this study are shown in Table 3, and the assumed speeds in Table 4. Bus operating costs are taken from a study of AC Transit operations. The costs varying with bus hours include a capital cost for the vehicles themselves, based on a replacement cost of $42,000 in 1972, a twelve-year useful life and 6% interest rate. Following Keeler, Small, Viton et al. (1975), all capital costs are allocated to peak operations. The costs varying with vehicle miles, including fuel, oil and maintenance expenses, also include an administrative cost component of 5¢ per mile.

There remains one further level of service quality to discuss. The company operating the express buses charges a fare considerably higher than that charged by local services, and tries to ensure that all travellers are seated. We include this attempt as a requirement: profits are to be maximised subject to the constraint that riders per

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7 See Viton (1974) and Keeler, Small, Viton et al. (1975). Note that these costs are for conventional buses without air-conditioning or luxury seats. Fisher and Viton (1974) in their Appendix A-7 show that (with optimal scheduling) the increased costs per passenger of operating a more luxurious bus are small.
Table 3

Operating Costs

<table>
<thead>
<tr>
<th></th>
<th>Costs varying with bus hours&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Costs varying with vehicle miles&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Bus</td>
<td>$12.77/hour</td>
<td>$0.2557/mile</td>
</tr>
<tr>
<td>II. Automobile-Compact Car</td>
<td>$0.0513/mile</td>
<td>$0.0467/mile</td>
</tr>
<tr>
<td>Freeway operating cost&lt;sup&gt;d&lt;/sup&gt;</td>
<td>$0.0464/mile</td>
<td></td>
</tr>
<tr>
<td>Parking cost&lt;sup&gt;e&lt;/sup&gt;</td>
<td>$1.35</td>
<td></td>
</tr>
<tr>
<td>Auto occupancy&lt;sup&gt;f&lt;/sup&gt;</td>
<td>$1.07</td>
<td></td>
</tr>
</tbody>
</table>

<sup>c</sup> From Keeler, Small, Viton et al. (1975). Includes only gasoline, oil and maintenance costs. Taxes added: gasoline $0.11/gal; oil $0.06/gal. These cost concepts are compatible with the costs used in estimating the demand models. See Reid (1977), p. 42. All costs inflated by 1.0559.
<sup>d</sup> See Keeler, Small, Viton et al. (1975), p. 86.
<sup>e</sup> Ibid, p. 98. Inflation factor, 1.0559. Cost is one-way per auto occupant.
<sup>f</sup> Source: U.S. Census (1970). Journey to Work, based on Richmond to Oakland CBD (San Francisco–Oakland SMSA) travel.

Table 4

Speed Notation and Assumptions

<table>
<thead>
<tr>
<th>Trip Segment</th>
<th></th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential streets/city streets</td>
<td>Auto</td>
<td>ASPD</td>
</tr>
<tr>
<td></td>
<td>Bus</td>
<td>SPD</td>
</tr>
<tr>
<td>Expressway, or major arterial</td>
<td>All vehicles</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25 mph</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14 mph</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50 mph</td>
</tr>
</tbody>
</table>

bus do not exceed available seats. If we assume a 50-seat configuration the constraint is

$$\psi Q \sum_{g=1}^{G} P^b g d_g \leq 50$$

(2.12)

2.5. The full problem

We may now write down the full profit-maximising problem of the express-bus operator in the market of Figure 1. The data for the profit expression, with the exception of the market parameters $r$, $L$ and $XL$ and the distribution of individual types $d_g$—to be discussed in the next section—has already been given; nonetheless it may be convenient to set it out in one place.

The bus operator is assumed to set $\psi$, $R$ and the fare $f$ to maximise
\[ \pi = Qf \sum_{g} d_g P_g^b - C(x^b) \]

s.t. \( \psi \sum_{g} P_g^b d_g \leq 50 \)

where

\( Q \) = number of commuters
\( d_g \) = fraction of \( Q \) in group \( g \)
\[ P_g^b = \frac{1}{1 + e^{\alpha_g}} \]
\[ \alpha_g = \frac{-0.0372 - 2(AC - F) - 0.0322(DAIV) \cdot 2 - 0.0338 \left(60\psi R + \frac{\pi \cdot r \cdot 60}{3R}\right) - 0.322}{W_g} \]
\( AC \) = auto cost per trip (\$ per round trip)
\( F \) = bus fare (\$ per round trip)
\( W_g \) = post-tax wage of group \( g \)
\( DAIV \) = auto in-vehicle–bus in-vehicle time (minutes per round trip)
\( \psi \) = bus headway (hours)
\( R \) = number of residential routes
\( r \) = radius of feeder area (miles)

\[ C(x^b) = 2 \left( C_H \left( \frac{L}{S} + \frac{r + XL + 1}{SPD} \right) + C_M (L + r + XL + 1) \right) / \psi \]
\( C_H \) = bus costs varying with hours (\$12.77)
\( L \) = linehaul (expressway) distance
\( S \) = expressway speed (50 mph)
\( XL \) = circuity factor (miles; see section 2.3)
\( SPD \) = residential streets speed (14 mph)
\( C_M \) = bus costs varying with miles (\$0.2557)

3. THE POSSIBILITY OF PROFITABLE SERVICE

In addition to the costs of providing service, the physical layout of the service area, and the preferences of the commuters, the possibilities of profitable service will also depend on the characteristics of the people being served. The only relevant determinant of demand not under the control of the bus company, or fixed by the assumptions of the model, is the distribution of income. Table 5 shows the categories of wages (and an approximate distribution of pre-tax and post-tax incomes) used by Cluff (1978) in his study of the income distributional effects of pricing rules. It should be stressed that the income categories are only approximate, and are derived from the post-tax wages under U.S. average tax rates. Thus, they contain implicit assumptions
on the distribution of working spouses, number of children and income-tax deductions in the population.

Table 5 shows three different distributions of income $d_x$. The first, distribution I, is the distribution used by Cluff (1978) in his study, and closely approximates the actual distribution of income among Bay Area commuters. To investigate the way in which the possibility of profitable service will vary with different distributions of income, Table 5 also shows two alternative distributions. Distribution II corresponds to a population with poorer workers; thus, the weighted average post-tax wage is $6.2c/minute, as compared with the $8.0c/minute observed for Bay Area commuters. Distribution III corresponds to the reverse case, in which the higher-income commuters outnumber the others. In this case, 60% of the population has an income greater than approximately $14,500 (pre-tax) per year. Here the average post-tax wage is $11.2c per minute. Note, for comparison, that about 60% of families living in metropolitan areas had incomes exceeding $12,000 per year.  

The demand model of Table 1 was estimated by journey-to-work travel. Therefore, I limit the analysis to peak-period travel. Of the 55 express bus lines in the New York City area, less than half run any midday service, and less than a quarter offer weekend buses. Thus it is reasonable to focus on the peak trip only.

### 3.1. The basic case

For the analysis of the basic case we assume that the line-haul distance $L$ is 8 miles, the circuity parameter ($XL$) is 4 miles, and the radius of the circular residential area is 2 miles. These distances, and the speed assumptions of section 2, serve to determine the in-vehicle times required to complete a single commuting trip to the CBD. An auto trip, allowing 2.5 minutes in carpooling delay and parking time at the CBD end of the
<table>
<thead>
<tr>
<th>Commuters/Peak Hour</th>
<th>Income Distribution I</th>
<th>Income Distribution II</th>
<th>Income Distribution III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>System Profit per Hour</td>
<td>Routes per Hour Fare</td>
<td>Buses</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>5,000</td>
<td>1.29</td>
<td>8.1</td>
<td>$</td>
</tr>
<tr>
<td>4,000</td>
<td>1.32</td>
<td>8.2</td>
<td>$</td>
</tr>
<tr>
<td>3,000</td>
<td>1.48</td>
<td>8.1</td>
<td>$</td>
</tr>
<tr>
<td>2,000</td>
<td>2.05</td>
<td>10.5</td>
<td>$</td>
</tr>
<tr>
<td>1,000</td>
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<td>$</td>
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<tr>
<td>800</td>
<td>4.09</td>
<td>3.4</td>
<td>$</td>
</tr>
<tr>
<td>306</td>
<td>800</td>
<td>3.4</td>
<td>$</td>
</tr>
</tbody>
</table>

*Bus cost per round trip is $24.52. Parameters: L = 8, XL = 4, r = 2 miles.

A dash indicates that profitable service is not possible.

Approximate Richmond area distribution.
trip,\textsuperscript{10} takes 23 minutes, while the in-vehicle leg of the bus trip takes 33 minutes.

These speeds and distances closely approximate the distances and speeds of the express route linking Richmond and Oakland in the Bay Area. The assumptions lead to a time of 40 minutes to traverse the route of Figure 1: this is to be compared with a scheduled time for the Bay area route of 41 minutes.\textsuperscript{11}

Table 6 shows, for a variety of assumptions about hourly travel volume, when profitable service along the routes of Figure 1 is possible, and with which characteristics.\textsuperscript{12}

First, with high densities of commuters profitable service is always possible, irrespective of the distribution of income assumed. Substantial profits can be made: even in the case of high-income predominance (distribution III) the hourly system profit is about \$1,000. In the case of income distribution I, the overall market share of the bus system is 31.7%; for the different income groups, shares range from 51.1% of the lowest income class to 19.1% of the highest. For the other two income distributions, the range of market shares is within 3% of that of distribution I.

Second, with a declining number of commuters going to the CBD the possibilities of profit tail off rapidly. The approximate number of Richmond commuters going to work in the Oakland CBD is 1,000 per hour. With this density, the bus system makes a small profit only. For a low-income group of commuters, profits increase to \$48; for the (rich) distribution III, profitable service is no longer possible. When the number of commuters per peak hour falls below 800, profitable service is nowhere possible.

Third, the optimal fare to be charged varies with the residential density, but is relatively constant over the different distributions of income. In all cases where profitable service is possible, the fares are considerably higher than the Richmond–Oakland express bus fare of \$0.35; at the same time all fares are below the observed fares on New York City express buses, which are regulated at \$1.50. The regulated fare, however, is an upper bound. The results shown here indicate that profits could be increased by lowering the fare as the commuter density decreases; the reduction could easily be implemented.

The characteristics of service other than the fare are determined by bus headway and by the route coverage of the residential area. Focusing attention now on income distribution I, and recalling that headway on a given route is routes/buses per hour and that, under the assumptions given in the last section, average wait is half the headway, we see that waiting times vary from approximately 8 minutes at the highest density to 19 minutes at a density of 1,000 commuters per hour, the lowest density allowing for profitable service. In this last case buses run every 35 minutes; that is comparable to the headways now observed on the Richmond–Oakland express route.

The difference in quality of service lies in the extent to which the residential area is covered by bus routes, since this is the measure of accessibility. At present the express route goes straight through Richmond; that is equivalent in the model used here to two routes. By comparison, in the case of 1,000 commuters per peak hour, more than three routes are optimal. The effect of this increased accessibility is that the walk to

\textsuperscript{10} These assumptions correspond to those in Keeler, Small, Viton \textit{et al.} (1975).

\textsuperscript{11} AC Transit Schedules (1975).

\textsuperscript{12} The estimation of the nonlinear minimisation program of section 2.5 was done on the DEC-10 at the University of Pennsylvania, using the program ACDPAK written by Michael Best of the University of Waterloo. See Best (1975) and Best and Ritter (1974).
the nearest bus line is reduced by a factor of 1.6. Walk time in this case is 19 minutes. As residential density increases, access time falls; at the highest density considered, the average commuter lives only 8 minutes from the nearest bus route.

### 3.2. Profitability under alternative conditions

Part of the reason for the unprofitability of the bus system when commuter densities are low lies in the assumed circuitry factor $XL$. In the basic case discussed above, the bus was assumed to travel an extra 2 miles at the arterial speed (12 mph) while the competing auto mode was able to cover the same distance at the expressway speed (50 mph). This pattern is characteristic of the Richmond–Oakland express service, but it need not be typical. In New York, the express route linking Co-op City in the Bronx with mid-Manhattan has a comparatively short collection route (from DeKruyt Place to Alkott Place) and then approaches Manhattan on the Bruckner and Major Deegan Expressways.

Table 7 shows, for income distribution I, the possibilities of profitable service in such a situation. The circuitry factor is set to zero at both ends of the route, so that some of the perceived disadvantage of the bus mode—a lengthy period of slow travel—disappears. The most striking conclusion to be drawn from this table is that profitable service is now possible under a much wider set of circumstances. Whereas in the previous case a commuter density of 1,000 or more per peak hour was necessary, here profitable service is sustainable when as few as 400 commuters per peak hour travel between home and the CBD. From 1970 census data (Census Tract 462.01), we observe that the number of work trips originating in Co-op City and ending in the New York City CBD is 1,073, 21% of all Co-op City work trips. This corresponds to an approximate peak-hour density of 500 to 700 commuters. Even for these relatively low commuter densities, the results of Table 7 show that profitable service is possible; it is interesting that the profit-maximising fare is generally below the observed express-bus fare.

Table 8 investigates the effect on profitability of a slower expressway speed, which
may be due to either of two causes. The first is that there is underinvestment in highway capacity, and thus excessive congestion. Keeler and Small (1977) have investigated the question of optimal expressway investment; they conclude that optimal speeds should be between 48 and 55 mph. Thus the assumption of a 50 mph speed in the cases discussed above may be thought of as corresponding to an optimal level of expressway investment. Table 8 assumes an expressway speed of 30 mph, and may be considered as corresponding to underinvestment in expressways. The other possible cause of the 30 mph speed is that all traffic travels on a major arterial rather than an expressway. In this case, the notion of an "express bus" means only that on the line-haul portion (L) of the route no stops are made. These two interpretations of the case of 30 mph "expressway" speed may have wider relevance to the actually observed operating conditions of many bus companies than the cases discussed in Tables 6 and 7.

Table 8a shows the possibilities for profitability and the service characteristics

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13 Keeler and Small (1977) for a 6% interest rate on Urban-Suburban or Central City freeway.
14 Keeler and Small (1977) also find that for, a social-welfare maximum, congestion tolls of about 3.3¢/auto-mile and 15.2¢/auto-mile are optimal (see note 13). The model discussed in this paper has no congestion pricing, as congestion pricing is not generally found.
Table 9

Residential Speed 7 mph*

<table>
<thead>
<tr>
<th>Commuters/ System Profit Fare Routes Buses Overall</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Hour</td>
<td>per Hour</td>
<td>per Hour</td>
<td>per Hour</td>
<td>per Hour</td>
</tr>
<tr>
<td>5,000</td>
<td>$48</td>
<td>$0.85</td>
<td>4.3</td>
<td>8.8</td>
</tr>
<tr>
<td>4,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Circuit Factor (XL) = 0

<table>
<thead>
<tr>
<th>Commuters/ System Profit Fare Routes Buses Overall</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Hour</td>
<td>per Hour</td>
<td>per Hour</td>
<td>per Hour</td>
<td>per Hour</td>
</tr>
<tr>
<td>5,000</td>
<td>1,585</td>
<td>1.30</td>
<td>8.7</td>
<td>35.8</td>
</tr>
<tr>
<td>4,000</td>
<td>1,138</td>
<td>1.25</td>
<td>7.8</td>
<td>19.0</td>
</tr>
<tr>
<td>2,000</td>
<td>348</td>
<td>1.14</td>
<td>5.8</td>
<td>16.2</td>
</tr>
<tr>
<td>1,000</td>
<td>56</td>
<td>1.00</td>
<td>4.1</td>
<td>8.2</td>
</tr>
<tr>
<td>800</td>
<td>15</td>
<td>0.94</td>
<td>3.6</td>
<td>6.3</td>
</tr>
<tr>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Income distribution I.

when there is a circuit factor (XL) of 4 miles; Table 8b gives the same information when this disadvantage is absent. The results of this case, when compared with those of Tables 6 and 7, confirm our belief that expressway speeds have little effect on profitability. The reason is that, as noted earlier, all traffic is assumed to move at the same speed on the expressway (non-stop bus service), and it is only differences in service characteristics that affect demand. Generally, profitable bus service is possible with 30 mph speeds just when it is possible with 50 mph speeds, though profits and service characteristics differ slightly.

Much more important than linehaul speeds is the assumed speed in residential or city streets. That buses must stop to pick up and discharge passengers along their routes, and consequently passengers must spend longer in the vehicle than they would in a car, is thought to constitute a serious disadvantage for transit. Tables 9a and 9b investigate this disadvantage, assuming that the attainable speed is 7 mph, as opposed to the 12 mph in the basic case. The results are just what we might expect. When the slow speed is combined with the 4-mile circuit factor, profitable service is barely possible at the extremely high commuter density of 5,000 persons per hour going from the residential area to the CBD, and impossible at lower densities. This case may be thought of as corresponding to the case of heavily congested residential streets. With part of the bus disadvantage removed, Table 9b shows that profitable service is possible when more than 800 commuters travel per peak hour. Profits, however, are much reduced from those possible at 12 mph speeds (see Table 7).
4. CONCLUSIONS

Previous research fails to explain when and where profitable bus service might be possible. Thus, the examples of profitable service, which disprove the view that bus operations must run at a loss, raise the question how far other operators could implement their own profitable services. This paper has attempted to provide a theoretical framework within which we can analyse the question.

Commuter behaviour is modelled as a discrete mode choice by utility-maximising consumers. Two modes are offered: the private car, with fixed characteristics, and a system of buses. The bus operator acts as a duopolist, setting fare and quality of service to maximise profits in a given residence-to-CBD travel market.
For a particular set of consumer preferences (demand functions expressed as mode-choice probabilities) estimated for Bay Area commuters, the empirical estimates of the last section show that profitable service is possible over a wide variety of commuter travel densities, income distributions, and assumptions about the physical layout of the service area. Figure 2 summarises these results, displaying maximum potential system profit per hour as a function of some of these variables; the actual profit-maximising service characteristics which yield the given profits are shown in the detailed tables of the last section.

These results open the very real possibility that selected portions of a transit system may be operated at a profit, either by franchised companies, or by the transit system itself in order to reduce an overall system deficit. In the light of recent measures of tax-reduction, this latter possibility may be not without interest.

Two related points are worth noting. First, if in a given situation profits are not possible, one cannot therefore decide that service ought not to be provided. In no case examined here was profitable service possible with 200 commuters per peak hour. This is not to say that in low-density markets no service should be offered. The reason lies in the structure of the model used here: the model involves only the maximisation of profits of producers, and does not discuss at all the associated benefits to consumers. Thus we cannot say whether a decrease in profits might not be accompanied in certain cases by an offsetting net benefit to consumers. Indeed, since when buses are not full an additional passenger can be transported at zero cost, one may argue that marginal-cost pricing requires a zero fare and an associated deficit, to be made up from taxes. In a sector with short-run economies of density (or with indivisible and lumpy supply) the presence or absence of profits is no sure guide to economic efficiency.

Secondly, just because in a given instance profitable service is possible, there is no inference that it ought to be provided by an unregulated single franchise. Two issues are at stake here. First, for the reasons noted above, an unregulated profit-maximiser will not in general provide the welfare-maximising output and price. In this instance, some form of regulation may be desired in order to modify the behaviour of the bus carrier. In New York City fares are regulated at upper limits, and the regulated fare is the one that universally prevails among the express-bus operators. Where the traffic is too small to allow profitable service, various forms of subsidy have been proposed to induce private carriers to enter the market (SPUP (1977)). The evaluation of some proposals for regulation and subsidy will be discussed in a later paper.

A further issue is the choice between a public and a private carrier. If a market is opened up to private enterprise, then service will be provided on all those routes where profitability is possible, and only on those; it will be left to a municipal system to provide service at a deficit on the remaining routes. By skimming the cream (the potentially profitable routes), private operators may force even lower service levels and greater deficits on the municipal system. Two important questions arise: first, when will cream-skimming be possible? And second, what can we say about the welfare implications of cream-skimming? The possibility cannot be dismissed that, from a net welfare standpoint, cream-skimming, by providing profitable service in one market, may increase overall social benefits. These questions will be discussed in a future paper.
## APPENDIX

### Specification of Detailed Model

**MODEL 9 (Mode 1—auto; mode 2—bus)**
Model: Multinomial logit, fitted by the maximum likelihood method

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Estimated Coefficient</th>
<th>$T$-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost divided by post-tax wage, in cents + per minute</td>
<td>-0.0343</td>
<td>5.60</td>
</tr>
<tr>
<td>On-vehicle time, in minutes</td>
<td>-0.0233</td>
<td>2.80</td>
</tr>
<tr>
<td>Walk time, in minutes$^a$</td>
<td>-0.0240</td>
<td>3.67</td>
</tr>
<tr>
<td>Transfer wait time, in minutes$^a$</td>
<td>-0.0536</td>
<td>2.39</td>
</tr>
<tr>
<td>Number of transfers$^a$</td>
<td>0.0249</td>
<td>0.205</td>
</tr>
<tr>
<td>Headway of first bus, with a ceiling of 8 minutes, in minutes$^a$</td>
<td>0.109</td>
<td>3.30</td>
</tr>
<tr>
<td>Headway exceeding 8 minutes of first bus, in minutes$^a$</td>
<td>-0.0113</td>
<td>1.02</td>
</tr>
<tr>
<td>Family income with ceiling of $7,500, in $ per year$</td>
<td>0.000113</td>
<td>1.25</td>
</tr>
<tr>
<td>Family income minus $7,500 with floor of $0 and ceiling of $3,000, in $ per year$</td>
<td>0.000130</td>
<td>0.928</td>
</tr>
<tr>
<td>Family income minus $10,500 with floor of $0 and ceiling of $5,000, in $ per year$</td>
<td>-0.0000707</td>
<td>0.928</td>
</tr>
<tr>
<td>Length of residence in community, in years$^b$</td>
<td>0.121</td>
<td>3.82</td>
</tr>
<tr>
<td>Number of persons in household who can drive$^b$</td>
<td>0.550</td>
<td>3.64</td>
</tr>
<tr>
<td>Auto alone alternative dummy$^c$</td>
<td>-3.80</td>
<td>5.07</td>
</tr>
</tbody>
</table>

Likelihood ratio index 0.487
Log likelihood at zero -534.4
Log likelihood at convergence -274.3
Percent correctly predicted 83.53

Values of time saved as a percent of wage:
- On-vehicle time: 68%
- Walk time: 70%
- Transfer wait time: 157%

All cost and time variables are calculated for the round trip. Dependent variable is alternative choice (one for chosen alternative, zero otherwise). Sample size: 771.

Source: Train (1976).

$^a$ The variable is zero for the auto alternative, and takes the described value for the other alternative.

$^b$ The variable takes the described value for the auto alternative and zero otherwise.

$^c$ The variable is one for the auto alternative and zero otherwise.

## REFERENCES

September 1980

Oakland Tribune. Oakland, California.