THE PRICE-DISCRIMINATING PUBLIC ENTERPRISE, WITH SPECIAL REFERENCE TO BRITISH RAIL

By S. D. Trotter*

1. INTRODUCTION

Many public enterprises, by virtue of having a considerable degree of either natural or legal monopoly power, are in a position to practise market segmentation and price discrimination; in the last fifteen years few have pursued them with such enthusiasm, at least in public, as British Rail. Much of the early literature on discriminatory pricing was indeed framed in terms of railways and other public utilities, but this is hardly a sufficient explanation. In this paper we consider when in general we would expect a public enterprise to adopt price discrimination, and in particular why British Rail’s market situation has apparently lent itself so readily to that policy.

To this end we look in the next section at the particular circumstances facing British Rail, concentrating on the passenger market only, and in the third section discuss a simple model (the main theoretical analysis is contained in the Appendix). The fourth section then considers the results of the theoretical work, and how far they might be applied in practice; the paper concludes with a brief summary.

2. BRITISH RAIL: THE BACKGROUND

All nationalised industries have to satisfy certain statutory requirements, and in the case of British Rail there are both financial and service criteria to be met. On finance, the 1978 White Paper on Nationalised Industries required that “break-even is to be achieved after receipt of grants” (Chancellor of the Exchequer

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(1978), page 38). More recently an “external financing limit” has been set for the Board. The requirement for the level of services remains that of the Railways Act 1974: “The British Railways Board shall from 1 January 1975 operate its railway passenger system so as to provide a public service which is comparable generally with that provided by the Board at present” (Direction by Secretary of State, 19 December 1974, under Section 3 of the Railways Act 1974).

Achieving both these objectives at once has become more difficult in recent years; this has been due to three main factors. One has been the economic recession, which has reduced the demand for travel in general, and for business travel in particular. A second factor has been the fall in the level of government financial support under the Public Service Obligation, which has meant that an increased proportion of total receipts has had to come directly from rail passengers themselves. Finally there has been a continuing increase in competition: general trends of rising car ownership and motorway mileage, for example, have continued and, more particularly in the case of British Rail, there have been the effects of the deregulation of long-distance coach services in October 1980. (For a good analysis of this from a British Rail standpoint see Bleasdale (1983).)

Though the coach market is relatively small compared with the total rail market, the impact on British Rail has nevertheless been quite significant. The main effect has been in the price-sensitive leisure market; passenger journeys by express coaches have increased from around eight million in 1980 to 13.3 million in 1983. Bleasdale (1983, page 516) suggests a direct loss of revenue to Inter-City services of around £15 million (on a £450 million turnover), but points out that there are also more complicated indirect effects. Rail discount fares have been lower than they might otherwise have been — the initial reductions in some “Saver” fares were very large — and this will have reduced revenue; on the other hand, additional new travel will also have been generated. The net effect on revenue appears to be small. Geographically the effects have been unevenly spread; the main impact has been in the Western Region, and to a lesser extent on the very long-distance routes, where British Rail finds itself caught between airline competition for the time-sensitive market and coach competition for price-sensitive customers.

In addition to these external influences, British Rail’s strategy has been determined by the particular features of its own market. The main ones are the uneven loading of trains at different times of the day or week, and the fact that certain identifiable sectors of the population were using rail very little: students, for example, had transferred in large numbers to coaches in response to lower fares, and families increasingly tended to travel by car.

Thus the situation facing British Rail in the absence of discrimination — the status quo ante, so to speak — is one of increased competition, declining or at best static external support, and a service being used mainly at certain times by certain sectors of the population. The choice of how best to approach this situation is to a large extent made for the Board already by the particular nature of the demand for travel (by any mode, not just rail): travel is a means to an end, and we assume that passengers do not derive any inherent utility or disutility from rail travel itself, only from its abstract characteristics such as price and

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quality of service. Demand will therefore depend on the demand for the purpose
for which the journey is being made; pace Robert Louis Stevenson, people travel
only to arrive.

The two prerequisites for the pursuit of price discrimination are that different
sections of the market should manifest different demand characteristics, and
that they can be effectively separated. As far as the first of these is concerned,
distinct demands for rail travel do indeed exist: on price alone they range from
the London commuter who is often "locked in" to rail, at least in the medium
run, to the person making an optional pleasure trip at the weekend, whose
demand may be highly price-elastic. Equally, different consumers' relative weight-
ings of price and quality of service — whether speed, or frequency, or comfort —
will vary considerably, and this too can be exploited.

Segmentation of the market is helped by some of the other features of travel:
as a commodity it is both perishable and non-transferable. An empty seat, once
carried, is gone for ever; also customers cannot buy travel at cheap times and save
it up for subsequent use. Nor can it be exchanged among consumers, as it is a
personal service: there is little scope for one person to travel instead of another
to take advantage of a cheaper fare. Thus in general arbitrage is unlikely.

To take full advantage of this situation, however, would require a monopoly
position in the market, and we have seen that here British Rail is less well
favoured. In the commuting markets of London and the South East rail enjoys
considerable monopoly power; elsewhere, and in other markets, the car, the bus,
the coach and the aeroplane are all in contention too, and British Rail's freedom
of action is correspondingly much reduced.

On the cost side, too, the scope for action is limited. The statutory require-
ment to maintain a service “comparable generally with that provided . . . at
present” means that the timetable, and therefore costs, are fairly fixed. Even
apart from this, the long lead times involved in any major changes to the time-
table mean that action on the revenue side is much quicker to bear fruit. The
result tends to be that “most fares decisions are less a question of weighing up
revenue against cost than of weighing up the extra traffic (and benefits) a par-
ticular promotion will achieve against the loss of revenue from existing passengers
diverting to the lower fare” (Nash (1982), page 140).

In sum, then, British Rail's situation is that of an industry with relatively fixed
costs, selling a perishable and non-transferable product in a market where identi-
ifiable groups of consumers have differing demands. The traditional literature
concludes immediately that discriminatory pricing is the appropriate policy; we
turn now to whether this is also true for a non-profit-maximising public enter-
prise. For now we assume that there are no constraints on pursuing such a policy
if it is theoretically optimal to do so; how reasonable this is is discussed in
Section 4.

1 The rigidity implied by this should not be exaggerated: the Minister for Transport said at
the time that “the direction does not imply that there will be no changes in the coverage and
quality of the passenger services the Railways Board provides”. (Written answers, Hansard,
19 December 1974, Volume 883, Column 607.)
3. THE THEORY OF THE PRICE-DISCRIMINATING PUBLIC ENTERPRISE

Most recent published work on the subject of price discrimination, which stems from the contributions of Pigou (1932) and Joan Robinson (1933), is concerned with the case of the private profit-maximising firm. In these circumstances it is a standard result that, if different markets do exist and can be separated, it is optimal for the firm to practise discrimination (and may be beneficial to society too if output is raised). The literature has expanded and developed this original result in several directions, but little attention has been given to whether it still holds for organisations under different ownership and with different goals.

One exception to this general neglect is Baumol and Bradford’s important 1970 paper, which does have some bearing on the matter; though they do not specifically include discrimination when they consider the maximisation of social welfare, they point out that the pricing rule they derive is similar to that which would be obtained from profit maximising with discriminatory pricing. More recently, work by Nash (1978) and Glaister and Collings (1978), reported in this Journal, has considered different possible maximands for public transport operators. Nash, in the context of bus services, does distinguish between peak and off-peak services; and Glaister and Collings, who are thinking mainly of London Transport, consider an operator providing separate services to three different markets. But neither of these articles includes price discrimination in the sense of charging different prices to different people for the same service. It is this most clear-cut example of discrimination that we concentrate on here: charging more than one price for a ticket that confers the same right to occupy a seat on a particular train.

Cooter and Topakian (1980), in their article on the pricing decisions of a public transport system, also consider a range of possible objective functions. However, they treat passengers from each station in a local transport network (the Bay Area Rapid Transit system in the San Francisco area) as a separate and single market with its own distinctive cost function, so they are not mainly concerned with possible price discrimination. Here this will be considered as an essentially demand-side phenomenon, consisting of something more than a reflection of differing costs; in the case discussed above, the cost of providing a seat on a train is independent of who subsequently sits in it.

In view of this apparent gap in the literature, it seems worthwhile to consider what a theoretical approach predicts if a public enterprise like British Rail pursues a goal other than profit maximisation, and is permitted to practise discriminatory pricing. In the remainder of this section we outline a simple model, and then use it to examine the consequences of the pursuit of different, non-profit objectives, subject to plausible constraints. The bulk of the more technical detail is relegated to the Appendix.

3.1 The Model

We consider a single product or service being sold in two different markets, with inverse demand curves $f_1(q_1)$ and $f_2(q_2)$. The price in each market ($P_i = f_i(q_i)$) therefore depends only on the quantity sold in that market; the implicit assump-
tion is that cross-elasticities of demand are zero. This is clearly an over-simplification of reality, but we follow the writers mentioned above in making it to keep the results tractable. In particular we should bear in mind that it rules out any effect on one group's demand from resentment at a cheaper price being available to another group, a point we return to later.

The cost of supply is $C(q_1 + q_2)$, $C' > 0$, and does not depend on the quantity of any other product the firm may be selling in these or any other markets. Nor is it affected by which of the two markets an additional unit of output is sold in; implicitly, advertising and selling costs are being taken as equal in the two markets. This may well be rather unrealistic; for example, filling otherwise empty seats on businessmen's trains by encouraging their wives to take a day out by the same train may require much more promotional expenditure than selling the original business travel tickets. The cost side of our model differs from that of Glaister and Collings, in which costs are determined separately in each market.

Three main maximands will be considered: profit (mainly for comparison), output and a measure of social surplus. While clearly not exhausting all the possibilities, this list does include those likely in practice to be set for a public enterprise, at least in the current framework of operations in the United Kingdom.

### 3.2 Maximising profit or revenue

We start by considering profit maximisation, initially with no constraints, as this is the case dealt with in the mainstream literature and forms a useful benchmark for the later results.

Profits are given by

$$\pi = q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2)$$

and maximising this expression with respect to $q_1$ and $q_2$ separately gives the familiar condition

$$MR_1 = MR_2 = MC,$$

where $MC$ is the marginal cost of producing another unit of output for either market, and therefore has no subscript. Writing $\eta_i = [f_i(q_i)/q_i]$, $[\partial q_i/\partial f_i(q_i)]$ for the elasticity of demand in market $i$, marginal revenue can be expressed as $P_i(1 + 1/\eta_i)$. Equal marginal revenues thus generally mean unequal prices:

$$P_1(1 + 1/\eta_1) = P_2(1 + 1/\eta_2)$$

or

$$P_2 = \frac{\eta_2(1 + \eta_1)}{\eta_1(1 + \eta_2)} P_1.$$  \hspace{1cm} (3)

So if, for example, $\eta_1 = -1.2$ and $\eta_2 = -1.5$, then $P_2 = \frac{1}{2} P_1$.

If a profit-maximising enterprise faces an inelastic demand ($|\eta| < 1.0$) in either or both markets, it will continue to raise price and reduce output there until either the absolute value of the elasticity reaches unity or only one unit is demanded in that market. Thus a reasonable constraint for society to impose on a public enterprise whose goal remained profit maximisation would be a minimum
specified level of output. Consider the maximisation of profit subject to the constraint \( q_1 + q_2 \geq \bar{Q} > 0 \); that is, it is the total output we are concerned with, and not its distribution between the two markets. Then we can consider the Lagrangean expression

\[
Z = q_1f_1(q_1) + q_2f_2(q_2) - C(q_1 + q_2) + \lambda(q_1 + q_2 - \bar{Q}),
\]

(4)

where \( \lambda \) represents the shadow price of output in terms of profit: profit will increase by \( \lambda \) for every mile the output constraint \( \bar{Q} \) is reduced.

The most likely result (see the Appendix) is that the output constraint will bind and the results will be

\[
MR_1 = MR_2 = MC - \lambda
\]

(5)

which is obviously a generalisation of the previous result. Again, there is no reason to expect prices to be equal. Marginal revenue is kept below marginal cost to the extent necessary to fulfil the output requirement; if no binding floor to output is specified, then \( \lambda = 0 \) and we revert to equation (2).

So, as expected, the enterprise that wants to maximise profits will price differentially according to the elasticity of demand. A related objective would be to maximise total revenue, subject perhaps to some profit or output constraint. Although perhaps unlikely to be set as a goal by the government, it might well be an objective favoured by the management of a nationalised industry, for the sort of reasons advanced by Baumol (1959) and Williamson (1964) and on the basis of the inclusion of firm size or sales, for example, in the utility functions of managers. Thus it might be seen as something of a de facto objective, rather than a de jure one.

In the case where costs are fixed, maximisation of revenue or of profit amounts to the same thing. So the pricing section of British Rail might take the output decision as given — taking the timetable as already set by the operating department, for example — and simply maximise revenue; then if the inequality in (4) is reversed, and costs are taken as fixed, (5) is changed to

\[
MR_1 = MR_2 = \lambda
\]

(5')

and it will again pay to separate the markets. This sort of behaviour seems quite plausible in an industry subject to strict cash limits and with relatively fixed costs in the short run; during the 1970s, according to Bleasdale (1983, page 515), “BR developed its pricing policy to maximise net revenue from price-sensitive markets”.

More generally, revenue maximisation could be a target in its own right. Taken as it stands, it would mean raising output until the marginal revenue in each market was equal to zero; in practice, the likelihood of a sizeable deficit due to neglect of costs means that some constraint is likely to be imposed. Two possibilities are a minimum absolute profit (Williamson (1964, page 79) analyses this case) and a minimum rate of return on turnover. We can combine both of these in a general inequality

\[
q_1f_1(q_1) + q_2f_2(q_2) \geq (1 - \delta)C(q_1 + q_2) + \pi,
\]

setting \( \delta = 0 \) for the first equation and \( \pi = 0 \) for the second. Maximising total

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revenue subject to this constraint will lead to the general condition

\[(1 + \lambda)MR_1 = (1 + \lambda)MR_2 = \lambda(1 + \delta)MC\,.
\]

(6)

Once more we find there is no reason to expect equal prices.

3.3 Output maximisation

Profit maximisation, constrained or not, is perhaps an unlikely aim for a public enterprise, in the United Kingdom at least; indeed, one reason advanced for nationalisation in the first place is that profits are the wrong goal for basic industries meeting common needs such as fuel or transport or communication. The main value of the preceding section lies in the provision of comparisons with the results of other policies.

Achieving maximum output is one alternative: maximising passenger-miles has been adopted as a target by London Transport (see Glaister and Collings (1978) and the references therein). It was also put forward by Sir Peter Parker, when Chairman of British Rail, in his 1978 Haldane lecture. There he considered various possible "ways to run a railway", including "the system . . . effectively working at the present . . . living within the Government's financial constraint, of cash limits and of trying to 'beat this contract' by . . . (improving) our efficiency and productivity" (1978, page 9). To do this it is necessary to develop a set of management objectives: "so we have been moving towards . . . 'trying to maximise passenger miles within a given financial constraint'" (page 10).

It is of course necessary to impose some constraint to prevent the problem from becoming degenerate: output maximisation alone can be achieved simply by setting a zero fare — or even better, indeed, by paying people to travel (though in practice even very large external benefits in reducing congestion or fuel consumption would not justify an infinite subsidy). Imposing a minimum absolute profit level (which need not be positive) as a constraint on the output level achieved brings us close to the sort of "financial target" which is actually set for public enterprises; a profit level of zero clearly corresponds to a "break-even" constraint. Maximising the appropriate Lagrangean leads to the result

\[MR_1 = MR_2 = MC - 1/\mu\,.
\]

(7)

where \(\mu\) is the shadow price of profit in terms of output.

If the required profit is a percentage rate (\(\delta\)) on turnover, it is a simple matter to show that the result becomes

\[MR_1 = MR_2 = (1 + \delta)MC - 1/\mu\,.
\]

(7')

In both (7) and (7') equal marginal revenues will again mean that the conditions at (3) above will hold for prices and elasticities.

Glaister and Collings (1978, page 308) point out that the symmetry in these results itself indicates that we may expect to find price discrimination and cross-subsidisation under maximisation of output, as we know this will occur when profits are the goal. In both cases there will be an incentive to use an inelastic market to produce revenue and an elastic one to produce output, whichever is the target and whichever the constraint. The results of their numerical examples
<table>
<thead>
<tr>
<th>Objective</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Profit</strong></td>
<td>$MR_1 = MR_2 = MC$</td>
</tr>
<tr>
<td><strong>Revenue</strong></td>
<td>$MR_1 = MR_2 = 0$</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>$P_i = P_1 = 0$</td>
</tr>
<tr>
<td><strong>Separate</strong></td>
<td>$P_i = P_1 = 0$</td>
</tr>
<tr>
<td><strong>Consumer’s</strong></td>
<td>$P_i - P_1 = \phi(MR_1 - MC)$</td>
</tr>
<tr>
<td><strong>surplus</strong></td>
<td>$i = 1, 2$</td>
</tr>
<tr>
<td><strong>Social</strong></td>
<td>$P_i - MC = -\phi(MR_1 - MC)$</td>
</tr>
<tr>
<td><strong>surplus</strong></td>
<td>$i = 1, 2$</td>
</tr>
</tbody>
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of output maximisation suggest that such an outcome is indeed quite plausible.

So far in this section we have considered the two markets together. We could also consider an enterprise required to maximise output in each market separately subject to a local profit constraint. Clearly the total cost must be divided in some way between the two, and an obvious way to do this is to assign it to each market in proportion to the output there. If we do this we find the optimality condition becomes

$$MR_1 + 1/\mu_1 = MR_2 + 1/\mu_2 = MC,$$

where the $\mu_i$ are the Lagrangean multipliers on the two separate profit constraints. (All these results reappear in Table 1.)

3.4 Social Welfare

In an ideal world the levels of output and prices would be set so as to maximise the welfare of the people in general, that is, their "well-being" or utility. Even if that economist's dream may never be achieved, it is worth seeing what the conditions are that would rule there; we may as well at least know what Utopia would look like.

The usual measure of welfare that is used in this context is consumer's surplus; producer's surplus may be added to take into account both parties to the deal. Consumer's surplus is only a theoretically correct measure of welfare if income effects are zero — Marshall himself (1920, page 842) noted that "We assume that the marginal utility of money to the individual purchaser is the same throughout". As long as the product in question accounts for only a small part of total expenditure by the consumer, we shall not be introducing a serious error if we use consumer's surplus.²

Consumer's surplus is a measure of the difference between the valuation placed by the consumer on the quantity he acquires (measured by the area under the demand curve to the left of the chosen quantity) and the amount he actually has to pay for it (the rectangle described by the axes and the equilibrium price and quantity lines). The excess of the former over the latter arises because the valuation of the intra-marginal units is higher than that of the marginal unit — that is, the price.

The results of simple maximisation of consumer's surplus are obvious: prices will be set at zero. In practice, however, the resulting financial deficit, like that due to output maximisation alone, is likely to prove unacceptable, and some form of financial constraint would be imposed to prevent this. Again we could specify some minimum — possibly non-positive — level of profit that must be earned. If we do so, we can show in the usual way that we must have

$$\frac{(MR_1 - MC)/P_1}{(MR_2 - MC)/P_2} = \frac{1/\eta_1}{1/\eta_2}.$$  

² A good discussion of this issue can be found in Willig's paper, where he shows that "observed consumer's surplus can be rigorously utilised to estimate the . . . correct theoretical measures of the welfare impact of changes in prices and income of an individual" (1976, page 589).
This is reminiscent of the result obtained by Baumol and Bradford (1970); their solution, however, has prices in place of the marginal revenues in (9). They initially considered the maximisation of a general social welfare function \( Z(P_1, \ldots, P_n) \) subject to an absolute profit constraint \((\pi)\), and derived the marginal condition \( \frac{\partial Z}{\partial P_i} = \lambda \frac{\partial \pi}{\partial P_i} \). They then took the compensating variation corresponding to the price change \( \frac{\partial Z}{\partial P_i} = -q_i \) as the derivative of the benefit function in the marginal condition above, and manipulated it to get their answer. Thus the difference between their result and equation (9) follows from the choice of a different social welfare function.

If instead of a budget constraint we impose a required output level — it will need to be a maximum if we are not to find ourselves in the zero price situation of section 3.3 — then we find

\[
P_1 - MR_1 = P_2 - MR_2
\]

or

\[
P_2 = \frac{\eta_2}{\eta_1} P_1.
\]

In this case, if Market 2 is the more elastic one, then the equilibrium price in Market 2 will be the higher. The rationale for this is that the gain in consumer’s surplus for a given rise in output — this is the constraint making itself felt — is greater in the inelastic market, and so price will be dropped further (or raised less) in this market.

A wider measure of welfare is the sum of producer’s and consumer’s surpluses, the former being the difference between receipts and cost of production: that is, what was termed profit above. Differentiation of the maximand in this case yields

\[
P_1 = P_2 = MC.
\]

Because of the joint cost function, we have the familiar (non-discriminatory) case of marginal cost pricing.

Maximisation of this objective subject to a budget constraint gives

\[
\frac{P_1 - MC}{P_2 - MC} = \frac{MR_1 - MC}{MR_2 - MC}
\]

or

\[
\frac{(P_1 - MC)/P_1}{(P_2 - MC)/P_2} = \frac{1/\eta_1}{1/\eta_2},
\]

which is exactly the Baumol-Bradford result, with \( MC_1 = MC_2 \). It is interesting that we can also arrange the last form to get

\[
MC = \frac{P_1 P_2 (\eta_1 - \eta_2)}{P_1 \eta_1 - P_2 \eta_2},
\]

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3 “Summarising, the traditional welfare function employing an equally weighted sum of consumers’ and producers’ surpluses . . . is an appropriate measure of social benefits whenever income effects are not too large and the policy variations under consideration are not too drastic” Crew and Kleindorfer (1979), page 13. These conditions seem likely to be met in the case of rail travel.

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which can also be obtained from equation (9). In that case we maximised consumer's surplus alone, which requires lowering $P_1$ and $P_2$ until the profit constraint binds. For social surplus we wish to lower prices as far as possible towards marginal cost; the profit constraint will stop us at the same point as before.

The two forms in (12) mean that we can express the result by saying either that the mark-up of price over marginal cost is proportional to the difference between the latter and marginal revenue, or that each price is set "so that its percentage deviation from marginal cost is inversely proportional to the item's price elasticity of demand" (Baumol and Bradford (1970), page 267). Alternatively, we may follow Ylä-Liedenpohja (1981, page 48) and, defining $\eta^* = \eta_i(1 + \phi)/\phi$ (where $\phi$ is the shadow price of profit), rearrange (12) (or (A20)) to get the rule

$$P_1 (1 + 1/\eta^*_1) = P_2 (1 + 1/\eta^*_2) = MC$$

(13)

This is a modification of the profit-maximiser's rule of equating marginal revenue and marginal cost, with $\eta^*$ (a measure of elasticity adjusted to take into account the budget constraint) replacing $\eta_i$ in the definition of the former. The effect is to overstate the marginal revenue, as $(1 + \phi) > \phi > 0$.

If, finally, we were to maximise social surplus subject to achieving some minimum output level, then we find that

$$P_1 = P_2 = MC + \phi.$$  

(14)

In this case prices are not discriminatory, and differ from marginal cost because of the requirement on output.

All our results so far, and a few not discussed here, may be drawn together for the sake of comparison in Table 1.

3.5 Conclusions from the model

Various results can be seen from Table 1. Firstly, a public enterprise will not always want to discriminate, even if allowed to do so. Unconstrained maximising of social surplus, for example, leads to marginal cost pricing; in our formulation of the problem, that means that prices are also equal to each other. (Note that possible differential selling costs have not been considered here.)

Nevertheless, discrimination does feature fairly frequently in the table. With the exception of unconstrained output maximisation, the profit, revenue and output combinations in the first three rows are all based on equating marginal revenues. The solutions are thus discriminatory, and relative prices will depend on the elasticities. Social welfare goals, by comparison, lead to a mixture of discriminatory and non-discriminatory results, according to what constraints are placed on their attainment; generally, no constraints mean no discrimination.

Finally, any attempt to work backwards from these results to the aims of a public enterprise ("if prices are of this pattern, and the enterprise is behaving rationally, then the target must be this one") is made more difficult by the coincidence of some of the results. From results to goals is not a one-to-one mapping. Some cases are the duality results mentioned earlier; others are not even linked in that way.
4. THE THEORY IN PRACTICE

The theoretical discussion in the previous section (with the fuller analysis in the Appendix) has shown that price discrimination can be expected from a public enterprise in a wide variety of circumstances. We consider now the extent to which these results are applicable to British Rail.

A general problem in any practical application of the theory is the need to decide where to draw the line between price discrimination and product differentiation. In the British Rail context, first and second class travel are different goods. To some extent so are peak and off-peak services: the main difference lies in the frequency of service, as stopping patterns are often very similar and the rolling stock identical. It is really a question of steering between failing to make any allowance for (often considerable) differences in levels of comfort and speed, on the one hand, and ending up with a *reductio ad absurdum* on the other, by treating every single service as a different good with its own price and demand curve. Quite where the line is drawn is inevitably somewhat arbitrary, and practical convenience or availability of data may be as good a criterion as any other.

Availability of data may be a more general constraint on how far the theory can be put into practice. Even if the various sub-markets can be distinguished and separated, actually determining the elasticities necessary for accurate differential pricing may not be possible, as the characteristics of interest are rarely directly observable. There are also administrative limits to the number of ticket types it is practical to issue or have to check.

So, whereas according to the theory the objective function could be maximised with a fine enough classification of passengers and journeys, in practice this is not possible anyway. British Rail generally works with a threefold classification: journeys which have to be made regularly, journeys which have to be made but not regularly, and those which do not have to be made at all. In the first group we find travel by commuters, regular trips at the same time each day. Business travel is the main (but not sole) component of the second category (and is also the travel least likely to be paid for by the traveller himself). In the third sector we find leisure travel, shopping trips to London, and so on. Related to this division by journey purpose is the use of the “time away” principle in pricing: the longer the traveller is away from home, the smaller the proportion of the total cost made up by travel, and therefore the less elastic will be the demand for the travel component. Hence weekend returns are cheaper than tickets which are valid for a month.

Both these classifications are designed to take into account differences in journey-related elasticities of demand; the theoretical analysis showed that prices and elasticities will generally be inversely related. Different consumers' relative weightings of price and service quality will also vary and can be exploited; for example, the introduction of High Speed Trains on the East Coast main line was followed by an element of “real” pricing on that route. To prevent what British Rail terms “abstraction” from the full fare market, the validity of discounted fares is restricted. For instance, day return tickets are not permitted on some peak morning trains: that is to discourage businessmen from making use of them. (The

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persistence of some degree of abstraction is in fact evidence that market segmentation is never perfect.)

Parallel to all this is differentiation by type of passengers, epitomised by the concept of railcards, which allow certain categories of travellers to qualify for cheap tickets by buying the cards. (Here we simplify the issue by considering the cards only as a method of identifying and separating the markets — ignoring their other function of raising sales revenue directly — so that cardholders and non-cardholders form separate markets facing different prices.) As most holders of railcards qualify by age or occupation, possible abstraction between markets is much less, though there is no doubt some loss of revenue from students attending interviews and businessmen aged over 65.

Railcards may, however, create more resentment, and the public relations aspect should not be neglected. There is anecdotal evidence at least to suggest that people do resent the feeling that “everybody except me” is apparently eligible for some form of cheap travel. This is compounded by the fact that British Rail is unable to offer secret discriminatory prices, as an hotel for example might; offers have to be promoted, to all “eligible” customers, and passengers can and do compare notes on the fares they have paid. (A similar situation arises in the case of airlines.) British Rail is aware of this problem, but how far resentment is actually translated into reduced demand remains an unanswered empirical question. Though the railcard user does have to “buy in” to get his discount, the relative price of the card is so low that one journey is often enough to recoup the initial outlay. In addition, by their very nature they discriminate by personal factors, which may be more reprehensible than differentiating according to the time of day or purpose of journey (which does not rule out some people from ever taking advantage of it), and may be more easily rationalised to the traveller who sees an empty off-peak train but finds it hard to perceive the different demand curve of the pensioner or student). Suggestions have been made for the introduction of a railcard open to all passengers, with a rather higher charge for the card than under the present schemes (see The Times letters page, 23 November 1979 and 9 November 1981).

A related issue is the possibility of resentment and loss of business due to the complexity of the menu of fares on offer. The Serpell Committee noted that the “range . . . is large and confusing” (Department of Transport (1983), page 27) and commended the Board’s plans to introduce a simpler structure; more recently the Central Transport Consultative Committee (1984, page 4) concluded that “the whole fare structure is in need of simplification to make it readily comprehensible and usable by staff and passengers”. (For a full discussion see the Committee’s 1977 report, Fares Fair.) Even if only a limited number of the total offers are relevant to a particular person, the scope for confusion is still very definitely present. (Any short-term gain to British Rail if a passenger fails to take advantage of a cheap fare offer may well be outweighed in the longer run by the loss of goodwill and potential future custom.) Excluding these considerations from the theoretical analysis inevitably reduces the value of its results.

Criticism of the current system has focused largely on this mixture of confusion and resentment at the range of fares on offer; the best answer is to con-
vince the full-fare passenger that he does in fact gain from the reduced fares available to others, through a rise in total revenue, leading to either a better service or a fall in (taxation-financed) public support for the industry. Some doubts have also been raised whether some of the marginal traffic is in fact being carried at a loss; this will happen if marginal revenue is pushed below marginal cost.

Whether the fact that a nationalised industry is publicly owned helps or not is not clear. On the one hand, it may be easier for it to claim that discrimination is in the "public interest" than it would be for a private profit-maximising firm, whose motives are generally assumed to spring solely from self-interest. The pursuit of some social welfare goal may be put forward to justify the action, if only at a rather paternalistic level. Alternatively, however, an element of moral unfairness may creep in, so that what is acceptable in a private firm, which somehow no one expects to behave "well", becomes unacceptable in a public enterprise which is supposed to be there as a public service. In the case of British Rail, too, perhaps queues of people at ticket offices clutching their railcards are more visible and therefore cause greater resentment than do, say, discounts received by large firms which are negotiated behind closed doors.

What this amounts to formally is that demands by different consumers are not independent, as theory tends to assume; one man's demand is affected by the prices and quantities available to his neighbour. Whether or not this is "rational" in the economic sense is of no great relevance to the manager on the ground. It may be efficient for him to charge different fares to different people, but their notions of equity may carry greater weight.

On the whole, the pattern of British Rail's prices does reflect that suggested by the theoretical analysis, with fares inversely related to elasticities of demand. The exception is that one would expect higher fares, not discounted season tickets, in the commuting market of London and the South East, where demand is the least elastic of all. Such a policy could accord with the theoretical results of Section 3.3, raising revenue in this market and using lower fares to stimulate the more elastic demand for travel elsewhere. Raising prices to commuters has been proposed, for example by the Serpell Committee (and particularly in the Minority Report), but the ramifications of any large change mean that it is unlikely to be made by British Rail acting alone.4

The Serpell Report also argued that the reductions in "Saver" fares are too large, particularly in the longer term when reductions in capacity can be made. Bleasdale (1983, page 516) defends the low initial fare levels on the grounds that the ticket was thereby well established in the market; any final judgement must be a commercial or empirical one. British Rail's view is certainly that its reduced fares policy is financially beneficial, as the various promotions bring in added net revenue; reports that BR is working on a simplified fares structure suggest that it is also aware of the limitations of the policy.

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4 For further discussion of this "fundamentally bad job" see CTCC (1977), paragraphs 90–99.
5. SUMMARY AND CONCLUSIONS

This paper has brought together two previously rather separate subjects: the pursuit of different possible objectives by a public enterprise, subject to plausible constraints, and the effects of practising price discrimination. The main result of the analysis is that embodied in Table 1: many possible strategies will lead to an equating of marginal revenues in different markets, and hence to discriminatory pricing. The fact that we do observe this in a nationalised industry is not therefore a very useful guide to the objective of the enterprise, apart perhaps from ruling out some choices which were probably less likely anyway.

The consideration of British Rail showed that it is an industry whose circumstances are particularly suited to the application of price discrimination, and its practice thereof is indeed more extensive than that of any other nationalised industry (at least in publicly quoted prices to individual consumers). With a relatively fixed timetable, and therefore costs, the main weight of policy tends to fall on prices and marketing. This seems likely to increase in the future: British Rail's policy has shifted recently, partly as a result of the appointment of a new chairman, and the new thrust is "to turn the railways away from its 150-year-old tradition of being engineering or resource-led, into a more rounded, commercially led business enterprise ... to show that ... British Rail is market-led, a consumer-orientated business" ("BR's quiet revolution gets going", The Guardian, 21 March 1984).

Just how far differential pricing should be pursued remains an open question. The model discussed here, if taken literally, would mean that every separate demand would be individually catered for. Given the constraints of competition and administrative practicality, that demands are not independent between markets, and that there is at least strong casual evidence of some loss of custom from consumer resentment, such a policy is no longer optimal or even feasible. Where to strike the balance, and how efficiently price discrimination is actually practised, are questions for empirical solution.

APPENDIX

A.1 The model

Recapping briefly, we consider a single product or service being sold in two separate markets with inverse demand curves $f_1(q_1)$ and $f_2(q_2)$; there are no cross-price effects from one market to the other. More specifically, we will assume that both functions are monotonically decreasing, and that the elasticities of demand are non-zero; that is, we assume $-\infty < f_i''(q_i) < 0$ ($i = 1, 2$), where the prime denotes differentiation. No restrictions are placed on the second derivative, $f_i''(q_i)$. Costs are a function of the joint output in the two markets: $C = C(q_1 + q_2)$, with $C' > 0$. The marginal cost of an additional unit of output (MC) is therefore independent of which market it is destined for.
The main maximands we will consider are profit, output, and a measure of social surplus. Before we begin, however, some more technical matters should be cleared up. First, we wish to rule out corner solutions, so that we can make use of the methods of calculus, and also to avoid the need to analyse the second-order conditions, which are both rather intractable and less economically enlightening. Thus we make the simplifying assumption that the functions are well-behaved—generally concave with convex constraints—and that the required conditions therefore hold. Second, the fact that cost is a strictly increasing function of the level of output means that we are excluding any investment effects. In other words, we are concerned with the current period only, with fixed capital.

A.2 Profit maximisation

For the “benchmark” case of simple profit maximisation, we have

\[ \pi = q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2), \]  \hspace{1cm} (A1)

and maximising this expression with respect to \( q_1 \) and \( q_2 \) separately gives

\[ \frac{\partial \pi}{\partial q_i} = \left[q_i f'_i(q_i) + f_i(q_i)\right] - C'(q_1 + q_2) \hspace{1cm} i = 1, 2 \]

where the prime denotes differentiation. The expression in square brackets is simply the marginal revenue in market \( i \) (\( MR_i \)), so equating the partial derivatives to zero yields the standard result for an optimum:

\[ MR_1 = MR_2 = MC \]  \hspace{1cm} (A2)

or (see main text)

\[ P_2 = \frac{\eta_2 (1 + \eta_1)}{\eta_1 (1 + \eta_2)} P_1. \]  \hspace{1cm} (A3)

If, as is perhaps more likely, profit maximising is subject to supplying some minimum level of total output, then we can consider the Lagrangean expression (4) in the main text, where \( \lambda \) represents the shadow price of output in terms of profit:

\[ Z = q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) + \lambda(q_1 + q_2 - \bar{Q}). \]  \hspace{1cm} (A4)

From this we can set up the Kuhn-Tucker conditions:

\[ \begin{aligned}
\frac{\partial Z}{\partial q_i} &= q_i f'_i(q_i) + f_i(q_i) - C'(q_1 + q_2) + \lambda \leq 0 \hspace{1cm} q_i \geq 0 \\
q_i [q_i f'_i(q_i) + f_i(q_i) - C'(q_1 + q_2) + \lambda] &= 0 \\
\lambda(q_1 + q_2 - \bar{Q}) &= 0 \\
\lambda (q_1 + q_2 - \bar{Q}) &= 0
\end{aligned} \hspace{1cm} i = 1, 2 \]

Two main cases can be identified, according to whether the constraint is binding (\( \lambda > 0 \)) or not (\( \lambda = 0 \)).
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Case I ($\lambda > 0$) i.e. $q_1 + q_2 = Q$
Then either (a) $q_1, q_2 > 0$ and $MR_1 = MR_2 = MC - \lambda$
or (b) $q_1 = \tilde{Q}$, $q_2 = 0$ (without loss of generality)

Case II ($\lambda = 0$) i.e. $q_1 + q_2 \geq \tilde{Q}$
Now either (a) $q_1, q_2 > 0$ and $MR_1 = MR_2 = MC$
or (b) $q_1 \geq \tilde{Q}$, $q_2 = 0$ and $MR_1 = MC$, $MR_2 \leq MC$.

In the latter case, $q_2 = 0$ means $MR_2 = P_2$, so the condition is $P_2 \leq MC$; this is intuitively appealing, as it means that no output will be sent to Market 2 if the price at which demand becomes positive is less than the marginal cost (at the output level determined in Market 1). In practice it seems likely that both markets will be served, so the general result can be written

$$MR_1 = MR_2 = MC - \lambda,$$

which is obviously a generalisation of the previous result, and again there is no reason to expect prices to be equal.

In the case where one market has an elastic demand and the other an inelastic one, it would still be open to the enterprise to serve the second market only. It could then be desirable to specify a minimum quantity to be sold in each market separately. If these outputs are denoted $\tilde{Q}_1$ and $\tilde{Q}_2$, the objective function now incorporates two constraints:

$$Z = q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) + \lambda_1 (q_1 - \tilde{Q}_1) + \lambda_2 (q_2 - \tilde{Q}_2).$$ (A4')

The relevant conditions are now (writing $MR_i$ for $q_i f'_i(q_i) + f_i(q_i)$)

$$\frac{\partial Z}{\partial q_i} = MR_i - MC + \lambda_i \leq 0 \quad q_i \geq 0$$

$$q_i (MR_i - MC + \lambda_i) = 0$$

$$\frac{\partial Z}{\partial \lambda_i} = q_i - \tilde{Q}_i \geq 0 \quad \lambda_i \geq 0 \quad \lambda_i (q_i - \tilde{Q}_i) = 0$$

Three distinct cases can be identified:

Case I: $\lambda_1 = \lambda_2 = 0$; i.e. $q_i \geq \tilde{Q}_i$, $i = 1, 2$
In this case either (a) $q_1, q_2 > 0$ and $MR_1 = MR_2 = MC$
or (b) $q_1 > 0$, $q_2 = \tilde{Q}_2 = 0$ (this only occurs if a minimum output is specified in one market only). Again we have $MR_1 = MC$ and $MR_2 = P_2 \leq MC(q_1)$.

Case II: $\lambda_1 > 0$, $\lambda_2 = 0$; i.e. $q_1 = \tilde{Q}_1$, $q_2 \geq \tilde{Q}_2$.
Now either (a) $q_1, q_2 > 0$ and $MR_1 = MC - \lambda_1$ and $MR_2 = MC$
or (b) $q_1 > 0$, $q_2 = \tilde{Q}_2 = 0$, so $MR_1 = MC - \lambda_1$ and $P_2 \leq MC(q_1)$

Case III: $\lambda_1 > 0$, $\lambda_2 > 0$ $q_1 = \tilde{Q}_1$, $q_2 = \tilde{Q}_2$
In this case $MR_1 = MC - \lambda_1$ and $MR_2 = MC - \lambda_2$.

As above, we may put forward the general solution

$$MR_1 + \lambda_1 = MR_2 + \lambda_2 = MC;$$ (A5')

marginal revenues are no longer generally equal; nor are prices.

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Maximising total revenue subject to a profit constraint is represented by
\[ Z = q_1 f_1(q_1) + q_2 f_2(q_2) + \lambda \left[ q_1 f_1(q_1) + q_2 f_2(q_2) - (1 + \delta)C(q_1 + q_2) - \bar{\pi} \right] \]
giving Kuhn-Tucker conditions
\[ \frac{\partial Z}{\partial q_i} = MR_i + \lambda [MR_i - (1 + \delta)MC] \leq 0 \quad q_i \geq 0 \]
\[ q_i [MR_i + \lambda [MR_i - (1 + \delta)MC]] = 0 \quad i = 1, 2 \]
\[ \frac{\partial Z}{\partial \lambda} = TR_1 + TR_2 - (1 + \delta)C(q_1 + q_2) - \bar{\pi} \geq 0 \quad \lambda \geq 0 \]
\[ \lambda [TR_1 + TR_2 - (1 + \delta)C(q_1 + q_2) - \bar{\pi}] = 0 \]

\((TR_i)\) represents the total revenue in market \(i\). Then there are again two main cases depending on whether the constraint is binding.

Case I: \(\lambda = 0\) (constraint not binding)
Then \(q_i > 0\) and \(MR_i = 0\) (unless \(q_i = MR_i = 0\) if there is no demand in market \(i\)).

Case II: \(\lambda > 0\) and the constraint binds.
Then either \(q_1, q_2 > 0\) and \((1 + \lambda)MR_1 = (1 + \lambda)MR_2 = \lambda (1 + \delta)MC\)
or \(q_1 > 0, q_2 = 0\) and \((1 + \lambda)MR_2 = (1 + \lambda)\bar{P}_2 \leq (1 + \lambda)MR_1 = \lambda (1 + \delta)MC(q_1)\)
or \(q_1 = q_2 = 0\), that is \((1 + \lambda)\bar{P}_1 \leq \lambda (1 + \delta)MC(0)\)
and \((1 + \delta)C(0) = -\bar{\pi} \).

In the last two cases one or both markets are not supplied at all, as the demand price for even one unit is too low.

Again we can write the most likely general condition as
\[ (1 + \lambda)MR_1 = (1 + \lambda)MR_2 = \lambda (1 + \delta)MC. \]  \((A6)\)

A.3 Output maximisation

If the enterprise is maximising output — passenger-miles, say, for British Rail — subject to earning some minimum profit level \(\bar{\pi}\), then the Lagrangean is
\[ Z = q_1 + q_2 + \mu \left[ q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) - \bar{\pi} \right], \]  \((A7)\)
where \(\mu\) is the shadow price of profit in terms of output.

As this is the dual of the problem in equation \((A4)\), the result will be the same. Ruling out the case of \(\mu = 0\) on the grounds of non-satisfaction, and assuming both markets are actually served, we get
\[ MR_1 = MR_2 = MC - 1/\mu. \]  \((A8)\)

If the required profit is a minimum percentage rate on turnover, then the constraint can be written \(q_1 f_1(q_1) + q_2 f_2(q_2) \geq (1 + \delta)C(q_1 + q_2)\). (This requirement is similar to that laid down for the Post Office in the 1978 White Paper — see Chancellor of the Exchequer (1978), Appendix B.) It is then simple to show that the solution now becomes
\[ MR_1 = MR_2 = (1 + \delta)MC - 1/\mu \]  \((A8')\)
In both (A8 and \(A8'\)) equal marginal revenues will again mean that the conditions at (A3) above will hold for prices and elasticities.

If we consider the case where sector managers are delegated to maximise their own outputs, and each is given his own profit constraint \(\pi_i\), and we allocate the cost proportionately, we have two maximands:

\[
Z_1 = q_1 + \mu_1 \left[ q_1 f_1(q_1) - \frac{q_1}{q_1 + q_2} C(q_1 + q_2) - \bar{\pi}_1 \right]
\]
\[
Z_2 = q_2 + \mu_2 \left[ q_2 f_2(q_2) - \frac{q_2}{q_1 + q_2} C(q_1 + q_2) - \bar{\pi}_2 \right]
\]

Joint maximisation of these leads to the following four marginal conditions, where we assume the constraints do bind:

\[
\frac{\partial Z_1}{\partial q_1} = 1 + \mu_1 \left[ MR_1 - \frac{q_1}{q_1 + q_2} C'(q_1 + q_2) - \frac{q_2}{(q_1 + q_2)^2} C(q_1 + q_2) \right]
\]
\[
\frac{\partial Z_1}{\partial q_2} = \mu_1 \left[ \frac{q_1}{(q_1 + q_2)^2} C(q_1 + q_2) - \frac{q_1}{q_1 + q_2} C'(q_1 + q_2) \right]
\]
\[
\frac{\partial Z_2}{\partial q_1} = \mu_2 \left[ \frac{q_2}{(q_1 + q_2)^2} C(q_1 + q_2) - \frac{q_2}{q_1 + q_2} C'(q_1 + q_2) \right]
\]
\[
\frac{\partial Z_2}{\partial q_2} = 1 + \mu_2 \left[ MR_2 - \frac{q_2}{q_1 + q_2} C'(q_1 + q_2) - \frac{q_1}{(q_1 + q_2)^2} C(q_1 + q_2) \right]
\]

Setting \(\partial Z_1/\partial q_2 = \partial Z_2/\partial q_1 = 0\) gives \(C = (q_1 + q_2)MC\); substituting this and setting \(\partial Z_i/\partial q_i = 0\) produces

\[
1 = \mu_1 (MC - MR_1)
\]
\[
1 = \mu_2 (MC - MR_2)
\]

or

\[
MR_1 + 1/\mu_1 = MR_2 + 1/\mu_2 = MC.
\]

This result is the same as that in \((A5')\), though the two problems, while related, are not exact duals of each other. Again the normal outcome will be discriminatory pricing.

Note that there is one feature unique to this case: we now have two decision-makers, whose actions will impinge on each other through the joint-cost structure. Some assumption will have to be made about whether each manager takes into account the likely reaction of his colleague to his own decisions. For example, in the case where neither does, we can derive the reaction functions of the two managers, which show what output will be chosen by one for any given decision by the other. If we have large fixed costs and then increasing returns to scale in the industry, the functions will look as shown in Figure 1. For decreasing returns
the curves will slope down. In either case $E$ will be a stable equilibrium if Sector 1's curve is steeper than Sector 2's. This is clearly true for the example shown; whether it is true in practice, and how realistic the initial assumption is that one manager will not anticipate his colleague's response, will obviously vary from one industry to another. This whole framework is in any case only a crude approximation to British Rail's concept of sector management; given that, Figure 1 might well apply, and so point $E$ is an equilibrium.

A.4 Social surplus

If our objective is simply consumer's surplus, but constrained by an absolute profit level of $\pi$, we can write the maximand as

$$Z = \int_0^{q_1} f_1(q) dq - q_1 f_1(q_1) + \int_0^{q_2} f_2(q) dq - q_2 f_2(q_2) + \phi [q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) - \pi]$$ (A11)

where $\phi$ is the shadow price of profit. The first-order conditions are then
\[
\begin{align*}
\frac{\partial Z}{\partial q_i} &= P_i - MR_i + \phi(MR_i - MC) \leq q_i, 
\quad i = 1, 2 \\
q_i[P_i - MR_i + \phi(MR_i - MC)] &= 0 \\
\frac{\partial Z}{\partial \phi} &= q_1P_1 + q_2P_2 - C(q_1 + q_2) - \tilde{\pi} 
\quad \geq 0, 
\quad \phi \geq 0 \\
\phi[q_1P_1 + q_2P_2 - C(q_1 + q_2) - \tilde{\pi}] &= 0
\end{align*}
\]

If \( \phi = 0 \) the constraint is not binding, and we have the case discussed above. The constraint binds when \( \phi > 0 \); then \( P_i - MR_i = \phi(MC - MR_i) \). It is possible that there will be no output to market \( i \); then \( f_i(0) = MR_i(0) \) and the condition for that market becomes \( 0 \leq \phi(MC - P_i(0)) \), i.e. \( P_i(0) < MC \) and we have the situation where no units can be sold. The general case, however, is

\[ MR_i - P_i = \phi(MR_i - MC) \quad i = 1, 2 \tag{A12} \]

or eliminating \( \phi \)

\[ \frac{MR_1 - P_1}{MR_2 - P_1} = \frac{MR_1 - MC}{MR_2 - MC}. \tag{A13} \]

The left hand side of this can also be written as

\[ (P_1/\eta_1)/(P_2/\eta_2). \]

Substituting this expression and rearranging gives

\[ \frac{(MR_1 - MC)/P_1}{(MR_2 - MC)/P_2} = \frac{1/\eta_1}{1/\eta_2}. \tag{A14} \]

This is similar to Baumol and Bradford’s (1970) result, except that we have marginal revenues in place of prices on the left hand side.

Replacing \( MR_i \) in (A13) and rearranging gives

\[ MC = \frac{P_1P_2(\eta_1 - \eta_2)}{\eta_1P_2 - P_1\eta_2}. \tag{A14'} \]

There is some vestige of marginal cost pricing here, reflecting the nature of the maximand, but the price term has now become quite complex.

Changing the profit constraint to a required percentage on turnover again leads to a rather similar result. Now the general first-order conditions are

\[ MR_i - P_i = \phi[MR_i - (1 + \delta)MC], \quad i = 1, 2 \]

where \( \delta \) is again the required mark-up, or

\[ \frac{MR_1 - P_1}{MR_2 - P_2} = \frac{MR_1 - (1 + \delta)MC}{MR_2 - (1 + \delta)MC}. \tag{A13'} \]

An analogous result to that of Baumol and Bradford follows as before.

If instead of a budget constraint we impose a required output level – it will
need to be a maximum if we are not to find ourselves in the zero price situation of section 3.3 — then we optimise
\[ Z = \int_0^{q_1} f_1(q) dq - q_1 f_1(q_1) + \int_0^{q_2} f_2(q) dq - q_2 f_2(q_2) + \phi [\bar{Q} - q_1 - q_2]. \quad (A15) \]

Now we have the following situation:
\[
\begin{align*}
\frac{\partial Z}{\partial q_i} &= f_i(q_i) - [f_i(q_i) + q_i f'_i(q_i)] - \phi \leq 0 \\
q_i &\geq 0 \\
q_i [q_i f'_i(q_i) + \phi] &= 0 \\
\phi [\bar{Q} - q_1 - q_2] &= 0.
\end{align*}
\]

\[ i = 1, 2 \]

Again if \( \phi = 0 \) the price will be zero. When \( \phi > 0 \), the usual condition will be \( P_i - MR_i = \phi \) \( (i = 1, 2) \); if a market is not served \( (q_i = 0) \) the condition \( P_i - MR_i \leq \phi \) reduces to \( 0 \leq \phi \). Eliminating \( \phi \), the general result is thus
\[ P_1 - MR_1 = P_2 - MR_2 \quad (A16) \]

which gives us
\[ P_2 = \frac{\eta_2}{\eta_1} P_1. \]

A wider measure of social surplus is the sum of producer's and consumer's surpluses. The maximand in this case simplifies from
\[ Z = \int_0^{q_1} f_1(q) dq - q_1 f_1(q_1) + \int_0^{q_2} f_2(q) dq - q_2 f_2(q_2) \]
\[ + \{q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2)\}, \]

where the term in square brackets is the producer's surplus, to
\[ Z = \int_0^{q_1} f_1(q) dq + \int_0^{q_2} f_2(q) dq - C(q_1 + q_2). \quad (A17) \]

Differentiation then gives
\[ \frac{\partial Z}{\partial q_i} = f_i(q_i) - C'(q_1 + q_2) \quad (i = 1, 2) \]

or
\[ P_1 = P_2 = MC. \quad (A18) \]

Therefore, as the cost function is a joint one, no price discrimination occurs in this case (the familiar one of marginal-cost pricing).
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Maximisation of this objective subject to a budget constraint is represented by the expression

$$Z = \int_0^{q_1} f_1(q) dq + \int_0^{q_2} f_2(q) dq - C(q_1 + q_2)$$
$$+ \phi[q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) - \bar{\pi}]$$ \hspace{1cm} (A19)

differentiation then gives

$$\frac{\partial Z}{\partial q_i} = f_i(q_i) - C'(q_1 + q_2) + \phi(MR_i - MC) \leq 0 \quad q_i \geq 0$$

$$q_i [f_i(q_i) - MC + \phi(MR_i - MC)] = 0$$ \hspace{1cm} i = 1, 2

$$\frac{\partial Z}{\partial \phi} = q_1 f_1(q_1) + q_2 f_2(q_2) - C(q_1 + q_2) - \bar{\pi} \geq 0$$ \hspace{1cm} \phi \geq 0

$$\phi[TR_1 + TR_2 - C(q_1 + q_2) - \bar{\pi}] = 0$$

If the constraint does not bind, then both prices are again equal to marginal cost. Otherwise the general result, unless \( q_i = 0 \), is

$$P_i - MC = -\phi(MR_i - MC)$$ \hspace{1cm} i = 1, 2 \hspace{1cm} (A20)

or, in the special case where \( q_i = 0 \),

$$P_i - MC = -\phi(MR_i - MC).$$

Eliminating \( \phi \) from the general result gives

$$\frac{P_1 - MC}{P_2 - MC} = \frac{MR_1 - MC}{MR_2 - MC}$$ \hspace{1cm} (A21)

or

$$\frac{(P_1 - MC)/P_1}{(P_2 - MC)/P_2} = \frac{1/\eta_1}{1/\eta_2},$$

which is exactly the Baumol-Bradford result with \( MC_1 = MC_2 \).

Again, if the constraint is of the percentage form, the outcome at (A21) is changed to

$$\frac{P_1 - MC}{P_2 - MC} = \frac{MR_1 - (1 + \delta)MC}{MR_2 - (1 + \delta)MC}.$$ \hspace{1cm} (A21')

Finally, maximising the sum of the two surpluses subject to an output constraint is represented by

$$Z = \int_0^{q_1} f_1(q) dq + \int_0^{q_2} f_2(q) dq - C(q_1 + q_2) + \phi(\bar{Q} - q_1 - q_2).$$ \hspace{1cm} (A22)
Then
\[
\begin{align*}
\frac{\partial Z}{\partial q_i} &= f_i(q_i) - C'(q_1 + q_2) - \phi \leq 0 \quad q_i \geq 0 \\
\{ & \quad i = 1, 2 \\
q_i[f_i(q_i) - C'(q_1 + q_2) - \phi] &= 0 \\
\frac{\partial Z}{\partial \phi} &= \bar{Q} - q_1 - q_2 \geq 0 \\
\phi(\bar{Q} - q_1 - q_2) &= 0
\end{align*}
\]
Assuming the constraint is binding, so that we do not just revert to the result in (A18), the general result is
\[
P_1 = P_2 = MC + \phi; \quad (A23)
\]
prices are not discriminatory in this case.

REFERENCES


