OPTIMAL PRICING AND SUBSIDIES
FOR SCHEDULED TRANSPORT SERVICES

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This paper develops the discussion of issues raised in two recent contributions to this Journal, Larsen (1983) and Frankena (1983), on optimal pricing and subsidies in scheduled transport services.

Larsen in his note attempted a formulation of the theory of optimal pricing of scheduled services as developed in Mohring (1972), Turvey and Mohring (1975) and Jansson (1979), to show more explicitly the part played by demand in determining the optimal price. There were, however, some deficiencies in his approach. One was that the relationship between vehicle-kilometres supplied and passenger-kilometres accommodated was taken as given by operating practice, whereas the logic of the Turvey/Mohring approach is that it is something determined in the optimum solution. Secondly, his analysis took no explicit account of the costs passengers impose on each other, which were of particular importance to Turvey and Mohring (1975). Thirdly, he took no account of the potential external benefits (and costs) of public transport, which could affect the optimum price and certainly could feature prominently in the debate on public transport subsidies.

This paper shows first how these deficiencies can be overcome within the context of a relatively simple analytical framework. Thus section 2 considers a situation corresponding to that considered by Larsen; the costs passengers impose on each are added to the model in section 3, and the discussion is extended to cover other externalities in section 4.

However, though all this analysis provides an indication of the level of subsidy required in an optimal situation, merely paying that amount of subsidy would obviously not be enough to ensure that all or indeed any of the potential benefits of the subsidy were realised. That would depend on a number of factors, including the form of the subsidy, the nature and objectives of the concern operating the service, and its relationship to the subsidising body. Frankena (1983), building on earlier work including Frankena (1981), Nash (1978) and others, explored some of the problems involved, but with what he regarded as somewhat negative results. In section 5 Frankena's discussion is reviewed and developed on

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1 These costs are taken account of in the more detailed model developed by Jansson (1979 and 1984), but the approach used in this paper suggests implications which are not apparent from Jansson's approach.
the basis of the analysis in the earlier sections, and it is argued that even without detailed information on the underlying behavioural relationships it is possible to draw conclusions about what might be the most beneficial approach to subsidy policy.

Section 6, in drawing together the conclusions of the paper, notes that they are not inconsistent with the results of some recent empirical work on the benefits of public transport subsidies, and considers some of the more general issues raised.

Throughout, the focus of attention is on a monopoly supplier of a network of reasonably homogeneous passenger services. To develop the analysis for a more competitive situation is the task of ongoing research.

2. OPTIMAL FARES AND SUBSIDIES: THE SIMPLE CASE

As in Larsen (1983) and Frankena (1983), but using Frankena's notation, the operator's cost function is taken to be of the form

\[ C = C(R, M) \]  

where \( R \) is the number of passenger-kilometres accommodated (that is, ridership) and \( M \) is the vehicle-kilometres of service provided. One problem here is that the capacity of vehicles is likely to impose a constraint on the relationship between \( R \) and \( M \). With standard vehicles\(^2\) the constraint would take the form

\[ R \leq \beta M \]  

In practice, the inequality tends to be satisfied rather than the equality; but, rather than assume a fixed load factor as Larsen does, it is here assumed -- to allow the load factor to be determined by operating conditions -- that

\[ C_R \to \infty \text{ as } R \to \beta M \]  

The cost function can then be represented diagrammatically by a family of "short-run" average and marginal cost curves of the type illustrated in Figure 1, where the dashed lines indicate the capacity constraints. The marginal cost curves may not, in practice, approach the capacity constraints asymptotically as illustrated; but an asymptotic form is assumed here because it avoids certain complications without materially affecting the analysis, particularly that of subsequent sections.

Further, if, as we may not unreasonably suppose in the light of empirical findings, there are no economies or diseconomies of scale, there will be a horizontal "long-run" average (and marginal) cost curve touching the short-run average cost curves at their minimum points.\(^3\) It may also be noted that, with condition (3), \( C_R \) can

\(^2\) The possibility of varying vehicle size introduces further complications (see Jansson, 1980, and Walters, 1982), which are not considered here.

\(^3\) A functional form meeting all these requirements would be

\[ C = aM + \alpha M \left( \frac{R}{\beta M - R} \right)^b \]

where \( a, b, \alpha > 0 \)

but with any constant-returns-to-scale form of (1) the minimum points of the short-run average cost curves would have the same value of \( M/R \).
be everywhere non-negative as Larsen assumes, but $C_M$ need not necessarily be positive because the cost of additional vehicle-kilometres may be offset by the savings from reduced overloading.\footnote{In terms of Figure 1, $C_M$ would be negative where the short-run marginal cost curve was above the long-run marginal cost curve.}

The demand function used by both Larsen and Frankena expresses demand as a function of the (average) fare per passenger-kilometre and vehicle-kilometre, representing the level of service provided. The latter variable is included for the
standard reason that it influences passengers’ demand through its effect on (among other things) the walking and/or waiting times involved in journeys. An alternative and slightly simpler way of modelling this effect is to specify demand (in terms of passenger-kilometres) as a function of generalised cost \( G \) or, writing it for convenience in inverse form,

\[ G = G(R) \text{ where } G'(R) < 0 \]  

Then, since generalised costs are the sum of the fare charged and other costs (that is, time costs and other disutilities) and since these other costs are a declining function of vehicle-kilometres, the inverse demand function can be written as

\[ F = G(R) - T(M) \text{ with } F_R = G'(R) < 0 \text{ and } F_M = -T'(M) > 0 \]  

where \( T(M) \) is the non-fare component of generalised cost. In practice \( T \) may also be a function of \( R \); but, since that implies that passengers impose costs on each other, consideration is deferred to the next section.

With the demand function (5), the first order conditions for maximum net social benefit require:

\[ G(R) - T(M) - C_R = 0 \text{ or } F = C_R = SM C \]  

and

\[ -R T'(M) - C_M = 0 \text{ or } C_M = -R T'(M) \]  

When these conditions are satisfied, the operator’s surplus of revenue over costs can be expressed as

\[ F R - C = C_R R - C(R,M) \]

But with \( C(R,M) \) a linear homogeneous function, using Euler’s Theorem

\[ C_R R + C_M M = C(R,M) \]

and therefore, using (7),

\[ F R - C = M R T'(M) < 0 \]

since \( T'(M) < 0 \).

Net social benefit maximisation then requires a fare equal to “short-run” marginal cost and a total subsidy of \(-M R T'(M)\). The latter is simply a reflection of the decreasing social costs from passenger transport operation originally identified by Mohring (1972). As far as the operator is concerned the optimal solution involves a situation like that illustrated in Figure 1, where the level of service is that associated with the demand curve \( D_2 \) and “short-run” marginal cost curve \( SM C_2 \), giving \( F_0 \) and \( R_0 \) as the optimal fare and ridership respectively and \( S_0 \) as the operator’s cost per passenger-kilometre. It will be noted that the load factor is less than that required to minimise costs per passenger-kilometre, but a unique value for it can be obtained only by making unduly restrictive assumptions about the form of \( T(M) \).

Following Larsen, these results can be expressed in terms of the fare elasticity of demand \( (e_F) \) and the service elasticity of demand \( (e_M) \). Since
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\[ e_F = \frac{F}{G'(R)R} \quad \text{and} \quad e_M = \frac{T'(M)}{G'(R)R} \]

the required subsidy becomes \(- (e_M/e_F)F \) \( R \) or alternatively

\[ F = \frac{C/R}{1 - e_M/e_F} \]  \hspace{1cm} (9)

Equation (9) corresponds to Larsen's expression for the optimal price except that his numerator is, in the context of his model, an expression for the operator's marginal cost with respect to an increase in the level of demand. With the variable load factor in the present formulation there is no corresponding concept; but the results here are of perhaps more significance, since they give an indication of the operator's overall position, suggesting that, in an optimal situation (since \( e_F < 0 \)) the higher \( e_M \) is relative to \( -e_F \) the smaller the proportion of costs that should be covered by revenue.\(^5\) Indeed, if, as empirical studies seem to suggest, \( e_F \) is typically around \(-0.3 \) and \( e_M \) is possibly, but perhaps less certainly, in the range 0.6 to 0.7 (see Lago et al., 1981), and if the same relative values would also apply in an optimal situation, only about a third of total costs would be covered by revenue from fares. But even with different values, as long as \( e_M > -e_F \) at the optimum, less than 50% of costs would be covered by revenue.

3. PASSENGER CONGESTION COSTS

The natural starting point for discussion of the costs passengers impose on each other — which are essentially a form of congestion costs — is the analysis of Turvey and Mohring (1975), who argued that, with a given level of service, an increase in the number of passengers increases the costs to passengers of using the service. The extra passengers increase delays at stops as more people get on and off; they increase congestion within the vehicles; and, in view of the stochastic nature of demand, they increase the chance of not being able to travel on a particular trip because the vehicle is already fully loaded. As Jansson (1979) has pointed out, operating practices may reduce the variation in some of these costs, since operating schedules have to make a reasonable allowance for delays at stops, and intermediate timing points on a route may reduce the possible saving of time on lightly loaded journeys. Even so, there are likely to be positive marginal costs as capacity loadings for particular services are approached. Passenger congestion costs of this nature are, of course, among those reflected in the demand functions facing operators. An increase in \( M \) with a given number of passenger-kilometres reduces these costs and thus increases the amount passengers are willing to pay for their journeys, pushing the demand curve upwards and reinforcing the effect of reduced walking and/or waiting time already taken into account.

\(^5\) This result also reflects the assumption of constant costs. With increasing costs, as might be expected, the optimal fare would be raised relative to costs, but with decreasing costs it would be reduced.
This problem of passenger congestion costs could be dealt with by inserting a passenger-kilometres variable into the function representing the non-fare component of generalised costs. However, to allow the effects of these costs to be examined separately from those already considered, we have incorporated them by amending the demand function to

$$F = G(R) - T(M) - P(R,M)$$  \hspace{1cm} (10)

where $P(R,M)$ represents the passenger congestion costs per passenger-kilometre, which are then an increasing function of $R$ and a declining function of $M$ (that is, $P_R > 0$, and $P_M < 0$).

With this demand function, the first order conditions for maximum social net benefits require

$$G(R) - T(M) - [P(R,M) + R P_R ] - C_R = 0$$

or

$$F = R P_R + C_R$$  \hspace{1cm} (11)$$

and

$$-R T'(M) - R P_M - C_M = 0$$

or

$$C_M = -R [T'(M) + P_M]$$  \hspace{1cm} (12)

The optimal fare now therefore reflects the operator’s marginal cost plus the difference between the marginal and average passenger congestion cost. In other words it should include a congestion tax component to internalise all the costs that passengers impose on each other.

Using (11) and (12) we now write the expression for the operator’s surplus as

$$F R - C = R (P_R R + P_M M) + M R T'(M)$$

It would seem reasonable, however, to suppose that increasing vehicle-kilometres and passenger-kilometres in the same proportion would leave $P(R,M)$ unchanged (that is, it would be a homogeneous function of degree zero with $P_R R + P_M M = 0$), so that again

$$F R - C = M R T'(M)$$  \hspace{1cm} (13)

But, though this is the same (negative) expression as before, it would involve different values of $F$, $R$ and $M$.

Expressing the required subsidy in terms of elasticities is less straightforward because of the extra term in the demand function, but the expression for the optimal fare in terms of elasticities (that is, the equivalent to (9) above) becomes

$$F = \frac{C/R + R P_R}{(1 - e_M/e_F)}$$  \hspace{1cm} (14)

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6 The effect of passenger congestion costs would be to raise $M$ relative to $R$, but the effect on $F$ would be indeterminate.
In other words, the effect of passenger costs on the optimal fare is to raise it relative to the operator's costs per passenger, but by less than the "congestion tax" component of the fare.

The optimal situation, taking account of passenger congestion costs, is illustrated in Figure 2. The relevant points are labelled as in Figure 1, but for the sake of clarity, only the demand and "short-run" marginal cost curves associated with the optimal value of $M$ are shown (as $D_0$ and $SMC_0$ respectively). The dashed curve $SMSC_0$ shows for each value of $R$ (when $M$ is optimal) marginal operating costs plus the marginal passenger congestion costs not reflected in the movements of the demand curve (that is, that part which needs to be covered by the congestion tax component of the fare). The overall optimum position is thus found where this curve intersects the demand curve.
4. OTHER EXTERNALITIES

The arguments for public transport subsidies arising from the factors discussed in the preceding sections have long been recognised in the academic literature; but discussions of actual policy tend to focus more on the potentially favourable external benefits of public transport, such as its effect on traffic congestion costs, accident costs and other environmental costs. So it would seem pertinent to consider the effects of including these factors in the analysis.\(^7\)

Turning first to traffic congestion costs, since public transport may contribute to them, \(X\) can be defined as the (external) costs imposed by the operation of public transport on other road users, so that

\[
X = X(R,M)
\]

where \(X_R < 0\), to the extent that more use of public transport means a reduced use of other vehicles, and \(X_M > 0\) to the extent that an increase in public transport mileage \((R\) constant) simply increases total traffic. Both partial derivatives will of course have zero values in the absence of traffic congestion, and \(X_M\) will also be zero where the public transport operates on a reserved track (for example, a railway — unless the railway is also congested).

Expressed in this form, these traffic congestion costs add further elements to the conditions for net benefit maximisation. In particular

\[
F = R P_R + C_R + X_R
\]

or, where \(X_R < 0\), the optimal fare is reduced by an amount reflecting the effects of increasing \(R\) on the level of traffic congestion. Further

\[
F R - C = (X_R R + X_M M) + M R T'(M)
\]

The net effect of traffic congestion on the level of subsidy therefore depends on the effects of public transport on traffic congestion at the margin. If, for example, attracting passengers to public transport through lower fares reduces traffic congestion (that is, \(X_R < 0\)) and increasing public transport service has little effect on congestion (for example, where \(X_M = 0\)), then the optimal subsidy exceeds \(-M R T'(M)\). If, however, when more passengers are attracted by lower fares there is no effect on other traffic (that is, \(X_R = 0\)) and increased public transport services add to traffic congestion, then the optimal subsidy will be less than \(-M R T'(M)\).

In the case of bus services, empirical studies have suggested that the cross-elasticity of demand between bus fares and other traffic may be as low as 0.05 (see Lewis, 1977). As recent estimates suggest an average \(R\) to \(M\) ratio in the United Kingdom of around 12 (see Department of Transport, 1984), \(X_R R + X_M M\) would be negative as long as the ratio of car passenger-kilometres to bus passenger-kilometres was greater than 3. Since, in metropolitan districts at least, the ratio appears to be more than 6 (see Glaister, 1984, Table 1), and since in

\(^7\) Throughout this section the presumption is that it is not practicable to internalise these costs through an appropriate pricing system for road space. In other words, what is at issue here is what is sometimes referred to as the "second-best" argument for public transport subsidies.
Great Britain as a whole estimates suggest it is around 10 (see Department of Transport, 1984), it seems not unreasonable to conclude that in practice $X_R R + X_M M$ is more likely to be negative than positive, and that the effects on traffic congestion of increasing the use of public transport would increase the optimal subsidy.\footnote{If ratios of $R$ to $M$ were higher in congested areas than elsewhere, a negative $X_R R + X_M M$ would be even more likely.}

Certain other externalities can be handled in exactly the same way. For example, accident costs and environmental costs would also be decreasing functions of $R$ and increasing functions of $M$. Again, there could be some indeterminacy in the net effect on the overall level of subsidy. In the case of accident costs the level of subsidy would be more likely to be increased relative to $-M R T' (M)$, because the term equivalent to $X_R$ would reflect the effects of reduced accident costs as pedestrians, cyclists and motor-cyclists, as well as motorists, were attracted through lower fares to a mode of transport which has a lower accident rate.

In all these cases increasing public transport vehicle-kilometres imposes external costs as well as benefits; but there may be other external benefits which have no compensating costs. For example, individuals may derive utility from being within reach of public transport because it provides standby facilities, or because it not only makes other places more accessible to them, but also makes them more accessible to other people, such as their friends and relations. Benefits of this nature, though often intangible, would tend to add to the amount of subsidy in an optimal situation.

5. IMPLICATIONS FOR POLICY

The preceding analysis simply indicates the amount of subsidisation required when (first best) optimal pricing and output decisions are made by a public transport operator; but, as suggested in the introduction, paying that amount of subsidy clearly need not guarantee a position of maximum net benefit. The actual effect of the subsidy would depend on how it was paid, the nature of the concern operating the service, and the relationship of that concern to the authority paying the subsidy. Frankena (1983) examined some of the problems involved with the help of an adaptation of the Spence-Shesinski diagram, and the same approach is used here to provide the framework for an extension of his discussion.

In this diagram a convenient initial point of reference is the operator’s profit-maximising position. With the cost function as expressed in equation (1) above, and taking account of passenger congestion costs, the first order conditions for profit maximisation can be expressed as

\[
\pi_R = G(R) + R G'(R) - T(M) - P(R, M) - R P_R - C_R = 0
\]  
\[(17)\]

and

\[
\pi_M = -R T' (M) - R P_M - C_M = 0
\]  
\[(18)\]
In Figure 3, again using Frankena's notation, the curve \( \pi_R = 0 \) is the locus of points satisfying (17), and this shows the profit-maximising level of \( R \) for each value of \( M \) (that is, the level at which marginal revenue and "short-run" marginal costs are equal). Similarly, \( \pi_M = 0 \) is the locus of points satisfying (18), and can be interpreted as showing the profit-maximising value of \( M \) for each value of \( R \) (where the additional revenue generated by a marginal vehicle-kilometre is equal to its cost).
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In view of the restrictions on the relevant partial derivatives suggested in section 2, these curves will be upward sloping where the direct second order conditions for a maximum are satisfied. The overall profit-maximising position is then found where the two curves intersect, provided (to meet all the second order conditions) that \( \pi_R = 0 \) is steeper than \( \pi_M = 0 \), as at point \( N \). Iso-profit contours can then be added to the diagram showing combinations of \( R \) and \( M \) yielding particular (less than maximum) profit levels; and in Figure 3, \( \pi^0 \) is drawn to represent the break-even contour. Likewise \( \pi^\circ \) is drawn to represent the iso-profit contour when profits are what they would be if net social benefits were maximised. Since the previous discussion suggests that profits would then be negative, \( \pi^\circ \) is drawn outside \( \pi^0 \).

The first order conditions for net social benefit maximisation, which are conditions (15) and (16) from above when everything is included, are similarly illustrated in Figure 3 by the curves \( V_R = 0 \) and \( V_M = 0 \). \( V_R = 0 \) lies to the right of \( \pi_R = 0 \), because maximisation of net social benefit requires the fare plus the marginal external benefits of public transport use (reflected in \( X_R \)), rather than marginal revenue, to be equal to short-run marginal cost. In contrast \( V_M = 0 \) will lie below \( \pi_M = 0 \) to the extent that expanding public transport services imposes external costs (reflected in \( X_M \)) by adding to traffic congestion and the like. These curves will also be upward sloping and, as long as \( V_R = 0 \) is steeper than \( V_M = 0 \), will intersect at the social-benefit-maximising position, shown as \( H \) in the diagram. \( H \), of course, also lies on \( \pi^\circ \).

Figure 3 can now be used to consider what form subsidies need to take under various conditions to make it possible to reach the (first best) social-benefit-maximising position or some (second best) budget constrained alternative. For example, if the operator were a profit-maximising private concern, in the case illustrated in Figure 3 it could be induced to move from \( N \) to \( H \) by an appropriate subsidy per passenger-kilometer (equal to the difference between marginal revenue and marginal social cost at \( H \)), coupled with a tax per vehicle-kilometre equal to \( X_M \). The net effect of these would probably be to add substantially to the operator’s profits, but any excess profits could then be removed by an appropriate lump-sum tax. However, if the subsidising authority were unable or unwilling to pay the amount of subsidy required to reach \( H \), an appropriate combination of smaller subsidies and taxes would induce the operator to move to some “second best” position on the dashed line \( HEN \) in Figure 3. This is the locus

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9 With the less specific functions used in Frankena (1983) these curves could be negatively sloped, though it can be argued that that result would be unlikely. See, for example, Clarke and Else (1983) or Else (1982).

10 The slope of these contours is \(-\pi_R/\pi_M\), so they are horizontal where they intersect \( \pi_R = 0 \) and vertical where they intersect \( \pi_M = 0 \).

11 Again the more specific functions used here enable us to draw more definite conclusions about the relationship between \( V_M \) and \( \pi_M \) than is possible with Frankena’s formulation, even though the present model also includes externalities.

12 A subsidy per passenger-mile would push \( \pi_R = 0 \) to the right, and a tax per vehicle-mile would push \( \pi_M = 0 \) downwards.
of points of tangency between the iso-profit curves and welfare contours like \( W \), which can be constructed round \( H \) to connect combinations of \( M \) and \( R \) yielding the same net benefits. Of the points along \( HEN \), point \( E \) on the breakeven iso-profit curve would not, theoretically, need a net subsidy, as the amounts paid out in passenger-kilometre subsidies could potentially be recouped through vehicle-kilometre and lump-sum taxes and still leave the operator covering its costs. Points along \( EH \) would require some net subsidy, but points between \( N \) and \( E \) could (again potentially) provide the authority with an opportunity to benefit both transport users and taxpayers.

Alternatively, if the transport operator were a publicly owned concern subject to the ultimate control of the authority paying the subsidy (for example, British Rail or one of the Passenger Transport Executives of the metropolitan counties in the United Kingdom), there would appear to be a number of ways of reaching a desired position, whether it was at \( H \) or elsewhere on \( HEN \). One way would be to require the undertaking to maximise its profits and then apply the tax/subsidy policies outlined above. A second would be to impose the tax per vehicle-kilometre as above and then require the operator to maximise ridership (that is, passenger-kilometres) subject to a budgetary constraint which incorporated the required level of subsidy.\(^{13}\)

However, as Frankena suggested, an obvious practical problem is that the subsidising authority may not be sufficiently well informed about the relevant cost and demand functions to estimate the required levels of subsidy, whether the aim is a "first" or a "second best" optimum. Recently developed assessment procedures (for example, Department of Transport, 1982) may help to reduce some of the uncertainties, but may not be directly applicable in all cases. Further, even where the authority owns and controls the operator, its control may not be sufficiently effective, because of the resource and information costs involved, to prevent the operator from pursuing its own objectives. But similar considerations must apply to almost any area of policy, and one potentially useful function of economic analysis is to indicate what may be suitable policies in the light of all the uncertainties. For that the analysis needs to be taken a stage further.

Where owners' control is restricted, a well established approach is to analyse the behaviour of concerns in terms of the maximisation of some managerial utility function, in which size has been considered to be one of the main arguments, subject to meeting budgetary requirements.\(^{14}\) One of the most commonly used indicators of the size of transport undertakings would appear to be the number of vehicles operated; and, since this is likely to be closely correlated with the number of vehicle-kilometres operated, there may be some tendency on the part of public transport operators to attempt to maximise vehicle-kilometres subject to the relevant budgetary constraint. Of course, vehicle-kilometres may not be the only argument in the utility function; the number of passengers (or passenger-

\(^{13}\) Where the operator serves markets with differing elasticities of demand, this conclusion needs qualifying on lines suggested by Glaister and Collings (1978).

\(^{14}\) See Jackson (1982) for a recent discussion of the development of this approach and its application to both public and private sector organisations.
kilometres) may be another, since managers may in general terms prefer more passengers to fewer because they may feel more passengers will enhance the status and social usefulness of their jobs.\textsuperscript{15} Nevertheless, once a respectable load factor on their services has been achieved, their willingness to sacrifice vehicle-kilometres for passenger-kilometres may be quite low. In terms of Figure 3 therefore there may be some tendency for an operator to seek to operate at a point, depending on the budgetary constraint, close to \( \pi_R = 0 \). Thus, if it were required to break even, a point in the vicinity of \( B \) would be its preferred position.\textsuperscript{16}

In these circumstances, the tax/subsidy combinations suggested above may not yield positive benefits. This can be seen with the help of Figure 4. In this diagram the situation is simplified a little by taking the case in which there are no external costs associated with public transport operation, so that \( \pi_M = 0 \) and \( V_M = 0 \) coincide and there is no need for a congestion tax per vehicle-kilometre. The effect of a subsidy per passenger-kilometre, which in the case illustrated would be all that would be necessary to induce a profit-maximising concern to move towards \( H \), would push the curve \( \pi_R = 0 \) to the right, by an amount depending on the level of subsidy, to a position such as that indicated by the dashed curve \( \pi' = 0 \). The breakeven iso-profit curve would be simultaneously pushed outwards to \( \pi'' \) but it would now have a zero slope at the point where it cut \( \pi' = 0 \). The vehicle-kilometre maximising concern required to break even (to take the case with the simplest managerial utility function) would then move from \( B \) to \( B' \), which, given the upward-sloping convex shape of the welfare contours in that region, could easily bring reduced net benefits. Moreover, if the subsidy took the form of a relaxed budgetary constraint, as seemed appropriate with concerns maximising passenger-kilometres, reduced benefits would be all the more likely.

Now, though the authority may not know whether the behaviour of the operators will actually prevent the potential benefits of any subsidies from being realised, the analysis suggests that there is a case for giving subsidies in a way which restricts the opportunities of operators to over-expand their services. From that point of view the "cheap fares" approach favoured by some metropolitan authorities in the United Kingdom has something to commend it.

The level of fares, or more strictly the fare per mile (\( F \)), is not, of course, shown explicitly in Figures 3 and 4, but the form of the demand function implies that increases in \( M \), with \( F \) constant, would generate increases in \( R \). In other words, the locus of combinations of \( R \) and \( M \) obtainable with any given value of \( F \) would be an upward sloping curve like the dotted lines \( F_B \) drawn through \( B \) and \( F_H \) drawn through \( H \) in Figure 3. Thus points above \( F_B \) would require a higher level of fares than at \( B \), and points below a lower level. If therefore an operator, whatever its initial position, were required to reduce its fares and given

\textsuperscript{15} Frankena (1981) also includes the level of fares in his managerial utility function, on the ground that it is a source of electoral support; but it is probably less relevant in the U.K., where transport managers are not subject to popular election. For a more empirically based discussion on this issue, see Cooter and Topakian (1980).

\textsuperscript{16} If other variables in the managerial utility function led to X-inefficiency, the operator required to break even would move closer to \( N \) in Figure 3.
a subsidy designed to allow it to do no more than maintain existing service levels at the reduced fares, it would move, in terms of Figures 3 and 4, horizontally to the right. Unless this move took the operator to the right of \( V_R = 0 \) (in which

\[17\] Again there might be uncertainty about the precise level of subsidy, but estimates of the costs of maintaining existing services may be felt to provide a less speculative basis for subsidy negotiations.
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case the optimal level of subsidy would almost certainly be exceeded), it would necessarily produce positive net benefits.\footnote{As capacity constraints are built into the cost function, there would be no capacity problems at any point to the left of \( V_R = 0 \).}

The control over fares implied by a cheap fares policy can therefore be seen as a way of increasing the possibility that any potential benefits from subsidised public transport will be enjoyed by the community. Nevertheless certain limitations of this policy should be noted. One is that it may not take sufficient account of passengers' willingness to pay for the benefits of a higher quality service. In terms of Figure 3 that would happen if a point were reached below the constant fares contour \( F^H \). A second is that, where an operator provides a wide variety of services and has a complex fare structure (like British Rail, for example), and particularly if, as in British Rail's case, some services are meant to be financially self-supporting and others not, an appropriate cheap-fares policy may be difficult to devise. It may then be better to adopt some form of subsidy policy which leaves the operator to fix his fares in the light of his more detailed knowledge of the market. But for undertakings operating reasonably homogeneous services on a local scale, and where the relationship between the authority and the operator is in any case rather closer than with national concerns, the balance of advantage would more clearly lie with "cheap fares" policies.

6. CONCLUDING COMMENTS

The analysis in sections 2 to 4 would appear to suggest that, even if one takes a fairly conservative view of the effects of externalities, subsidies to maximise net benefit could amount to 60% or more of the operating costs of passenger transport undertakings — that is, if the relevant elasticities are indeed of the orders of magnitude which empirical studies have suggested in the region of the optimal position. This may seem, to some at least, a surprisingly high figure. It certainly implies much larger subsidies than have usually been paid in the United Kingdom, except in South Yorkshire. Nevertheless it does not seem to be unduly out of line with the results obtained in recent cost benefit studies which have approached the problem from a different angle. For example, the work of Glaister and others on the metropolitan counties (Glaister, 1984) has suggested that only in South Yorkshire were the net benefits of subsidies at the margin close to zero in 1980/81; in other areas with lower relative subsidies there appeared to be significant marginal net benefits. Dodgson's study of Merseyside (Dodgson, 1983) also suggested positive marginal benefits.

Further, the discussion in section 5 suggests that control of fares may be a way of preventing the benefits of public transport subsidies from being dissipated in the provision of excessive service. Again, the empirical evidence appears to support this: Glaister's study suggests that in some metropolitan areas in the United Kingdom positive net benefits would follow from reduced levels of service as well as from lower fares. By contrast in South Yorkshire, where, to a far greater extent than elsewhere, the main thrust of policy since 1975 has been to avoid any
nominal fare increases\textsuperscript{19} (which means of course that fares have fallen substantially in real terms), there is no similar suggestion that provision of service has been excessive.

Of course, as is widely recognised, the fact that subsidies may yield positive benefits at the margin is not a conclusive argument for subsidisation. There are a number of reasons for this. First of all, there is the important question of the opportunity cost of public funds — whether greater benefits can be obtainable from alternative, public or private, uses of the funds. Secondly, there is the question whether there may be other ways of realising the potential benefits.

The second question arises because a substantial part of the efficiency case for public transport subsidies appears to rest on the phenomenon of decreasing social costs, and because, as both Glaister and Dodgson have pointed out, the benefits of subsidies accrue largely to the people who use the service. Other public-utility-type industries with decreasing costs in the United Kingdom (for example, telephones, electricity supply) operate under policy constraints which reflect a (government) value judgment that the costs of supply should in general be covered by charges to users. Reconciliation of this constraint with the requirement of economic efficiency is then (where possible) attempted through the use of two-part or multi-part tariffs. Now it might be argued that, because public transport lacks the direct link between suppliers and users which exists in other public utilities, the most practical way of collecting the “lump sum” component of a notional two-part tariff is through taxation (that is, through central taxes for a national network like the railways and through local taxes for more local services). It might also be argued that the policy constraints applying to other public utilities should not necessarily apply to public transport, but that instead, public transport subsidies should be seen as an instrument for pursuing distributional or social objectives. Nevertheless, more widespread use of arrangements of the “travel card” type, through which individuals can buy the right to travel at relatively low fares, might be worth considering as a way of developing two-part tariff systems for public transport. Fares for individual journeys could then be more closely related to marginal social costs without imposing excessive requirements for subsidy. That approach need not, of course, preclude the payment of some subsidy reflecting the more general benefits of public transport. Nor need it preclude the pursuit of distributional or social objectives. Indeed, allowing relatively disadvantaged groups to purchase their travel cards on more favourable terms might be more cost-effective than blanket subsidies as a means of pursuing distributional objectives.

REFERENCES


\textsuperscript{19} A simplification of the fare structure in March 1984 brought a small increase in the average fare, but whether further changes would be avoided in 1985 was not clear at the time of going to press.
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