DEMAND FOR UNLIMITED USE TRANSIT PASSES

By Lawrence B. Doxsey*

Transit passes for unrestricted use have become an accepted and sometimes prominent component of the fare structure for many transit agencies in the United States. A report prepared in 1976 for the Urban Mass Transportation Administration (UMTA) on the status of transit fare prepayment found that 41% of the 146 systems surveyed offered some form of transit pass (Hershey et al., 1976). Almost 25% offered monthly transit passes. In the intervening years monthly passes in particular have become increasingly popular in the industry. Nearly every large U.S. transit undertaking either offers passes or has seriously considered introducing them.

In introducing transit passes, operators have commonly expected to maintain or increase ridership without losing revenue. In the survey conducted for the UMTA report, no operator with an unrestricted pass programme reported a fall in ridership attributable to the pass, and 55% indicated a ridership increase. Only 2% could identify a reduction in revenue, though another 27% were uncertain about the effect on revenue. These responses appear to be a firm endorsement of transit passes, and may well be representative of the beliefs on which operators decide whether to introduce them; but they were generally subjective perceptions based on aggregate ridership and revenue trends over the period covering the introduction of passes, rather than the results of formally isolating the independent effects of passes on either ridership or revenue.

In this paper an attempt is made to identify the factors which lead an individual user to buy a pass. Demand for transit passes is examined through analysis of the individual's underlying demand for transit. An important purpose of the paper is to provide information for operators assessing advantages and disadvantages of introducing or continuing a transit pass.

Some theoretical study on the demand for transit passes has been undertaken by Lago and Mayworm (1982). They, however, attempted to treat the demand for transit passes in isolation from a cash fare alternative, emphasising instead the pure income effect of purchase of a pass. While valid, this emphasis is relevant only to the extent that transit demand is highly income elastic and/or the absolute income effect is large. Neither is likely to be true. Furthermore, because they do not cover the influence of the cash fare alternative, their analysis treats only one part of the problem. Related

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empirical work has been conducted by Page (1981), though her data were inadequate for more than a superficial analysis.

The first part of this paper develops some of the microeconomic underpinnings of individual demand for transit passes. The model developed not only provides the basis for the subsequent empirical analysis, but in itself suggests that a transit property is likely to increase ridership but reduce revenue through issuance of transit passes. The second part of the paper presents an econometric model of individual choice on purchase of a pass.

MICROECONOMICS OF DEMAND FOR TRANSIT PASSES

It is essential, if one is to understand the problem of individual purchase of a transit pass, to realise that the choice whether or not to purchase a transit pass rests with the individual transit patron, and that the patron will choose to buy a pass only if he/she perceives a benefit from so doing. Thus, an operator cannot realise gains from selling transit passes unless the passes are offered under circumstances which provide a benefit to the patron. Specifically, unless patrons commonly attach a heavy weight to the convenience of holding a pass—and little weight to the inconvenience of obtaining it, the risk of losing it, and so forth—pass buyers will generally be those who expect the economic value of the trips taken with the pass to exceed its price.

This argument is illustrated in Figure 1, which depicts an individual’s demand for transit at alternative fare levels. Consider a cash fare level in the amount $OP$. At this fare the individual takes $ON$ transit trips and spends on these trips a total amount equal to $OPMN$. Without a transit pass additional trips beyond $ON$ will cost an additional $OP$ cents each. However, should the individual choose to purchase a transit pass, the marginal cost of transit trips falls from $OP$ to zero. As a result, assuming income effects to be negligible, the additional trips are represented by the distance $NQ$. Their economic value to the individual is represented by the area $NMQ$. If the price of the pass is set above the amount represented by the area $OPMN$ but below the amount represented by $OPMQ$, two results occur. First, in buying a pass the individual receives an economic gain equal to the difference between $OPMQ$ and the pass price. Second, the individual’s expenditure on transit, and hence transit revenue, increases by the difference between the pass price and $OPMN$. In general, the greater the economic gain to an individual from buying a pass, the greater is the likelihood that the individual will buy a pass. At a pass price below $OPMN$ the individual receives a greater economic gain, and thus has a greater incentive, and is more likely to buy. Both the individual’s expenditure on transit and the operator’s revenue are less with the pass than without it. At a pass price above $OPMQ$ the individual receives no economic gain with the pass, and so is likely to refrain from buying.

Selling passes can increase revenue, and thus be advantageous to the operator, if the price falls below the individual values of $OPMQ$ for a sufficient share of the purchasers. There are, however, three difficulties in achieving this goal. First, we have examined only the revenue implications of the induced trips and have ignored the additional cost of carrying them. If the cost of providing the additional trips exceeds the additional revenue they generate, the operator’s deficit will rise even while revenue rises. Note that the problem may not be very great if most of the induced trips are on
portions of the system with slack capacity and thus can be carried at very low marginal cost; and also that we have omitted the offsetting gains to the operator from such things as improved cash flow. However, it is likely that both revenue and cost impacts of these items are relatively small, and that their net effect is consequently also small.

The second problem is a bit more serious. It is well accepted that demand for transit is price inelastic, at least at observed fare levels. By implication, we will generally find that point \( Q \) is relatively close to \( N \), and therefore that \( NQM \) is relatively small. Therefore setting a pass price to lie between \( OPMN \) and \( OPMQ \) requires hitting a fairly small target. Also, if \( OPMN \) and \( OPMQ \) differ by little, the gain to be divided between the operator's increased revenue and the patron's increased economic benefit is
likewise small; this reduces the attraction of passes for each party and reduces the patron’s likelihood of buying a pass.

The third and final problem is the most severe of the three. The presentation so far has been focused on an individual’s demand for transit and on identifying the pass price which would generate a gain to be divided between the patron and the operator. That is, we have attempted to define the situation under which issuance of passes leads to a Pareto superior outcome. However, the ridership is made up of a large number of people each with his or her own values of $OPMN$ and $OPMQ$. The difficulty arises that the appropriate pass price will differ greatly over the population of patrons, while little or no price discrimination is possible.

To illustrate this problem it is convenient to divide users into three groups on the basis of the relationships between the price structure and their individual values of $OPMQ$ and $OPMN$. Group one is composed of patrons for whom the pass price exceeds their individual values of $OPMQ$. These people would realise an economic loss in buying a transit pass, and so are unlikely to do so. Because they don’t buy passes, their contribution to transit revenue is unaffected by the availability of passes. Group two encompasses patrons for whom the pass price falls between their individual values of $OPMN$ and $OPMQ$. Because members of this group realise an economic benefit in purchasing passes they are likely to do so. With pass purchase their contribution to transit revenue increases. However, following on the first difficulty discussed above, the revenue gain per purchaser will be small. Finally, there is group three, for whom the pass price falls below their individual values of $OPMN$. These people are most likely to buy passes, since they thereby reduce their expenditure on transit even if their individual demands are perfectly inelastic. Because patrons’ expenditure is operator revenue, this group’s purchase of passes works to the operator’s disadvantage. The fundamental disadvantage to the operator in offering transit passes is that only a small portion of the ridership will be so placed that buying a pass both increases the operator’s revenue and yields the patron an economic gain. With a pass priced near the common cash fare equivalent of 9 to 12 weekly trips, group three is likely to be larger than group two. Furthermore, because members of group three are those who make the greatest use of transit, the revenue loss per purchaser will tend to be large in proportion to the revenue gain per purchaser from members of the second group.

It should be emphasised that this conclusion does not depend on the actual pass price, but only on differences between patrons in levels of use of transit and in fare elasticities. Figure 2 presents a hypothetical distribution of monthly trip rates for a system’s user population. The solid line represents the distribution of levels of usage if all patrons paid cash fares. The dashed line represents the levels which would be realised if all patrons had transit passes. The dashed line is derived from the solid line by adding to it the distribution of trips induced by the purchase of transit passes. Both are drawn to peak in the neighbourhood of 40 trips a month. This is consistent with empirical results from many properties, and with the observation that many people use a transit system exclusively or primarily for commuting to and from work. It is convenient to express pass prices by their cash fare break-even equivalent. Thus, for example, with a $0.50 cash fare a $20.00 pass price implies a break-even at 40 trips. Note that, in light of the discussion of the valuation of induced trips, the break-even should not be taken as the pass price which leaves the patron equally well off with or without a pass, but rather as that which leads to equal expenditure in either case. For a
given breakeven level the proportion of patrons who would reduce their cash outlay through buying a pass can be represented in Figure 2 as the cumulative area to the right of that breakeven point and under the solid line. An upper bound on the share of patrons who would realise an economic gain through purchasing a pass is represented as the cumulative area to the right of the breakeven point and under the dashed line. It is an upper bound because it implicitly assumes that the individual values the induced trips at the fare level. However, we know that if the individual did so value the trips they would have been made even without a transit pass; so their value must be less than the fare level, and the dashed line is an upper bound.

Similarly, the difference between the two areas on the right of the breakeven level can be shown to be an upper bound on the share of patrons who belong in group two: those who, in buying passes, both increase their contribution to the operator’s revenue and realise individual economic gains. It should be evident that without a large component of induced trips the difference between the two areas will be quite small relative to the area under the solid line. This means that for every patron who spends more on transit through buying a pass there will several who spend less. Furthermore, if the right-hand tail of the distribution is long, there will be people who spend a lot less. In contrast, we have argued above that those who do spend more do so by only a small amount. The net effect is a fall in transit revenue.

Consider now the effect of different pass prices. At a relatively low price, say at a 30 trip breakeven, a large share of patrons will find it in their interests to buy passes; some will receive very large windfalls in so doing, and the drop in transit revenue will likewise be large. At a higher pass price fewer people will find advantage in a pass, the largest windfalls will be smaller, and both pass sales and the revenue loss will be reduced. However, the share of pass buyers for whom purchase implies an individual increase in expenditure remains small at any pass price. As the pass price is increased, fewer individuals find advantage in a pass, sales continue to fall, and the revenue loss abates.

To summarise, the combination of price-inelastic individual demand and wide dispersion of individual demands at any fare level generates an inverse relationship between pass sales and revenue.
EMPIRICAL ANALYSIS OF PURCHASE OF PASSES

In the remainder of this paper we present an econometric mode of the choice made by individual transit users between buying a pass and paying the cash fare. The model is estimated on disaggregate data, and follows the theoretical structure developed above. The source and nature of the data are first discussed; then follows an explanation of the relationship between the theoretical model and the model used for econometric estimation. Thereafter we present the results of estimation and suggest some basic implications for policy.

Origin and characteristics of the data

In March 1979 the fare on the MARTA system was increased from 15 cents to 25 cents. Simultaneously, an unrestricted use monthly transit pass was introduced at a price of $10.00. A policy of free transfers remained in effect after the changes in fare structure. The data used here were collected during May 1979, as the initial measurement for an Urban Mass Transportation Administration study of the influence of the pass in offsetting the additional transfer requirements caused by realignments of bus routes in July 1979, at the time of the opening of Atlanta’s heavy rail line.

The survey was conducted on-board over a period of three weeks. Bus runs were drawn to provide representation proportional to the number of boardings by passengers on the respective routes. To make up the determined quota of a route, runs were drawn randomly in morning peak, midday, afternoon peak, evening, Saturday, and Sunday. The final sample contained 4,651 observations.

The extensive use of field personnel gave substantial control over the sampling process. On each selected bus run the people interviewed were alternately people boarding with passes and boarding with cash or transfers. That is, from the beginning of the interviewer assignment the fifth pass boarder was surveyed, then the fifth cash boarder after the surveyed pass boarder, then the fifth pass boarder after the surveyed cash boarder, and so on. In the event of a refusal, the sixth boarder in the appropriate category was surveyed. Thereafter the sampling reverted to the person who would have been next to be selected if there had been no refusal. This process effectively stratified riders into pass and cash classes and provided a different sampling rate for each. Over the entire month of May approximately 17% of all trips were made with passes. The sample, on the other hand, was split about evenly between pass and cash trips. Thus the sampling rate from pass trips was about 4.8 times as high as that from cash trips.

Administration of surveys by interviewers appears to have had a large and favourable effect on the rate of response. Complete records were not kept, but it appears that the rate was at least 85% and probably over 90%. Additionally, the completion rate for the surveys was very high. Few questions had more than 10% non-response, and two-thirds of the surveys had answers to all questions.

The econometric model

Each transit user is faced with the choice whether to buy a transit pass or not. Therefore the appropriate statistical model is one of binary choice, wherein the
probability of pass purchase is related to characteristics of the observed individuals. In this case the estimated model is binary logit of the standard form:

\[
\text{Probability of pass purchase by person } i = \frac{1}{1 + e^{X_i' B}}
\]  (1)

where \( X_i \) is vector matrix of independent variables and \( B \) is a vector of estimated coefficients. Alternatively, \( X' B \) can be written as equal to the natural logarithm of the probability of buying a transit pass. The logit form was chosen largely to allow easy incorporation of weights to adjust both for the pass/non-pass stratification and for the fact that the probability of an individual user being selected in an on-board survey is directly proportional to his/her frequency of use of transit (Doxsey, 1982; Manski and Lerman, 1977).

The basic premise of the model is that an individual will choose to buy a pass only if he/she expects to be better off with it than without it. Ideally, the model should both include a measure of "better off" and reflect the influence of the process by which expectations are formed. Thus it would be sensitive to differences in expected benefit, to differences in the certainty of expectations, and to differences in the individual assessment of each. The estimated model approximates each of these criteria, probably doing best in meeting the first, for which a direct measure is computed.

Here the expected net consumer surplus associated with pass purchase is used as the measure of benefit; net consumer surplus is defined as the total consumer surplus realised with a pass less that realised by paying cash. As part of the survey, each respondent was asked how frequently each week she/he rode MARTA buses for work and for non-work trips. Each was also asked how his/her use for work purposes had changed since the date of the fare increase. Together, these two sets of questions provide estimates of the individual's use of MARTA before the fare increase. It was a drawback that the survey could not be conducted till May 1979, the third month of pass sales. The passage of ten to thirteen weeks from the date of the fare increase and introduction of passes caused at least some reduction in the accuracy of answers about bus travel before 1 March. Because it also excluded those people who abandoned MARTA as a consequence of the fare increase, the study population consists of people who elected to ride the transit system at the fare structure of 25 cent cash fare and $10.00 pass price.

The totals of weekly MARTA trips taken by each respondent before and after the fare increase were used to construct one of the fundamental variables for the empirical model. For comparability with the period of validity of a pass, the responses were converted from a weekly to a monthly base by multiplying by 4.2. Because a pass eliminates the need for any payment on boarding, a pass holder's use of MARTA represents his/her demand for transit with a zero marginal fare. Thus, for each respondent the data provided observations of demand at two price levels. For pass holders the observations were at prices of fifteen cents and zero cents. For cash-paying riders the observations were at fifteen and twenty-five cents. From the available pair of points a transit demand curve was estimated for each individual, and from it the net consumer surplus associated with pass purchase was computed. At an individual level this requires assuming no change in any argument of the demand function except fare. This assumption is likely to have been violated in at least some cases. About 1% of the cash boarders indicated that they were making more trips at the 25 cent fare than they made at the 15 cent fare. These changes were almost certainly caused by changes in
factors other than fare. People who reduced their usage for reasons other than fare could not be identified. Presumably these, too, were few in number.

An exponential form was used for estimation of individual demand curves. It can be written as:

\[ q = q_o e^{b(p_o - p)} \]  

(2)

where

- \( p \) = the cash fare level
- \( p_o \) = the cash fare at some arbitrary reference level
- \( q \) = the quantity demanded at fare \( p \)
- \( q_o \) = the quantity demanded with a price of \( p_o \)
- \( e \) = the base of the natural logarithm
- \( b \) = a coefficient determining the specific demand curve.

The exponential form was chosen on the basis of results from experiments in free transit (Donnelly et al., 1980; Connor, 1982). These experiments clearly showed individual demand to be convex to the origin as fare approaches zero. The exponential form reflects this behaviour. For each individual, a demand curve was estimated by using the observed pair of price quantity observations to fix a value of \( b \). The value of \( b \) is related to the demand elasticity in that \((dq/dp)(p/q) = -bp\). Thus at each fare level a larger value of \( b \) is equivalent to a more elastic demand. A fare of 15 cents was used for \( p_o \) because each respondent had a quantity observation at that level. Once estimated, \( b \) becomes a parameter which, together with \( p_o \) and \( q_o \), fully defines the individual demand curve.

An assumption of perfectly inelastic demand (\( b = 0 \)) was imposed on those individuals whose responses otherwise indicated upward-sloping demand curves, and on those pass buyers who had previously not used transit. These increases in use of transit were assumed to have been caused by factors other than the fare change. Since induced trips were pervasively a small share of the whole, these artificial restrictions on elasticity are unlikely to have been detrimental. Because there was no indicator for exogenous decreases in use they were not given special treatment.

Individual net consumer surplus from pass purchase was calculated by integrating under each demand curve from a price of zero cents to one of 25 cents and subtracting the $10.00 price of the pass. Thus a value to be referred to as individual savings from pass purchase was defined as:

\[ \text{Savings} = \int_0^{0.25} q_o e^{b(0.15 - p)} \, dp - \text{(pass price)} \]  

(3)

It may appear that through the savings variable the marginal fare faced by the individual and the level of individual demand are erroneously treated as exogenous to the choice, but this is not so. Rather, the savings variable is written as endogenous to the cash fare and pass price set by the operator and to the individual's transit demand function. It is not the demand function, but the actual level of demand, which is jointly determined with the choice of method of payment. Similarly, while the marginal fare actually faced is endogenous, the alternative marginal fares presented by the operator are exogenous. Thus the factors determining the savings variable, as well as the savings variable itself, are indeed exogenous to the choice.

The value of savings is positive for heavy users of the system and for some of those moderate users who are highly responsive to fare, and negative for light users of the
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system. The theoretical development in the first part of the paper suggests that the value of savings has a large impact on the individual decision to buy a transit pass. Computing the value directly will make possible an empirical measurement of its role and, hence, a test of the theoretical model.

For several reasons, all generally pertaining to individual uncertainty of expected use of transit, non-linear forms were considered in specifying the relationship between savings and the log-odds pass purchase. Most important, the demand from which the savings were calculated is best interpreted as the expectation of demand from an associated probability distribution. In the face of individual uncertainty about the volume of use of transit, initial dollars of expected gain or loss may have an influence on purchase different from that of subsequent units. In view of the individual consumer's uncertainty about demand, an increase in the expected consumer surplus is accompanied by a decrease in the probability of suffering any loss. If the higher moment effect has an influence, the relationship will be non-linear when expressed in terms of expected surplus alone. Furthermore, if the individual distributions of expected usage are skewed, responses to gains and losses will be asymmetric. Finally, the effects of factors such as the inconvenience of obtaining a pass, which is a once-a-month cost, and the convenience of using it, which recurs at each trip, may be reflected in the influence of the savings variable and may well introduce non-linearities.

One variable was included as a proxy to provide a direct reflection of individual uncertainty about the amount of travel: this was the proportion of bus trips taken for non-work purposes. Acceptance of the variable as a measure of uncertainty depends on the assumption that the number of work trips to be made in a given month is known ahead of time with greater certainty than is the number of non-work trips. For comparability over all respondents, this variable was based on usage before the fare increase and introduction of passes.

An important hypothesis for the fare integration study which gave rise to the data was that a pass could aid intramodal integration, so that people who regularly had to transfer would have a greater tendency to buy passes than people who seldom or never had to transfer. Because of MARTA's free transfer policy, the only transfer disadvantage offset by a pass was the inconvenience of requesting a transfer slip from a driver. The model includes dummy variables for the first transfer needed during the surveyed trip and inclusively for all additional transfers. It should be clear that measuring transfers on the surveyed trip only did not provide a full reflection of the stated hypothesis. However, the more regularly a person transfers, the more likely is it that the surveyed trip will include a transfer. Inversely, the people whose trips included transfers are more likely to be people who transfer regularly.

Though few demographic or socioeconomic variables are included in the estimated model, a substantial number were collected. These were available for estimation observations on gender, race, a six-class categorisation of age, a five-class categorisation of income, household size, and auto availability. It was expected that these variables might distinguish segments of the transit market with systematically different responses on purchase of passes. As one example, it has sometimes been argued that the up-front cost deters low-income people from buying monthly transit passes even if they would be economically better off with them. As another example, people with autos readily available might be less certain about their expected use of buses, and so might less readily purchase passes.


**Table 1**

*Estimation Results*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Value</th>
<th>t-statistic</th>
<th>Significance Level for Rejecting the Hypothesis that the Coefficient equals Zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.725</td>
<td>-4.41</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Gain for savings &gt; 0</td>
<td>0.217</td>
<td>7.96</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Loss for savings &lt; 0</td>
<td>-1.39</td>
<td>-10.8</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>(Loss)² for savings &lt; 0</td>
<td>0.123</td>
<td>7.76</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Share of trips for non-work if savings &gt; 0</td>
<td>-2.35</td>
<td>-6.12</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>One transfer</td>
<td>-0.021</td>
<td>-0.153</td>
<td>12%</td>
</tr>
<tr>
<td>Two or more transfers</td>
<td>0.302</td>
<td>1.70</td>
<td>91%</td>
</tr>
<tr>
<td>Age 40–59</td>
<td>0.415</td>
<td>2.67</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Age 60–64</td>
<td>0.816</td>
<td>2.48</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>Male</td>
<td>-0.216</td>
<td>-1.69</td>
<td>91%</td>
</tr>
</tbody>
</table>

$L(O) = \log$-likelihood with all coefficients set to zero = $-1954.7$

$L(C) = \log$-likelihood with constant only = $-1187.4$

$L(B) = \log$-likelihood with estimated coefficients = $-850.2$

$-2[L(B) - L(O)] = 2209.0$

$-2[L(B) - L(C)] = 674.4$

The number of observations used in estimation was 3,007. Observations with missing data and observations on people who had not heard of the pass were deleted from the sample.

**Econometric results**

The results of estimation are presented in Table 1. The model was estimated by maximum likelihood. The four columns list the variables, the coefficient estimates, the $t$-statistics, and the significance levels of the $t$-statistics. At the bottom of the table are summary statistics for the goodness-of-fit. Likelihood ratio statistics have been computed for the estimated coefficients relative both to the model with all coefficients set to zero and to the model with the constant alone. The likelihood ratio statistic is calculated as $-2[L(R) - L(U)]$, where $L(U)$ is the log-likelihood of the unrestricted model and $L(R)$ is the log-likelihood of the restricted model. The statistic is distributed chi-square with degrees of freedom equal to the number of restrictions. The statistic allows testing of the hypothesis that the unrestricted, estimated model cannot be distinguished from the restricted, reference model. From either reference model the null hypothesis can be rejected at well above a 95% confidence level.

The second through fourth entries comprise the savings variable, with savings expressed in dollars. The first of them is a linear measure of savings for cases where savings were greater than zero. The others are linear and squared measures of losses for cases where savings were less than zero. We also tested a second order term for
positive savings, but it made no contribution to the model. The coefficients suggest that a one-dollar increase in a positive savings value would increase the log-odds of pass purchase by 0.217. Since the relationship between the probability and the log-odds is highly non-linear, the implication for changes in choice probability of this and other coefficients depends on the initial probability level, and thus cannot be generally expressed. As the probability moves away from 0.5, increasing changes in log-odds are required to achieve constant changes in probability. For losses, because of the presence of the second order term, the effect of an increased loss depends on the initial level of loss. From the neutral point of no gain or loss, the effect of a dollar loss on the log-odds of purchase outweighs the effect of a dollar gain by 6.4 to one. Given the relationship between the probability and the log-odds, if the probability of purchase at the neutral savings point is less than 0.5, then a larger coefficient on losses than on gains is required to produce symmetric impacts on the probability of purchase. Indeed, the estimated mean purchase probability at the neutral point is just under 0.3, and the ratio of the probability impacts of an initial dollar loss to that of a dollar gain is about four to one. One implication of this is that users for whom purchase of a pass would simultaneously provide an individual gain and increase the operator's revenue are unlikely to make the purchase.

The negative sign on the second order term implies that succeeding units of loss affect the log-odds at a decreasing rate. The combined influence of the two loss terms is maximised at a loss of about $5.50. However, given the magnitude of the coefficient on the first order term, the total effect remains strongly negative up through the maximum possible loss of $10.00. Few respondents had measured losses of more than $8.00, and the fit appears to have been somewhat dominated by observations on people with small to medium losses. If so, the sign on the second order term may result from an already low probability of purchase as losses begin to accrue, and a rapid early abandonment of pass purchase in response to losses.

The significance level for the t-statistic on each of the savings variable components is well above 95%. In addition, Table 2 shows the result of a likelihood ratio test on the joint influence of the three variables, and hence on the contribution of the savings variable as a whole. The log-likelihood substantially increases (in absolute value) with the deletion of the savings variables. Net consumer surplus, expressed here as savings, clearly makes an overwhelming contribution to the decision on pass purchase. This is consistent with the theoretical structure developed in the first part of the paper.

The contribution of the share of trips for non-work purposes was also statistically significant for people whose expected savings from a purchase were positive. The variable had insignificant influence for respondents with negative values of savings. The magnitude of the coefficient is large, but that of the variable is bounded between zero and one, so the total contribution is, at most, modest. As explained above, the variable was entered as a proxy for the degree of certainty in expectation of frequency of travel. If the underlying premise of greater certainty about the number of work than of non-work trips holds, then this result suggests that the probability of pass purchase is influenced by the certainty of individual travel expectations, and that the second moment about the mean of expectations has a negative influence on purchase.

The hypothesis that transfers positively contribute to pass purchase is confirmed weakly, at best. The first transfer had little influence on the purchase decision. The coefficient on the second and subsequent transfers is significant at the 91% confidence
TABLE 2

Likelihood Ratio Tests on Purchase Model Coefficients

<table>
<thead>
<tr>
<th>Deleted Variables</th>
<th>Likelihood Ratio Statistic</th>
<th>Confidence Level to Reject H_0*</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>850.2</td>
<td></td>
</tr>
<tr>
<td>Gain for savings &gt; 0</td>
<td>1067.0</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>(Loss)² for savings &lt; 0</td>
<td>433.6</td>
<td></td>
</tr>
<tr>
<td>One transfer</td>
<td>851.7</td>
<td>78%</td>
</tr>
<tr>
<td>Two or more transfers</td>
<td>3.00</td>
<td></td>
</tr>
</tbody>
</table>

* The hypothesis H_0 states that the model with deletions is undistinguished from the model without deletions. This is loosely equivalent to saying that the deleted variables made an insignificant contribution to the whole. In comparison with the t-tests above, the chi-square tests here are primarily for exploring the deletion of groups of variables.

level, but the coefficient value is relatively large. It implies that making two or more transfers has an impact equivalent to about $1.40 greater positive savings. Recall that the fare structure required no additional fare for transfers, and that it is therefore a pure convenience effect we are attempting to test. However, recall also that only transfers on the surveyed trip were recorded, so that the transfer variable has been applied to some respondents who rarely transfer and omits some who often require transfers. Despite the lack of influence of the first transfer, both variables were left in the model for completeness. Table 2 gives the result of a log-likelihood test on the joint contribution of the two variables. Only at a 78% confidence level can the hypothesis of no effect be rejected. In general, the need to transfer does not appear to have a strong influence on pass purchase.

Of all socio-economic, demographic, and related variables, only two (age categories and the gender distinction) exhibited any significant effect on pass purchase, and gender was significant at the 91% level. These variables identify some systematic differences in pass purchase behaviour among riders, but the results do not readily admit of behavioural interpretation. Note that together the two age variables bracket the 40 to 64 age group, and that, while the coefficient for the second is nearly twice that of the first, a test of the hypothesis that the two coefficients are equal allows rejection at only the 73% level.

Because there is interest in the transit and pass purchase behaviour of people with low incomes, several income relationships were explored. Neither low income nor low income related to household size or race had any meaningful impact on pass purchase. This strongly suggests that the arguments for passes of shorter duration to aid the budgeting of low-income people are misdirected.

Table 3 is a classification of the predicted versus actual choices of the respondents. A respondent was classified as a predicted pass buyer if his/her estimated pass choice
probability was 0.5 or greater. Overall the model did very well, with 90.3% of all respondents correctly classified. Nearly 87% of the prediction errors were pass buyers predicted to pay with cash. It is tempting, though not testable, to argue that this in some way reflects the unmeasured convenience attraction of the pass. In any event, the correct prediction of 83% of actual pass purchases is itself very good.

Table 3 reflects the model’s goodness of fit to the sample data. As the sampling rate was much higher among pass users, and as the model was relatively more effective in predicting the choices of cash users, the model would accurately predict more than 90.3% of choices by members of the population. Correctly predicting the choices of 83% of the 18,000 pass buyers and 97.4% of the 144,000 cash-paying riders would imply correctly predicting the choices of 95.8% of all the users.

The overwhelming influence of the savings variable on the individual decision on pass purchase has important implications for a transit operator. With inelastic individual demand, transit users will seldom receive the savings necessary to cause them to buy passes unless their expenditure on transit is lower with a pass than without it. Furthermore, even among those riders whose individual demand is elastic, many will be taking a sufficient number of trips to cause their expenditure on transit to be reduced through purchase of a pass. Since expenditure by users is also transit revenue, the reductions in expenditure which contribute to individual savings to bring forth pass purchases also appear as a reduction in transit revenue.

**The effect of changes in the pass price and cash fare**

If we estimate the choice decision as a function of individual savings, the cash fare and pass price enter only as parameters. The responsiveness to changes in savings can be readily identified from the coefficients, and both individual and aggregate demand elasticities can be calculated with respect to savings. However, because the pass price and cash fare are directly set by policy, while the levels of individual savings are not, it
is instructive to measure the influence on the demand for passes of changes in the cash fare and pass price.

Changes in cash fare and in pass price affect pass purchase through their influences on the savings variable. The change in savings resulting from a change in cash fare can be written as the derivative of expression (3) with respect to the cash fare. Evaluated at a given cash fare level, this is equivalent to the demand for transit at the given fare. Thus, at a cash fare level of 25 cents, the change in individual savings for a change in the cash fare can be written as:

\[
\frac{d \text{Savings}}{d \text{Fare}} = q_e e^{0.25 - \rho_s} > 0
\]  
(4)

The derivative of expression (3) with respect to pass price gives the effect on the savings variable from a change in the price of the pass. An increase (decrease) in the pass price is fully reflected as a decrease (increase) in savings. Thus:

\[
\frac{d \text{Savings}}{d \text{Pass price}} = -1
\]  
(5)

The elasticity of demand for passes with respect to fare is simply the product of the elasticity of pass demand with respect to savings and that of savings with respect to fare. Similarly, the elasticity of pass demand with respect to the pass price is the product of the elasticity of pass demand with respect to savings and the elasticity of savings with respect to pass price. With respect to each policy parameter a separate elasticity is calculated for each respondent. Market elasticities are developed by weighting individual elasticities according to the individuals' shares of the entire market (Domencich and McFadden, 1975).

Evaluated at a 25 cent cash fare and a $10.00 pass price, the market elasticity of demand for passes with respect to cash fare was computed to be 0.46, indicating that a 1% increase in cash fare would bring forth an increase of just under 4% in pass sales. The elasticity with respect to pass price was −1.57 so that a 1% increase in the pass price would reduce pass sales by a little more than 1.5%. Because the elasticity values depend heavily on the distribution over the population of the savings from pass purchase, they cannot be validly extrapolated much beyond the point at which they were evaluated. The values are particularly unstable with respect to variations in the relative levels of cash fare and pass price. The elasticities therefore serve better as measures of relative responsiveness under the past price structure than as tools for predicting the impact of substantial changes in either the cash fare or the pass price.

Introducing the two prices as policy parameters enables us to use the coefficient values of the estimated model to predict the effects of alternative prices on pass sales, ridership and revenue. Unlike an elasticity approach, this requires us to predict the purchase decision of each individual respondent as a step in predicting the aggregate impact of the alternative prices. Note, however, that the model implicitly excludes the possibility of mode shifts in response to price changes. The only shifts which are admitted are those between methods of payment, together with their consequent changes in transit. The model thus predicts choice of payment method conditional on continued use of transit. Suppressing a component of the response to the fare change limits the use of the model as a forecasting tool. For this reason the prediction
exercises are most accurate for minor variations in pass price and cash fare.

Applying the approach of constructing an aggregate forecast from predicted changes on individual choice probabilities, an increase in the pass price from $10.00 to $11.00 would have reduced pass sales from 18,000 to 14,550, reduced pass revenue from $180,000 to $150,000, but increased total revenue by $15,000; this is an increase in revenue of about $4.35 for each lost pass sale. The overall increase is attributable to an additional dollar in revenue from each of the patrons who continue to purchase a pass, plus the additional revenue from the cash fares of those who switch from a pass to cash. Similarly, a pass price reduction from $10.00 to $9.00 was predicted to generate pass sales of 21,665 but simultaneously to decrease revenue by $19,100. The average revenue loss of each additional pass sale would be about $5.20.

Finally, despite the cautionary notes on predicting the effects of large price changes, the results of eliminating the pass were projected. Without the pass total monthly revenue would have increased by $36,000 and total monthly boardings would have decreased by 168,000. These sums were, respectively 2.8% of total passenger revenue of the system and 3.1% of passenger trips, 20% of the expenditure and 16.2% of the trips by the 18,000 pass holders. The direct monthly impact on the operating deficit as a result of offering passes was the $36,000 lost revenue plus the cost of servicing the 168,000 additional trips.

CONCLUSIONS

It is evident that, above all other factors, individual savings dictate the decision on buying a pass. Furthermore, consumers' response to expected gains and losses is asymmetric in a way which works to the transit operator's disadvantage. From the fitted functional form, the asymmetry between gains and losses and the contribution of the non-work share variable, the certainty with which an expectation is held also appears to have an important influence. Thus people who do not expect an economic gain from the purchase of a pass, and even those who expect a small gain, are unlikely to purchase one. There are clearly exceptions, but the tendency of individual behaviour implies that a transit operator will suffer a loss in revenue as a consequence of introducing a pass. In light of growing budgetary difficulties, the contribution of a pass to the fare structure of a system should be carefully considered.

REFERENCES


