COSTS, ECONOMIES OF SCALE AND FACTOR DEMAND IN BUS TRANSPORT

An Analysis

By Joseph Berechman*

1. INTRODUCTION

Increasing attention is being paid to the economic underpinnings of bus transport services. Students of the industry and decision makers are asking questions pertaining to the production of services, the operators’ cost structure, and the impacts on performance of regulatory pricing and subsidy policies. This surge in interest is due to the fact that mass transit, and in particular bus transport, is regarded by some as the best option for alleviating many current problems such as energy shortages, traffic congestion, air pollution and urban decline; yet public transport is in general of poor quality, is produced inefficiently, and requires increasing subsidies. Moreover, regulatory and pricing policies, which are introduced to enhance efficiency and quality, by and large fail to do so and, in fact, are viewed as having a destructive effect on the provision of services. It is, therefore, most important to understand the economic structure of bus transport, so that the effects of public policies can be evaluated. This paper proposes a cost function approach to estimate empirically production conditions and the cost structure of bus transport.

Much of the published research done to date on these and related issues has been narrow in focus, using an outdated methodological framework for analysis. The single most frequently examined issue is the existence (or non-existence) of economies of scale in bus transport (Lee and Steedman, 1970; Koshal, 1970, 1972; Wabe and Coles, 1975; Button, 1977; Fravel, 1978). Relatively few studies take a broader view and analyse other production and cost factors (Nelson, 1972; Mohring, 1972; Veatch, 1973; Foster, 1973).

In part this narrow focus on economies of scale is the result of very simple analytical constructs which, in most cases, amount to the estimation of an average cost function of a single equation (e.g., Lee and Steedman, 1970; Koshal, 1970, 1972; Wabe and Coles, 1975), or several single equations, each containing a different set of

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explanatory variables (e.g., Foster, 1973; Veatch, 1973). A very few studies attempt to estimate a system of equations and relate the specification of these cost functions to the production process of the transit services (Nelson, 1972; Fravel, 1978; Berechman, 1980). None of the studies directly estimates the industry demand function for factors of production, or attempts to analyse factor substitution and price elasticities, which are of major importance for the design of efficient transit policies.¹

The general form of the estimated single cost function used in the previous studies is:

\[
\text{average cost} = f(\text{scale or size of producer, factor prices, demand setting}),
\]

where demand setting refers to urban factors such as population density, auto availability and income. Thus, the principal differences between studies are the specification of the cost function, the set of independent variables used, and the specific measure used to represent the scale of a producer.² For example, Koshal (1970, 1972), Foster (1973) and Wabe and Coles (1975) use a linear cost model. Fravel (1978) and Lee and Steedman (1970) use an exponential model transformed into a log-linear model for regression estimation. Nelson (1972) uses a system of three nonlinear equations (of transit demand and supply), which, for estimation purposes, are also transformed into log-linear simultaneous equations. The principal defect in these models (with the exception of Nelson) is that they lack a solid foundation of economic and transport analysis. Therefore, the conclusions that can be drawn from their results are limited in scope and value.

An important common feature of these studies is their almost exclusive use of cross-sectional data, in which each observation represents a particular bus operator. This practice appears to be the source of a number of computational problems. First, if all bus operators are treated as equal in the sample (i.e., receive the same weight, as in the Wabe and Coles study, for example), the analysis will produce average cost parameters which do not reflect the average cost per unit of output. The use of a deflating factor for eliminating the impact of size involves making some assumptions on the demand environment which, if incorrect, will generate further difficulties in estimation (Griliches, 1972).³ Second, there are likely to be wide differences in the composition and quality of services offered by different operators. In general, operating conditions across bus firms differ in form of ownership, fare structure, type of regulation imposed, and distribution of demand over time and space. Using in one sample observations on services produced in diverse environments may amount to combining apples with pears. Finally, factor prices may vary little among transit firms, and the resulting lack of variation may make accurate estimation difficult.

¹ Rail freight and passenger transport, and trucking freight, have received much more rigorous analytical treatment (see, for example, Harris, 1977; Spady and Friedlaender, 1978; Caves et al, 1980). But the market and production conditions of rail and trucking services are very different from those of bus transport.

² Griliches (1972) and Harris (1977) differentiate between scale of an operator and level of output, as in many instances output level is used to characterise size. I return to this point below.

³ Combining observations of very small and very large operators in a cross section analysis leads to statistical problems if error is related to size. The larger the observation the larger is the error associated with it (Johnston, 1972, p. 217).
An important finding common to all the studies is the almost total lack of economies of scale (Oram, 1979). They have all concluded that economies of scale in bus transit are insignificant and that service provision is characterised by constant returns over a broad range of sizes. For large systems, the industry is sometimes characterised by decreasing returns to scale. In contrast, the results of the empirical analysis carried out in this paper indicate the existence of economies of scale in the Israeli bus sector. This contradiction in findings can be attributed to differences in the data, to the particular organisational structure of the industry, and to the methodology used. It may also be attributed to the fact that the earlier studies examined primarily the cost differences associated with increase in the size of bus operators rather than with increases in their level of output, which is related to demand.

As has been stated, the main purpose of this paper is to analyse the cost structure of passenger bus operation, and in particular cost elasticities, the demand for factors of production, factor substitution and economies of scale. The model used for estimating these elements is a derivative of the generalised translog multiproduct cost function, and it is described in the following section. The data base consists of time series observations describing the inputs, outputs and other characteristics of the passenger bus sector in Israel. Section 3 provides a detailed description of the data base, with emphasis on the specific characteristics of the Israeli bus sector. The principal results of the empirical analysis are presented and discussed in section 4 of the paper. The final section will summarise the main findings and conclusions. A serious shortcoming of the empirical analysis is that, for lack of data, only two factors of production, labour and capital, can be considered. Needless to say, to analyse the bus sector operations fully it would be necessary to disaggregate these inputs further. The methodology used here would still be appropriate for analysis of the additional data.

2. METHODOLOGY

This section presents the analytical framework used for the empirical estimation of the cost structure of bus services. It is largely based on econometric developments in the area of production theory and duality.

Let

$$\Phi(Q, X) = 0$$  \hspace{1cm} (1)

represent an efficient transformation of a vector of inputs $X$ into a vector of outputs $Q$, where $\Phi$ is an implicit function. The duality theory states that, if $\Phi$ is strictly convex with regard to $X$, there exists a unique cost function which is dual to $\Phi$. It can be written as

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4 An important exception is the approach advocated by Mohring (1972) and Vickrey (1980), who regard passengers' time as a factor of production. Mohring's empirical study, while showing the existence of scale economies, does not explicitly explore demand for input factors and their substitution.
where $P$ is a vector of factor prices and $C$ is total cost. It is explicitly assumed that equation (2) represents cost-minimisation behaviour by the bus system management. That is, given the level of output and the prices of the factors of production, the transit operators will select that combination of inputs which will minimise their total costs of producing that output.5

For the purpose of analysing scale economies, factor substitution, and demand, an explicit estimable functional form of equation (2) is required. Assuming first homothetic cost function, homogeneous in factor prices, equation (2) can be factored into:6

$$\theta(P, Q) = \Psi(Q)\theta(P)$$

where $\theta(P)$ is homogeneous of degree 1, nondecreasing and concave with respect to $P$. A simple representative case of (3) would be

$$C(w, r, Q) = A_0 Q^a w^{\beta_1} r^{\beta_2}$$

where $w$ and $r$ are, respectively, labour and capital unit prices and $A_0$, $a$, $\beta_1$, $\beta_2$ are parameters. For estimation purposes, (4) can be written

$$\ln C(w, r, Q) = A + a \ln Q + \beta_1 \ln w + \beta_2 \ln r + \varepsilon$$

where $A = \ln A_0$ and $\varepsilon$ is the error term. Notice that homogeneity in prices requires that $\beta_1 + \beta_2 = 1$. For $a < 1$, the average cost decreases as $Q$ increases, implying scale economies.

A major problem with (4) is that if $C(w, r, Q)$ is a homothetic cost function, which meets the conditions of being nondecreasing and concave (with respect to prices), then, from the duality theorem, its ex ante production function is also homothetic. (See Varian, 1978, ch. 1, for mathematical exposition.) A homothetic production function implies that scale economies can be defined independently of factor proportions; that is a property which may not exist in the provision of transit service.7 Another theoretical problem is that of (non)linear factor separability in the cost function. A priori, there is no reason for cost to change in direct proportion to a given change in factor prices, as implied by equation (5). A change in factor price may affect the demand for other factors, which, in turn, will affect the total cost.

In the empirical section of the paper, model (5) will be estimated mainly for comparison purposes. However, the implications of these problems are that a cost function like (5) places a number of a priori restrictions on some important economic elements. For example, it assumes constant factor elasticity of substitution and

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5 It is also assumed that the transport companies face competitive factor markets. See below for a discussion of these assumptions in the context of the Israeli bus industry.

6 Shephard (1970) has shown that a homothetic cost function (2) can be written as $C = g(P)h(Q)$, where $h(Q) > 0$ for all $Q > 0$.

7 On the other hand it is plausible to assume independence between level of output and all unexplained factors represented by $\varepsilon$, because output is determined exogenously, by demand conditions.
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constant price elasticity of factor demand. For these reasons we wish to specify a cost function which does not assume such a priori restrictions on the underlying production structure. There are several such functions in the literature (for a review, see Fuss et al., 1978). In this study I use the translog cost function, which is quite well used and has known analytical properties (Christensen et al., 1973).

The general form of this function is:

$$\ln C(P,Q) = A_0 + \sum_{i} a_i \ln Q_i + \sum_{i} b_i \ln P_i + \frac{1}{2} \sum_{i} \sum_{j} \delta_{ij} \ln Q_i \ln Q_j + \frac{1}{2} \sum_{i} \sum_{j} \gamma_{ij} \ln P_i \ln P_j + \sum_{i} \sum_{j} \rho_{ij} \ln Q_i \ln P_j$$

with the symmetry conditions: \( \delta_{ij} = \delta_{ji} \); \( \gamma_{ij} = \gamma_{ji} \). Sufficient conditions for \( C \) to be homogeneous of degree one in \( P \) are

$$\sum_{i} \beta_i = 1, \sum_{j} \gamma_{ij} = 0 \quad (i = 1, \ldots, n), \sum_{j} \rho_{ij} = 0 \quad (i = 1, \ldots, m).$$

Notice that if \( \rho_{ij} = 0 \) for all \( i \) and \( j \), the function is a homothetic–homogeneous cost function.

Specifically, the following model is estimated.\(^8\)

$$\ln C = A + \alpha \ln Q + \beta_1 \ln w + \beta_2 \ln r + \frac{1}{2} \delta_2 (\ln Q)^2 + \gamma_1 \ln w \ln r + \frac{1}{2} \gamma_2 (\ln w)^2 + \frac{1}{2} \gamma_3 (\ln r)^2 + \rho_1 \ln Q \ln w + \rho_2 \ln Q \ln r$$

(6)

with the linear homogeneity conditions, \( \beta_1 + \beta_2 = 1; \rho_1 + \rho_2 = 0; \gamma_1 + \gamma_2 + \gamma_3 = 0 \), and the symmetry conditions (see above).

Differentiating (6) with respect to factor prices and rearranging, we get

$$\frac{w \partial C}{C \partial w} = \beta_1 + \gamma_1 \ln r + \gamma_2 \ln w + \rho_1 \ln Q$$

(7)

Similarly,

$$\frac{r \partial C}{C \partial r} = \beta_2 + \gamma_1 \ln w + \gamma_3 \ln r + \rho_2 \ln Q$$

(8)

From Shephard's Lemma it is known that the first partial derivatives of the cost function (with respect to factor prices) are equal to the cost minimising factor quantities necessary to produce \( Q \) units of output; that is,

$$\frac{\partial C(w,r,Q)}{\partial w} = L(w,r,Q) \quad \frac{\partial C(w,r,Q)}{\partial r} = K(w,r,Q)$$

\(^8\) This translog model is assumed to be an exact representation of the minimum cost function. As an alternative, it is possible to use it as a second order approximation at a point to an arbitrary twice-differentiable cost function; the main disadvantage is that test results hold only at point of approximation and not globally. For a discussion of these alternative approaches see Spady and Friedlaender, 1978.
where \( L \) and \( K \) are quantities of labour and capital used, respectively. Thus, from (7) and (8),

\[
S_L = \frac{wL}{C} = \beta_1 + \gamma_1 \ln r + \gamma_2 \ln w + \rho_1 \ln Q \tag{9}
\]

and

\[
S_K = \frac{rK}{C} = \beta_2 + \gamma_1 \ln w + \gamma_3 \ln r + \rho_2 \ln Q \tag{10}
\]

where \( S_L \) and \( S_K \) are the shares of total labour costs and total capital costs in the total cost. Equations (6), (9) and (10) form the system of equations to be estimated below.\(^9\) They provide information regarding scale economies, factor demand, substitution and price elasticities. Before we turn to the data base, estimation procedure and results, two more points should be made about the model.

The data base used in the present analysis contains time series data on costs, factor prices and output of the Israeli bus transit operation for the period 1972–79. Therefore, we must consider the possibility of technological changes over time. To test the hypothesis that no technological changes occurred during the period, two more parameters, \( \nu_1 \) and \( \nu_2 \), have been added to the cost function (6). Thus the cost function to be estimated is augmented by

\[
\ln C = [\text{eq. (6)}] + \nu_1 \ln T + \nu_2 \ln T^2 \tag{11}
\]

where \( T \) denotes time and the constraints \( \nu_1 = \nu_2 = 0 \) are imposed for testing the null hypothesis of no technological change. On the base of the data on Israeli bus transit, it was found that the parameters \( \nu_1 \) and \( \nu_2 \) are statistically not significantly different from zero; this implies that we cannot reject the null hypothesis of no technological change during the sample period.

The second point is that estimating the parameters of the cost function (6) can provide an estimate for the marginal cost of the bus sector, given its average cost. In general, \( \partial \ln C / \partial \ln Q = MC/AC \). Thus, from (6),

\[
MC = \frac{C}{Q} (\alpha + \rho_1 \ln w + \rho_2 \ln r + \delta \ln Q). \tag{12}
\]

The term \( \partial \ln C / \partial \ln Q \), which is the elasticity of total cost with respect to output, is also used below to provide an estimate of the degree of scale economies in the production of the services.

An important aspect of the production process which underlies the cost function model is the elasticity of substitution between the input factors. This element, denoted by \( \sigma \), measures the percentage change in the ratio of two factors (e.g., capital and labour) caused by a one per cent change in the relative prices of these factors, i.e.,

\[^9\] The cost share equations introduce no additional parameters into the cost system. By estimating them together with the cost function (6), the number of degrees of freedom is increased, without any increase in the number of parameters to be estimated.
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\[
\sigma = \frac{\partial (K/L)/(K/L)}{\partial (P_L/P_K)/(P_L/P_K)}
\]

where \( K, L, P_K, P_L \) are quantities and prices of capital and labour respectively. The importance of this measure lies in the fact that if \( \sigma > 0 \) the two factors are substitutes while \( \sigma < 0 \) indicates complementary inputs. \( \sigma = 0 \) indicates that the proportions of inputs in the production process are fixed. Given these results, our objective is to estimate empirically \( \sigma \) and, as a consequence, the technological relationships between the input factors in the production of bus transit services.

In general, there are various possible empirical definitions for the elasticity of substitution, \( \sigma_{ij} \), between any two factors \( i \) and \( j \). The most commonly used is the Allen partial elasticity of substitution (Allen, 1938), which is

\[
\sigma_{ij} = \frac{C^*(x_i/p_j)}{x_i x_j} \quad i,j = 1, \ldots, n
\]

where \( C^* \) is the (estimated) cost function, \( x_i, x_j \) are quantities of factors \( i \) and \( j \), and \( p_j \) is the price of factor \( j \). Uzawa (1962) had shown that the \( \sigma_{ij} \) are defined as

\[
\sigma_{ij} = C^* \left( \frac{\partial^2 C^*}{\partial P_i \partial P_j} \right) \left( \frac{\partial C^*}{\partial P_i} \cdot \frac{\partial C^*}{\partial P_j} \right)^{-1}
\]

where \( C^* \) is the cost function, \( P_i, P_j \) are factor prices, and \( \sigma_{ij} = \sigma_{ji} \) (\( i,j = 1,2,\ldots,n \)).

For the translog cost function, \( \sigma_{ii} = (\gamma_{ii} + S_i^2 - S_i)/S_i^2 \) and \( \sigma_{ij} = (\gamma_{ij} + S_i S_j)/(S_i S_j) \), where \( \gamma_{ij} \) are parameters and \( S_i, S_j \) are factor shares.

In the present two-factor model, the Allen partial elasticities of substitution equal the following:

\[
\sigma_{LL} = \frac{\gamma_1 + S_L^2 - S_L}{S_L^2} \quad (13)
\]

\[
\sigma_{KK} = \frac{\gamma_3 + S_K^2 - S_K}{S_K^2}
\]

\[
\sigma_{LK} = \sigma_{KL} = \frac{\gamma_1 + S_K S_L}{S_L S_K}
\]

where \( S_L \) and \( S_K \) are the factor shares, defined above.

From the above, the direct and cross price elasticities of demand for factors of productions, \( \varepsilon_{ij} \), are\(^{10}\)

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\(^{10}\) The price elasticity parameters, \( \varepsilon_{ij} \), are defined as \( \varepsilon_{ij} = \partial \ln x_i/\partial \ln P_j \) (\( i,j = 1,\ldots,n \)), where \( x_i \) is quantity of factor \( i \), and where quantity and prices of all others factors are held constant. Allen (1938) had shown that \( \varepsilon_{ii} = \sigma_{ii} \cdot S_i \) Note that \( \sigma_{ij} = \sigma_{ji} \), but \( \varepsilon_{ij} \neq \varepsilon_{ji} \).
\[ e_{ww} = \frac{\gamma_2 + S_L^2 - S_L}{S_L} \]
\[ e_{rr} = \frac{\gamma_2 + S_K^2 - S_K}{S_K} \]
\[ e_{wr} = \frac{\gamma_1 + S_L S_K}{S_K} \]
\[ e_{rw} = \frac{\gamma_1 + S_L S_K}{S_L} \]

These important components of the cost model will be estimated in section 4.

3. THE DATA BASE

The data used for the analysis in this study describe the Israeli bus transport sector. The Israeli sector is unique among the countries of the world in a number of ways, of which the most important are mentioned below. A detailed review of the sector can be found in Berechman (1980).

In patronage, number of daily trips performed, and geographical coverage, buses are the principal public mode of transport in Israel. Other modes such as trains, taxis and vans provide very limited services. About 85% of the total daily bus trips for all purposes are offered by two bus companies: Dan, which operates only in the Tel-Aviv metropolitan area, and Egged, which provides intraurban and interurban services elsewhere. The remaining 15% are provided by a number of relatively very small bus firms, most of which operate locally and are privately owned.

Institutionally the two principal bus companies are cooperative societies, in which each member works for the company and owns one voting share. The value of the shares changes over time to reflect appreciation in the value of the cooperative assets and changes in the number of members. Non-member employees are also used by the bus companies to meet their short and long-run needs for labour. In 1979 non-members formed about 40% of the total labour force of the two companies: their terms of employment mainly reflect market conditions. These terms are formally determined through collective bargaining and individual contracts.

This organisational form suggests that, like private enterprise, the bus firms in Israel wish to minimise their cost, and indeed that is supported by the evidence (Berechman, 1980). Accordingly in this analysis we consider the firms as selecting their factor inputs so as to minimise long-run total cost, given factor prices and the level of output.

Because the country is small and the population is densely concentrated along a narrow and short stretch of the coastal line, most of the daily trips performed (about 80%) are of intraurban type, including inner-city, metropolitan and suburban trips. Moreover many of the interurban trips are of short duration, often serving also population at the periphery of urban centres and thus having demand characteristics
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(e.g., peak off/peak ratio) similar to those of intratran vehicle trips. The principal implication of these facts for the present analysis is that demand and supply characteristics of interurban and intratran vehicle trips in Israel can hardly be distinguished in any systematic and meaningful way. On practical grounds the accounting reports of the Egged bus company, which operates both interurban and intratran services, do not differentiate between these types of trips, particularly with regard to the use of input factors and their associated items of cost. For these reasons the data used in these analyses are not disaggregated by trip type.¹¹

Government control of public transport includes the issuing of permits for operation on specific lines; setting the minimum level of services on regular lines; setting the bus fares; and providing lump sum subsidies to the bus operators (see Berechman, 1980, for an analysis of the subsidy policy).

The main results of this control are that the fare structure of bus trips in Israel is quite uniform and that intercity, metropolitan and many intratran fares are almost completely independent of trip length. In view of the scope of this paper, no attempt is made here to examine the motives and consequences of this policy, except to notice below its implications for the selected measure of output.

The data base used here is composed of quarterly observations for the years 1972–79 for the bus industry as a whole. A major advantage of using quarterly data is that the period of three months seems to be sufficient to enable the operators to adjust their supply of services meet fluctuations in demand and, as a consequence, minimise costs. The principal sources of data are the Quarterly Transport Statistics (QTS), 1972–79, the Statistical Abstract of Israel (SAI), 1979, and the Quarterly Prices Statistics (QPS), 1972–79. Other complementary sources are publications and reports published by the bus companies and the Ministry of Transportation.

One difficulty in cost studies is the selection of output variables which reflect the scale of operation of the sector (or of the individual bus operator). Yet, for the purpose of analysing long-run decisions on production, it seems appropriate to use a measure showing the amount of actual services produced, rather than the size of the producing units. For this reason the variable “revenue per passenger-kilometre” was proposed for this study. Data on bus passenger-kilometres, however, were, in general, unavailable. Consequently, gross revenue in fixed prices (1969 = 100) was used as the output variable.¹² Data on revenue in current prices, and the revenue-price index, were obtained from the QTS publications.

Data for the other real variables, labour (L) and capital (K), were also obtained from the QTS. Labour was measured by actual man-days worked, and capital by the number of buses in operation. The reasons for selecting the latter variable were that bus purchases constitute the principal capital outlay for the bus companies and that changes in the supply of services are, in the long run, effected through changes in the size of the bus fleet.

¹¹ Notice that none of the studies mentioned above estimated costs on the basis of trip type—probably for reasons similar to those given here.

¹² As stated above, bus fares in Israel are very uniform and, by and large, even interurban fares are not distance-related. Gross revenue actually measures the number of passenger trips times the unit fares, and the measure “price deflated gross revenue” approximates the actual number of passenger trips performed during the period studied.
The cost of labour ($w$) and the cost of capital ($r$) were measured as follows. Total labour cost (including wages, taxes and social benefits) per actual man-day worked for each quarter (1972–79) were obtained from QTS. These figures were then deflated by the general (not the transport industry) labour price index, which is reported in the QPS (using 1969 = 100). The results—labour costs in fixed prices—were converted for the analysis into an index.

Quarterly data on total expenditure on buses (including maintenance but excluding capital expenditure) are reported in the QTS. Dividing by $K$ and deflating these costs by the industrial inputs price index (1969 = 100) produced the cost of capital in fixed prices. This was then converted into an index.

Data on total cost ($C$) were obtained by using the accounting relationships $C = wL + rK$. Having the quarterly figures for $w$, $L$, $r$ and $K$, we produced the quarterly nominal figures for $C$. These were then deflated by the above vector of industrial input prices (1969 = 100) obtained from QPS. The result, in index form, was used as the $C$ variable for the analysis.\(^\text{13}\)

Table 1 contains these data, including data on factor shares. Of particular interest is the effect of inflation, which has multiplied the nominal prices of labour and capital during the sampled period by 19.4 and 20.4, respectively. In fixed prices, these increases were a much less dramatic 0.33 and 0.10. Some of the fluctuations in the real cost of labour, especially in the fourth quarters, can be attributable to some institutional peculiarities of the Israeli economy (e.g., a sharp rise, in the fourth quarter of each year, in the indices used for deflating current prices of labour and capital).

In section 2 a question was raised on the validity of the hypothesis of exogenous factor prices, that is, whether the prices of labour and capital are determined within the transit sector or external to it. A comparison of labour prices as reported in Table 1 with labour prices elsewhere in the transport sector (as defined by the Bureau of Statistics) shows that there are almost no significant differences. Similar comparisons of capital costs are more difficult, but one has to remember that buses and parts and materials are imported into the country, so there is little the bus companies can do to affect their prices. In general, therefore, cost shares depicted in Table 1 provide satisfactory estimates of cost elasticities with respect to factor prices. (See also Berechman, 1980.)

4. ESTIMATION AND RESULTS

Two different cost functions were estimated below. These are equation (5) and equations (6), (9) and (10). Hereafter these will be called model 1 and model 2, respectively.

\(^{13}\)To validate these figures, I have compared them with the estimated annual cost figures for the entire sector, computed as follows. The annual total cost derived from accounting reports by the largest bus company, Egged (59–63% of the market, in gross revenue and number of buses, 1972–79), was discounted by Egged's annual share of the market to produce an estimate of total annual cost of the industry. For all the years but one (1973) the two sets of figures compared quite reasonably (5–10% deviations). For 1973 the computed quarterly data were adjusted so that their sum would equal the sector's estimated total cost.
### Table 1
Factor Quantity and Price and Cost Data of the Israeli Bus Industry, 1972–79

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>K</th>
<th>L</th>
<th>Q</th>
<th>w</th>
<th>r</th>
<th>C</th>
<th>( S_L )</th>
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<td>1</td>
<td>4,139</td>
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<td>68.4</td>
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\( K \) = number of buses  
\( L \) = actual man-days worked (in thousands)  
\( Q \) = revenue in fixed prices (IL million)  
\( w \) = cost of labour, in fixed prices (IL x 100)  
\( r \) = cost of capital, in fixed prices (IL)  
\( C \) = total cost in fixed prices (IL million)  
\( S_L \) = share of labour in total cost  
*See text for explanation on the computation of variables.*
The estimation of model 1 required the use of ordinary least squares analysis, but the estimation of model 2 required a more complicated statistical procedure. The relatively large number of parameters to be estimated (equation 6) calls for the simultaneous estimation of the cost function and the share equations to increase the degrees of freedom without adding more parameters. However, since the cost-share equations sum to unity, their associated error terms are not mutually independent. Therefore, one equation is deleted for the joint estimation to avoid a singular variance-covariance structure.

A sound method of estimation under these circumstances is a modification of Zellner's (1962) procedure, which is a two-stage nonlinear iterative estimation process. In addition to providing efficient estimates of the parameters, it is invariant to which share equation is deleted. The estimates obtained at convergence are unbiased maximum likelihood estimates.

Table 2 presents the results of the joint estimates of the translog cost function and the labour share equation (model 2). For purpose of comparison the estimates of model 1 are also presented. The adjusted $R^2$, the Durbin-Watson statistic, the log likelihood function, and the $t$-values associated with the parameters, are all reported in Table 2.

A well-behaved cost function should meet two principal regularity conditions. Input price-coefficients should be non-negative to ensure concavity in prices (a sufficient condition for concavity), and each factor demand function should be strictly positive. Not all the price estimates, indicated in Table 2 (i.e., $\gamma_j$), are non-negative, and one has to verify that the Hessian matrix is negative semi-definite for concavity of the cost function (at least within a reasonable neighbourhood of observed prices). Using the above coefficient estimates, the Hessian matrix is negative semi-definite (i.e., $\partial^2 C / \partial p_i \partial p_j \leq 0$) for each sample observation, thus satisfying the concavity condition.

To test for positivity, the cost share equations (9) and (10) were fitted with the quarterly price data, using the above estimates. They were found to be positive for each quarter. (Note that $\rho_1$ and $\rho_2$ are statistically not different from zero.)

The questions of homotheticity of the production function in output and the separability of inputs were mentioned above as important motives for using the translog cost model. To test for these properties of the production function (which is dual to equation (6)), a likelihood ratio test is used. In this test the translog cost function is restricted so that $\rho_1 = \rho_2 = 0$, and ratio of the maximum value of the restricted likelihood function to that of the unrestricted function is computed. The

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14 The first stage of this process provides estimates of the variance-covariance matrix without the symmetry constraints. In the second stage the V–C matrix is held constant and the parameters are estimated with the symmetry constraints imposed. These estimates are iterated until the estimated V–C matrix is diagonal and the parameters estimates converge. For a detailed description see Christensen and Greene, 1976.

15 Notice that Model 1 essentially presents the cost function estimates corresponding to Cobb-Douglas production function technology. By assuming for equation (6) that $\beta_1 + \beta_2 = 1$, $a \neq 0$, and all other substitution parameters are zero, we obtain Model 1.

16 Under the null hypothesis ($-2 \log R$) is asymptotically distributed $\chi^2(n)$, where $R$ is the above ratio and $n$ is the number of restrictions.
test statistic value was 5.98, which, at 0.05 level, is on the border of the critical value. Thus, we cannot reject the null hypothesis that production is homogeneous in output.

To test for separability, the restrictions $\gamma_1 = \gamma_2 = \gamma_3 = 0$ are imposed. The test statistic was 16.32, which is greater than the $\chi^2$ critical value at level 0.05, thus implying that the hypothesis of linear separability of labour and capital cannot be accepted. If the above separability and homotheticity restrictions are jointly imposed on (6) (thus deriving model 1), the test statistic is 21.73, which is greater than the critical value at 0.05 level. Consequently, the hypothesis that the dual to model 2 production function is of the Cobb-Douglas type cannot be accepted.

I turn now to the measurement of factor substitution in the production of bus transport services. Using equations (13) and (14), the Allen partial elasticities of
substitution ($\sigma_{ij}$) and the factor demand price elasticities ($e_{ij}$) are computed for each year of the sampled period. Table 3 provides these estimates.

Several conclusions can be stated. The negative and small elasticity of substitution parameters between labour and capital suggest that these inputs are weakly complementary in the production of the services. This conclusion should be qualified, however, since a two-factor model such as this precludes complementarity of factors. An alternative explanation is that of a fixed factor proportions technology.\textsuperscript{17} This result is no surprise, given the technology of the bus sector, where each bus is operated by one driver. The cross-price elasticities of factors are also small; this indicates that increases in the price of labour will tend in the long run to reduce the demand for capital.

As could be expected, the demand for labour and capital in the bus industry is responsive to changes in their own prices, as indicated by their negative own-price elasticities. It should be noticed, however, that the variable used in the analysis as labour input was actual man-days worked, and not number of employees. Thus it is impossible to infer directly from the estimated values of $e_{wL}$ the actual impact of changes in labour prices on the size of the labour force. On the other hand, the derived values of $e_{rL}$, in part reflect the fact that the reported data on expenditure on buses, which were used for the computation of capital cost, include also elements of maintenance costs.

The cost elasticity of output $\partial \ln C/\partial \ln Q$ can be derived from equation (6),

\[
\frac{\partial \ln C}{\partial \ln Q} = \alpha + \delta \ln Q + \rho_1 \ln w + \rho_2 \ln r
\]  

\textsuperscript{17} Another possible econometric explanation is a missing variable. That is, if another factor of production (e.g. maintenance) were explicitly included in model 2, labour and capital could indeed be regarded as complementary while being alternatives to that third factor.
As the coefficients $\rho_1$ and $\rho_2$ were statistically not different from zero, the following simplified equation is computed:

$$\frac{\partial \ln C}{\partial \ln Q} = \alpha + \delta \ln Q$$  \hspace{1cm} (16)

Similarly, the marginal cost equation (12) can be written for computational purposes as:

$$MC = \frac{C}{Q} \left( \alpha + \delta \ln Q \right).$$  \hspace{1cm} (17)

The annual estimates of the cost elasticities and marginal cost appear in Table 4.

Several important points should be observed about the results of Table 4. The cost elasticities for every year are less than unity; this indicates scale economies in the production of bus transport services. Following the conventional approach (e.g., Caves et al., 1980), the degree of scale economies is measured as unity minus the cost elasticity. These results are given in column 2 of Table 4, and indicate that, in contrast with findings of previous studies, significant economies of scale were found in the Israeli bus sector. It is important to emphasise here that what was measured was economies of output related to passenger trips, and not economies of size of bus operators (e.g. size of rolling stock or vehicle-miles).
Since factor prices were statistically insignificant in the MC function (12), it seems that output alone determines the level of marginal cost (given AC, α and δ). It is evident from Table 4 that MC values are below the average cost values for all Q; this, of course, is expected under conditions of scale economies. The difference between AC and MC for a given level of output reflects the cost per unit output which would not be covered under a policy of marginal cost pricing. The range of these differences is between 0.47 and 0.75 in current prices and between 0.27 and 0.04 in fixed prices.

5. CONCLUSIONS

Various studies have been conducted in recent years on the cost function of bus transport. A review of these studies shows that most of them use simplistic analytical constructs which, in addition to being theoretically deficient, do not allow for analysis of the relationships between production cost on the one hand and output and input factor prices on the other. In particular, the demand for factors of production, factor substitution and price elasticities is not investigated. Also, it is impossible to deduce from these studies the analytical properties of the underlying technology production.

By using a general translog cost function under conventional restrictions from neoclassical theory, this study has investigated these issues. The data base represents the Israeli bus sector, which, in contrast to that of many other countries, is composed of privately owned bus firms. A number of conclusions from the analysis can be stated. The first conclusion is that the production of bus services probably cannot be accurately described by a Cobb-Douglas type technology. While the statistical analysis did not reject the hypothesis of homothetic production in output, it showed that the factors of production are not linearly separate.

Another conclusion from the analysis is that the technology of production of bus services is that of fixed proportions of factors (labour to capital). The own-price elasticities, which have the correct sign, are much larger for capital than for labour; labour is measured in units of actual man-days worked and not number of employees.

The analysis also revealed that economies of scale in the provision of bus service in Israel do prevail. This finding stands in direct contrast to the findings of many of the studies reviewed above, and it can be partly attributed to the use as output measure of gross revenue in fixed prices rather than variables such as bus-miles or bus-hours. The use of time series data rather than cross-section data may be another reason for this finding, since time series data do not require standardisation of observations by size of operator and demand environment. Another plausible explanation may be found in the specific structure of the Israeli bus sector, in particular its high degree of concentration and private ownership, coupled with the densely concentrated demand for travel. This suggests that the finding of economies of scale in the production of bus services cannot be easily generalised.

An obvious limitation of the analysis is the use of only two factors of production. The inclusion of fuel and repairs and maintenance as specific factors may provide more information on the production process, particularly on factors substitution and demand. Bus companies produce a variety of services such as city centre, metropolitan, suburban and express trips, which may differ in production charac-
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Joseph Berechman

teristics and cost. It is suggested, therefore, that a separate analysis be made for each type of service, if data are available.

Finally, it is desirable to compare the production of services by profit maximising (or cost minimising) bus transport systems with services produced by completely regulated and publicly owned companies. The principal question to be explored is whether type of ownership does affect the production technology and cost structure of bus services.

REFERENCES


Button, K. J. (1977): The Economics of Urban Transit. Saxon House, Ch. 5.


