CHOICE OF TRAVEL MODE AND THE
VALUE OF TIME IN GREECE

Some Robust Estimates

By S. K. Lioukas*

INTRODUCTION

In developed countries effort has been expended in determining appropriate values of
time for use in transport planning and in cost-benefit studies of transport investments.
In developing countries, however, there has been a dearth of research on the subject,
and in practice crude methods have often been employed (Howe, 1976). Estimation of
appropriate values of time has been greatly constrained because relevant information
has been scarce and costly to obtain. Therefore the common practice has been (1) to
transfer "western" values directly, often uncritically, and (2) to rely heavily upon
untested proportions of the market wage rates (the wage rate method).

These practices however have serious drawbacks. First, it is questionable whether
estimates can be transferred from developed to developing countries. Existing
research suggests that even within the same country model parameters (and thus
values of time) are far from transferable (Galbraith and Hensher, 1982). Values are
ultimately linked with the preferences and habits of a particular society. They also
depend on the physical characteristics of the geographical area and the transport
system concerned. Watson (1974, p. 153) expresses this common view: "Different
situations require different modelling efforts, and attempts to transfer results from one
area to another are fraught with danger." But whether results can be transferred
without serious error, and whether indeed values of time are universally valid, are
ultimately empirical questions; they cannot be answered without appropriate
estimates from developing countries around the world.

Secondly, reliance on market wage rates brings in practical and theoretical
complications. Under this method a value of time is derived from its link with the
production process. It is considered that travel is not an end purpose in itself, but

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results from the pursuit of other, possibly wealth-creating, activities. Therefore there is an opportunity cost associated with travel time, and reducing travel time can increase the time available for other economic activities. Market wage rates are typically used in developed countries for measuring the production which could be gained by reducing travel time. It is widely recognised, however, that in developing economies many market prices—including wages—are seriously distorted, and therefore wages provide an imperfect basis for measuring the production potential of man-hours freed by reducing travel time.

An elaborate calculation of "accounting prices" (as in the Little-Mirrlees method) would be required to find wages free of distortion, and to convert the marginal product of labour into accounting prices. Within this framework the marginal product of labour should, ideally, be estimated on a sector by sector basis, and should be valued as far as practicable at border prices. This exercise however is complex and would require extensive data on employment and budget surveys. As Little and Mirrlees (1974) point out, one cannot usually estimate all relevant accounting prices in the absence of an overall all-embracing framework. In this framework valuation of time would present extra problems, for it requires the determination of the uses to which savings in travel time would be put and an estimation of the economic value of the production potential of these savings.

Even when these difficulties are overcome, it is not at all obvious that the value attributed by people to time savings would correspond to the estimated production potential. Freed man-hours can be spent in social and cultural activities, which are highly valued by the society although they add very little to the measurable national product. Savings in travel time have a value even if they do not result in economically productive use of that time. Time and travel behaviour are intrinsically linked not only with economic but also with non-economic aspects of a society's activities and tastes. Quantifying these links is extremely difficult.

A more promising approach would be to estimate a behavioural value of time: that is, a value revealed by what the population of travellers does, or has done, in choosing between time savings and money savings. Revealed behaviour is the ultimate evidence of societal preferences and values. This approach avoids the direct determination of the many uses to which time savings are put and their direct economic valuation. Instead it observes actual choices in the market place and estimates the average time-cost trade-off implicit in these choices.

The present study is an attempt to estimate the value of time revealed by the population of travellers in choosing between alternative modes with different prices and journey times. The information available for the study shows passenger movements by each mode in various inter-urban routes. A multinomial logit model is used to simulate the allocation of traffic across modes. Parameters which best explain the observed relative market shares of transport modes are estimated. Special attention is devoted to price and time variables. The coefficients of these variables will indicate the responsiveness of travellers to differences in time and cost, and thus an implied value for time savings.

The aim in this study is to find broad average values of time which are comprehensive enough to be used for national transport planning. This contrasts with other approaches which seek to disaggregate values of time in sub-components, or for specific localities and/or sub-sections of the population. Disaggregation, though ideally
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desirable. presents problems when it is to be projected and generalised to the whole population.\textsuperscript{1} The purpose here is to estimate single national average values of time for two broad categories of travel, business and non-business. Some tests are also performed to investigate the sensitivity of these values to local income differences (measured by a car-ownership index) and distance. Results in this area are particularly important because (1) few studies of inter-urban mode choice are available (see Hensher and Stopher, 1979; Stopher and Meyburg, 1975); (2) previous efforts to separate the effects of journey time and price, and to estimate the implied value of time, have not always led to satisfactory results (see, e.g., Hensher and Hetchkiss, 1974; Watson, 1974).

The data used in the analysis are numbers of passenger trips between zone pairs and their modal split (or modal use according to the terminology suggested by Beesley, 1979). The Greek mainland was divided into appropriate zones on the basis of census information. Zones were generally defined around towns; as a result, inter-zonal trips are not very different from inter-urban trips.\textsuperscript{2} Data on inter-zonal trips, price, time, and other variables were collected from various sources and surveys. A description is given in Appendix I.

Similar aggregate zonal data have traditionally been used in regional and transport studies (e.g., Quandt and Baumol, 1966). Such data have become less fashionable recently because of the “aggregation bias” involved in using group averages as dependent and independent variables. The alternative would be to use purely “disaggregate data”, that is, data in which the basic unit of observation is the individual traveller. Disaggregate data, however, though better suited for the study of individual behaviour, are not without difficulties. First, they would be costly to collect in a country-wide inter-urban context. Extensive sample designs would be required to cover travellers on various routes (see, e.g., Westin and Manski, 1979; McFadden, 1979, 1980). Limitations of resources and data make this impracticable, especially for a developing country like Greece where no previous information of this kind exists. Second, parameters estimated from disaggregate data would have to rely on a sample of individual travellers. Generalising and projecting to the population as a whole would introduce an error. Aggregate zonal data tend to be more comprehensive, thus avoiding sampling errors. When the zones are small, as in the present study, the “aggregation bias” of zonal data is reduced and the behavioural validity of the estimated coefficients is enhanced (see Anas, 1981). It may be that, for practical purposes, the end result obtained from small aggregate units (zones) may not be inferior to that obtained from disaggregate data.\textsuperscript{3}

\textsuperscript{1} Recently, there has been some scepticism on the desirability of disaggregating values of time. For example, Beesley (1979) quotes a remark by the U.K. Department of the Environment (Economic Directorate Note): “The use of local values of time implies a much greater knowledge about the valuation of time than we could claim” (p. 461). Uncertainty and the difficulties of generalising cast doubts on the reliability and practical use of disaggregate values. As Beesley suggested in a private conversation with the author, for practical purposes it may be advisable to go back to general values of time.

\textsuperscript{2} In the present study the terms “inter-urban” and “inter-zonal” are used interchangeably. Strictly speaking, the study is concerned with inter-zonal trips.

MODEL SPECIFICATION AND ESTIMATION

The multinomial logit

In the general case where $M$ transport modes are available, the probability of choosing a particular mode, say mode $i$, depends on the relative price, time and other comparative advantages of this mode. The probability $P_i$ that an individual traveller will choose mode $i$ can be approximated with the observed relative frequency $f_i$ of this mode—or, equivalently, the market share of mode $i$. The $M$-dimensional logit model can be written as follows (see Theil 1969; Rassam et al., 1971):

$$f_i = \frac{\exp(a_i + b_i p_i + c_i t_i)}{\sum_k \exp(a_k + b_k p_k + c_k t_k)}$$  

(1)

where:

- $f_i \approx P_i$, the conditional probability/frequency of choosing mode $i$, $i = 1, \ldots, M$
- $p_i$ = the price of mode $i$ (fare plus other costs)
- $t_i$ = the overall journey time with mode $i$
- $a, b, c$ = mode-specific parameters to be determined
- $k$ = an indicator for the modes ($k = 1, \ldots, M$).

The multinomial logit model (1) has been widely applied in practice. Its main advantage is computational simplicity. At the level of the individual decision maker, the multinomial logit model can be derived from a micro-economic theory of consumer choice, under certain assumptions (see, e.g., Domencich and McFadden, 1975). This theory can also provide a multinomial logit model at the aggregate (zonal) level, when appropriate assumptions are introduced (see, e.g., Anas, 1981). The assumptions required to derive the model, though plausible in some situations, are fairly restrictive, and some investigators have attempted to develop more flexible models in the logit family (see Williams, 1977; Williams and Ortuzar, 1980; McFadden, 1980). However, the restrictive properties of the multinomial logit model may not be too serious from a practical point of view. Williams and Ortuzar (1980), for instance, find that the model is relatively robust to variations in certain assumptions.

Because of the complex fractional structure of model (1), it does not lend itself to easy estimation. A more popular version is the Nerlove–Press (1972) form, which restricts the parameters other than the constant terms to be equal across modes; i.e., $b_i = b$ and $c_i = c$ for all $i$. The advantage of this restriction is that the logarithm of the odds of a mode $i$ relative to another mode $k$ can be expressed as a linear function of price and cost differences, i.e.

$$\ln \frac{f_i}{f_k} = a_{ik} + b \Delta p_{ik} + c \Delta t_{ik}, \quad (i, k = 1, \ldots, M; i \neq k)$$  

(2)

The differences model can be derived from a micro-theory of consumer choice where: (1) a traveller’s evaluation of each mode is assumed to be a linear function of the mode’s price, time and other characteristics, with an additive error to account for individual variations in preferences; (2) the errors are independently and identically distributed, with Weibull distributions. No similar support has been found for other model specifications such as logarithms of price and time, or ratios. In this respect other specifications may be regarded as ad hoc.

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where:
\[ a_{ik} = a_i - a_k, \quad \Delta p_{ik} = p_i - p_k, \quad \Delta t_{ik} = t_i - t_k \]

Formulation (2) has also the advantage of allowing a unique value of time to be inferred from the data. A single estimate is deliberately sought in the present study in order to describe the population’s average money-time trade-off.\(^3\) The ratio \( c/b \) will give the implicit value placed by travellers on time savings \( \Delta t \), because \( c/b \) units of change in price difference produce the same effect on the dependent variable as one unit of change in time difference. Denoting the value of time with VOT, we obtain the following form for the relationship (2) above:

\[ \ln \frac{f_i}{f_k} = a_{ik} + b(\Delta p_{ik} + \text{VOT} \Delta t_{ik}) \]  
\[ (3) \]

The expression in parentheses gives the difference in the “generalised cost” of travel, that is, the cost including both money outlays and time converted to monetary terms via the VOT factor.

The constant term \( a_{ik} \) in (2) or (3) accounts for the inherent comparative advantage of mode \( i \) relative to mode \( k \). This advantage reflects characteristics of each mode such as comfort, image and service. Further variables could be introduced in (2) or (3) to capture these differences. However, little has been done so far in this direction, and the development and testing of models including further variables is a matter for future research.\(^6\) In the inter-urban context which provided the data for our analysis, it was felt that distance is an important variable. Any inherent advantages to the modes are likely to be related to distance. Direct distance, however, is naturally related to price and time variables, and therefore its inclusion in the set of explanatory variables would generate serious intercorrelations among the variables, leading to multicollinearity and poor estimation of coefficients. Already the problem of collinearity between \( \Delta p \) and \( \Delta t \) has troubled analysts (e.g. Hensher and Hotchkiss, 1974). For example, including kilometic distance would render the empirical application unreliable.

An intuitively satisfying way is to include distance as a dummy in the constant term, \( a_{ik} = a_{ik}' + d_{ik} D \), where \( D = 1 \) for short distance travel and \( D = 0 \) for long distance. This makes the reasonable assumption that the distance-dependent inherent advantage of a mode is different for short and long distances. For instance, if we compare car with rail, both \( a_{ik}' \) and \( d_{ik} \) are expected to be positive because there is an inherent advantage in car relative to rail (since it offers door-to-door delivery; this advantage is comparatively more important for short distances). By using a dummy \( D \) we effectively stratify by distance. Much experimentation is of course required to

\(^3\) Some experimentation was done with unrestricted specification of the form:

\[ \ln (\cdot) = a_{ik} + b_1 p_i - b_k p_k + c_1 t_i - c_k t_k \]

This form would give values of time specific to modes (e.g., trade-off between money savings in bus and time savings in rail). However, using level variables for price and time (as against differences) leads to serious problems of multicollinearity, and the results are difficult to interpret. Indeed, in the tests it was observed that some coefficients had the “wrong” signs or were statistically insignificant.

\(^6\) Excluding such variables does not affect the result unless omitted variables are significantly correlated with the variables included (omitted-variable bias).
determine whether variations in the upper limit for the “short-distance” category affect the result.

**Sensitivity to income and distance**

It is likely that travellers’ responses to price and time differences, and hence their value of time, depend on how affluent they are. Many investigators have recognised this possibility and have weighted price with the inverse of income. For example, a variable $\Delta p / Y$ was included instead of $\Delta p$, where $Y$ is an income variable. This would imply that the importance of price on mode choice decreases with income, or in other words that the value of time is proportionate to income. Others have multiplied travel time with income to reflect the presumption that high-income travellers are more concerned with lost time than low-income travellers. In terms of the coefficients of model (2) this would mean that $b = b' / Y$ and/or $c = c' Y$.

There is however a difficulty in deciding how income should be allowed to enter in the coefficients $a$ and $b$. Response to income may not be proportionate, and some investigators have used various powers of $Y$ (e.g. Westley, 1979). A more flexible method would be to use elasticity coefficients for describing the relationships $b = b(Y)$ and $c = c(Y)$. For instance, if $b = b'/Y^r$ and $c$ is independent of income, the value of time will be: VOT = $KY^r$ (where $K = c/b'$). When the elasticity coefficient $r$ is greater (smaller) than unity, the value of time increases faster (slower) than income. Train and McFadden (1978) show that under certain assumptions maximisation of a consumer’s utility function under time and income constraints leads to the following elasticity relationships: $b = b'/Y^r$ and $c = c' Y \rho$, where $r + \rho = 1$. This implies: VOT = $c'/b' Y$, i.e. a value-of-time proportionate to income. However, the assumptions underlying these relationships are rather restrictive, and it may be appropriate to allow the elasticities $r$ and $\rho$ to take values for which $r + \rho \neq 1$. Which values of the elasticity coefficients $r$ and $\rho$ are more appropriate, and whether values of time are significantly related to income, are empirical questions and should not be decided *a priori*. In the tests performed here income was approximated by the number of cars per thousand inhabitants for each zone. The auto/person ratio was the only indicator of local income available at this level in Greece.

Similarly, it can be argued that travellers’ responses to time differences $\Delta t$ depend on the total length of the journey (and thus on total time). In equation (2) it is assumed that the value of an incremental unit of time savings is constant for all levels of total time/distance: for example, a saving of ten minutes would have the same value whether it occurred in a short-distance or a long-distance journey. However, the value

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7 This result was obtained using a Cobb–Douglas utility function to represent a consumer’s indifference mapping for goods and leisure. It is apparent, however, that different functions may provide different ways of entering income: e.g., a constant-elasticity-of-substitution function would relax the condition $r + \rho = 1$, and thus would allow VOT to vary disproportionately with income.

8 This indicator discriminates well between problematic and other more developed areas in Greece. It should be noted however that the auto/population ratio may be biased (towards showing a positive effect) because it combines income and auto-availability effects on mode choice. People who own cars may prefer the car mode rather than bus or rail in their inter-urban trips. A wider model of simultaneous auto-purchase and mode-choice would be required to separate these effects, but this is outside the scope of the present study.

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attributed by people to time savings in short trips may differ from that of long trips: for example, time savings in long trips may be valued more because these trips are more arduous (Westley, 1979). Or they may be valued less if people committed to a long trip cannot use any time savings at the end of the journey and try to derive maximum pleasure from the journey itself. Watson (1974) has tested a proportionate relationship, assuming that travellers respond equally to the same proportion of overall time saved; a variable \( \Delta t/t \) was used, where \( t \) is the total journey time, averaged over all modes.

As with income, estimating an elasticity coefficient would show whether the effect is proportionate or disproportionate. Here kilometric distance \( D_3 \) is used as an indicator of total journey time (thus avoiding the problems of averaging the total time over all modes available; see Watson, 1974). The elasticity \( s \) of the value of time with respect to distance \( D_3 \) was estimated experimentally: that is, various values of \( s \) were assumed in the relationship \( VOT = KD_3 \), and the respective results (fit for the equation (2)) were compared.

It is noteworthy that this treatment of income and distance variables differs from many studies, which have a tendency to put all explanatory variables additively in the second part of the equation, often without proper justification. Here these variables are incorporated in (2) via the response coefficients \( b \) and \( c \). Income is assumed to affect both coefficients \( b \) and \( c \), while distance is likely to affect the time coefficient \( c \). This treatment, however, would require a tedious search method in estimation; each of the elasticities \((r, p, s)\) can take a range of values, and the "best fit" values have to be determined by experiment. Direct estimation is not possible, as existing estimation procedures only allow estimation of linear-in-parameters logit equations (2).

Estimation

Conventional maximum-likelihood methods have been commonly used for estimating logit-type models. More recent experience with disaggregate data suggests that maximum-likelihood estimators are unduly sensitive to outliers, i.e. observations with low calculated probabilities (see McFadden, 1979; Westin and Manski, 1979). An alternative approach is to develop more "robust" methods which are less sensitive to outliers, but very little has so far been done in this direction. Two robust methods appropriate for the observations available are used in the present study, to refine the results obtained by traditional least-squares methods. These methods are of wider applicability and can be used for analysing both aggregate and disaggregate data.

Ordinary least-squares (OLS) can at first be used for estimating logit models of the type (2). A problem, however, with the multinomial logit model is that there is more than one equation of the form (2). If we single out one mode as a base, there are \( M - 1 \) equations to be estimated from the data.\(^9\) It is desirable to estimate these equations simultaneously, because separate estimation of each equation may give different coefficients and because simultaneous determination may depict better the actual choice between \( M \) modes. In the present study, rail was chosen as the base mode, and

\(^9\) Note that equation (2) is symmetric in the sense that the same expression (with negative signs) is obtained if the inverse ratio \( f_i/f_t \) is used in the first part (see Theil, 1969).
dummy variables for other modes were used to achieve simultaneous minimisation of the squared residuals, as follows:

\[
\begin{bmatrix}
\ln \frac{B}{R} \\
\ln \frac{C}{R} \\
\end{bmatrix}
= \begin{bmatrix}
(a_B + d_B D) & 1 & 0 & \left(\frac{p_B - p_R}{p_B - p_R}\right) & t_B - t_R \\
(a_C + d_C D) & 0 & 1 & \left(\frac{p_C - p_R}{p_C - p_R}\right) & t_C - t_R \\
\end{bmatrix}
\]

(4)

where \(B = \text{bus}, C = \text{car} \) and \(R = \text{rail traffic (passengers)}\). This equation allows single coefficients \(b\) and \(c\) (and therefore a single value-of-time \(c/b\)) to be estimated from the data. At the same time it accounts for the unique advantage or disadvantage of each mode; this differs between short and long distances.

The OLS method, however, would give the same weight to small and to large inter-zonal flows. This may not be desirable, because (1) small flows are less reliable, (2) larger flows should ideally carry extra weight in determining the result. For this reason a weighted least-squares (WLS) procedure was employed, giving the following weights to individual observations (inter-zonal flows):

\[
w = (Tf_1, f_2, \ldots, f_M)^{1/2}
\]

(5)

where \(T\) is the total volume of traffic carried by all modes and \(f_i\) the relative market share of mode \(i\) in the particular inter-zonal traffic.\(^{11}\)

This weighting procedure is intuitively appealing. First, given the value of relative frequencies \(f\), more weight is given to larger zonal flows (large \(T\)). Secondly, the weight is zero for zone-pair flows which have one or more relative frequencies close to zero (e.g., if the rail market share is minimal, i.e. \(f_R \approx 0\), then \(w = 0\)). This is important, because the dependent variable (log of odds of other modes relative to rail) takes very large values in these cases, and hence cannot be handled in the computations. These observations behave as outliers. Moreover the dependent variable is highly sensitive to small changes in \(f\) for these extreme (negative or positive) values (see Theil, 1970). It is logical therefore to give less weight to such unstable observations. Given a value for \(T\), maximum weight is assigned to zonal flows where market shares are equal (for constant \(T\), \(w\) is maximum for \(f_R = f_B = f_C = \frac{1}{3}\), given that \(\sum f = 1\)).

However, application of the WLS procedure in the available data showed many outliers. The proportion of residuals outside \(\pm 3\) standard deviations was 10% for

\(^{10}\) The data used in this study include three cases with air links. These can be included in (4) in a similar way, or air traffic can be omitted (given the independence among alternative modes in the logit model, that omission may not affect the result). Here, air was included, considering four as against three modes for respective inter-zonal flows.

\(^{11}\) The idea of using this scheme came from a result by Theil (1970) or Warner (1962). This suggests that grouping data into \(T\)-sized aggregations (here inter-zonal flows) and using the observed mode frequencies in place of the theoretical probabilities of choice leads to an error of the logit dependent variable whose mean is zero and whose large-sample variance is \(1/w^2\). We should note, however, that this result refers to frequencies \(f\) and their sampling variation within each aggregation. Variation in equation (4), on the other hand, is cross-sectional, i.e. across all aggregations. Thus strictly speaking we cannot appeal to Theil's theory to justify the present scheme.
non-business travel and 4% for business travel. Analysis of the distribution of residuals suggested that it was symmetric, but flatter than the normal. Skewness was close to zero, but the kurtosis consistently above 3.

For this reason an iterative robust maximum likelihood (RML) method was adopted. This uses a non-linear optimisation procedure (Newton–Raphson search algorithm) to maximise the likelihood function. This method gives more efficient estimates, that is, estimates less affected by outliers, and hence by errors in model specification or data measurement. The RML method used here is well suited to fat-tailed distributions. It involves two types of weighting: (1) weights \( w \) are applied to the original observations (as in the WLS); (2) appropriate distribution functions for the error term are assumed which give less weight than the normal distribution to large errors (outlying observations). For the latter, two distribution functions were used, the inverse tangent function and the square root function. Both give weights to small errors almost identical to the normal distribution, but the weights to larger errors are smaller than in the normal distribution. The method is described in Appendix II.

RESULTS

Average values of time

Tables 1 and 2 show the results of the analysis for non-business and business travel, respectively. The coefficients estimated have the expected signs, and their magnitudes are sufficiently consistent across the various equations estimated. The level of explanation of modal shares, as measured by the adjusted \( R^2 \), is satisfactory. The reader should bear in mind that very low \( R^2 \) were achieved in similar types of analysis elsewhere. The results appear better when methods other than ordinary least squares are used. 12 Fewer observations were available for business travel, because the absolute numbers of passengers travelling for business purposes are much smaller than for other purposes. (Very small traffic flows were omitted; see Appendix I.)

The coefficients of price and time differences which are the focal point of this analysis are negative, suggesting that the higher the price and time characteristics of a mode relative to the base mode, the smaller its market share. The ratio of these coefficients gives the values of time (these are indicated in the last column). These values span from 27 to 43 drachmas per hour for non-business travel (Table 1) and from 93 to 174 drachmas per hour for business travel (Table 2), depending on the method of estimation. Expressed as proportions of the gross wage rate (see Table 3), these values are: 25–39% and 86–160% respectively. 13

These proportions lie within the limits suggested by other studies in developed

\[ 12 \text{ The } R^2 \text{ of the RML equations, however, are not strictly comparable with those of other equations.} \]

\[ 13 \text{ These values are likely to be biased because not all people have access to cars. Since the car tends to be the more expensive but faster mode, it is likely that people who have no access to cars (constrained choice space) may not have had the chance to choose a more expensive mode (here, car). In an unconstrained choice they might trade money for time, thus pushing their value of time upwards. But this hypothesis could be confirmed only if appropriate data were available (where auto-availability would be a controlled variable).} \]
### Table 1

**Estimated Logit Equations for Non-business Travel**

(Multinomial Logit Analysis; Base Mode = Rail; \( N = 102 \))

<table>
<thead>
<tr>
<th>Equation</th>
<th>Method of Estimation</th>
<th>Const.</th>
<th>Price Differ. ((\Delta p))</th>
<th>Time Differ. ((\Delta t))</th>
<th>Distance Dummy</th>
<th>( R^2 ) (% of gross wage rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Dummy: ( D = 1 ) for 0–50 kms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>OLS</td>
<td>1.342</td>
<td>-0.000397</td>
<td>-0.00263</td>
<td>1.518</td>
<td>2.497</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.231)</td>
<td>(0.00104)</td>
<td>(0.00106)</td>
<td>(0.526)</td>
</tr>
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<td>1.2</td>
<td>WLS</td>
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<td>-0.00237</td>
<td>-0.00169</td>
<td>0.368</td>
<td>0.949</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.141)</td>
<td>(0.00062)</td>
<td>(0.00093)</td>
<td>(0.440)</td>
</tr>
<tr>
<td>1.3</td>
<td>RML (1)</td>
<td>1.396</td>
<td>-0.00334</td>
<td>-0.00158</td>
<td>1.331</td>
<td>2.459</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.240)</td>
<td>(0.00119)</td>
<td>(0.00117)</td>
<td>(0.667)</td>
</tr>
<tr>
<td>1.4</td>
<td>RML (2)</td>
<td>1.401</td>
<td>-0.00335</td>
<td>-0.00158</td>
<td>1.340</td>
<td>2.457</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.262)</td>
<td>(0.00135)</td>
<td>(0.00129)</td>
<td>(0.793)</td>
</tr>
<tr>
<td>Distance Dummy: ( D^* = 1 ) for 0–75 kms</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1.5</td>
<td>OLS</td>
<td>1.114</td>
<td>-0.00339</td>
<td>-0.000221</td>
<td>1.112</td>
<td>2.211</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.257)</td>
<td>(0.00104)</td>
<td>(0.00112)</td>
<td>(0.408)</td>
</tr>
<tr>
<td>1.6</td>
<td>WLS</td>
<td>1.287</td>
<td>-0.00225</td>
<td>-0.00158</td>
<td>0.237</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.144)</td>
<td>(0.00062)</td>
<td>(0.00093)</td>
<td>(0.339)</td>
</tr>
<tr>
<td>1.7</td>
<td>RML (1)</td>
<td>1.181</td>
<td>-0.00280</td>
<td>-0.00127</td>
<td>1.017</td>
<td>2.117</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.253)</td>
<td>(0.00113)</td>
<td>(0.00114)</td>
<td>(0.465)</td>
</tr>
<tr>
<td>1.8</td>
<td>RML (2)</td>
<td>1.183</td>
<td>-0.00279</td>
<td>-0.00125</td>
<td>1.006</td>
<td>2.123</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.273)</td>
<td>(0.00126)</td>
<td>(0.00124)</td>
<td>(0.509)</td>
</tr>
</tbody>
</table>

Mean values: \( \Delta p = 19.6 \) drachmas, \( \Delta t = -97.8 \) minutes.

Figures in parentheses are standard errors.

OLS = Ordinary Least Squares; WLS = Weighted Least Squares; RML = Robust Maximum Likelihood—(1) tangent method; (2) square root method.

countries: that is, an average value of leisure time equal to 20–50% of the wage rate and a value of working time close to the wage rate (see Hensher, 1976). But there are several reasons why these comparisons should not be accepted uncritically. First, most of the studies in developed countries have focused on commuter travel, and results on interurban travel may be different—for example, Watson (1974) found a value of time savings for intercity non-business travel equal to 67.5% of the wage rate, though this estimate is somewhat unreliable. Secondly, modelling and/or data have been different (the binary model with disaggregate data has typically been used). Thirdly, methods of estimation have differed, and the results of the present analysis suggest that different methods of estimation can give different results. Fourthly, there are variations in the way in which the wage rate is defined and measured, and in how it is adjusted for social overheads.

The WLS method, although intuitively appealing, appears to give somewhat inferior results in the present case because of the increased number of outlying
CHOICE OF TRAVEL MODE AND VALUE OF TIME IN GREECE

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TABLE 2

Estimated Logit Equations for Business Travel
(Multinomial Logit Analysis; Base Mode = Rail; N = 79)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Method of Estimation</th>
<th>Const.</th>
<th>Price Differ. ($\Delta p$)</th>
<th>Time Differ. ($\Delta t$)</th>
<th>Distance Dummy</th>
<th>VOT (% of gross wage rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bus ($D_b$)</td>
<td>Car ($D_c$)</td>
</tr>
<tr>
<td>Distance Dummy $D = 1$ for 0–50 kms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 OLS</td>
<td></td>
<td>1.701</td>
<td>-0.00340</td>
<td>-0.00596</td>
<td>0.081 2.586</td>
<td>0.209 97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.244)</td>
<td>(0.00145)</td>
<td>(0.00229)</td>
<td>(0.673) (0.667)</td>
<td></td>
</tr>
<tr>
<td>2.2 WLS</td>
<td></td>
<td>1.491</td>
<td>-0.00251</td>
<td>-0.00679</td>
<td>0.147 0.690</td>
<td>0.553 149</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.199)</td>
<td>(0.00153)</td>
<td>(0.00232)</td>
<td>(0.608) (0.596)</td>
<td></td>
</tr>
<tr>
<td>2.3 RML (1)</td>
<td></td>
<td>1.762</td>
<td>-0.00370</td>
<td>-0.00576</td>
<td>-0.033 2.718</td>
<td>0.451 86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.331)</td>
<td>(0.00139)</td>
<td>(0.00253)</td>
<td>(0.799) (0.937)</td>
<td></td>
</tr>
<tr>
<td>2.4 RML (2)</td>
<td></td>
<td>1.769</td>
<td>-0.00368</td>
<td>-0.00571</td>
<td>-0.039 2.698</td>
<td>0.452 86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.364)</td>
<td>(0.00174)</td>
<td>(0.00209)</td>
<td>(0.836) (0.998)</td>
<td></td>
</tr>
<tr>
<td>Distance Dummy $D^* = 1$ for 0–75 kms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5 OLS</td>
<td></td>
<td>1.564</td>
<td>-0.00247</td>
<td>-0.00481</td>
<td>-0.241 2.465</td>
<td>0.341 107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.258)</td>
<td>(0.00135)</td>
<td>(0.00215)</td>
<td>(0.466) (0.450)</td>
<td></td>
</tr>
<tr>
<td>2.6 WLS</td>
<td></td>
<td>1.480</td>
<td>-0.00213</td>
<td>-0.00619</td>
<td>-0.165 0.887</td>
<td>0.574 160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.209)</td>
<td>(0.00101)</td>
<td>(0.00231)</td>
<td>(0.428) (0.411)</td>
<td></td>
</tr>
<tr>
<td>2.7 RML (1)</td>
<td></td>
<td>1.652</td>
<td>-0.00223</td>
<td>-0.00391</td>
<td>-0.172 2.501</td>
<td>0.460 97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.371)</td>
<td>(0.00122)</td>
<td>(0.00135)</td>
<td>(0.637) (0.697)</td>
<td></td>
</tr>
<tr>
<td>2.8 RML (2)</td>
<td></td>
<td>1.659</td>
<td>-0.00223</td>
<td>-0.00386</td>
<td>-0.180 2.491</td>
<td>0.461 96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.405)</td>
<td>(0.00121)</td>
<td>(0.00134)</td>
<td>(0.685) (0.783)</td>
<td></td>
</tr>
</tbody>
</table>

Mean values: $\Delta p = 91.7$ dr., $\Delta t = 96.6$ minutes.
Figures in parentheses are standard errors.
OLS = Ordinary Least Squares; WLS = Weighted Least Squares; RML = Robust Maximum Likelihood—(1) tangent method; (2) square root method.

TABLE 3

Workers’ Average Wage Rate (Year 1979)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>w_1 = 80.6 drs/hour</th>
<th>w_2 = 108.8 drs/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>A:</td>
<td>Directly paid average rate:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B:</td>
<td>Gross wage rate (social overheads included):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sources
B—A mark-up of 25% was used to cover payments for social security, family allowances, payment for non-working days and other social overheads. This mark-up was taken from a study of the cost of labour in manufacturing produced by the National Statistical Service of Greece for the year 1977.
observations. The two robust procedures (RML 1 and 2) provide results closer to OLS rather than to WLS. The values of time obtained with the robust procedures are somewhat more reliable. These values are:

1. non-business travel: VOT = 27–28 drachmas per hour
   (= 25–26% of the wage rate)
2. business travel: VOT = 93–105 drachmas per hour
   (= 86–97% of the wage rate)

Estimation of variances for the VOT is also possible, as follows:

\[ \text{Var} \ (\text{VOT}) = \frac{1}{b^4} \left[ b^2 \text{Var} \ (c) + c^2 \text{Var} \ (b) - 2bc \text{Cov} \ (b,c) \right] \]

Calculations provide values for the variance in the range from 0.034 to 0.063 (or, equivalently, standard errors: 11–15 drachmas per hour).

However, while computed variances give an indication of the possible spread in the VOT estimates, they do not necessarily provide a basis for determining confidence intervals, because the distribution of \( c/b \) is skewed (see Lianos and Rauzier, 1972). There are no standard statistical tables to test the significance of the ratio of two regression coefficients. Indicative “fiducial limits” can however be obtained by Fieller’s theorem (see Finney, 1978, p. 81). In the present case the upper and lower fiducial limits to VOT are given by the formula:

\[ \text{VOT}_u, \text{VOT}_l = \frac{1}{1 - g} \left( \text{VOT} - g \frac{\text{Cov} \ (b,c)}{\text{Var} \ (b)} \pm \left[ \text{Var} \ (\text{VOT}) - g/b^2 \left( \frac{\text{Var} \ (c) - \frac{\text{Cov}^2 \ (b,c)}{\text{Var} \ (b)}}{\text{Var} \ (b)} \right) \right]^{1/2} \right) \]

where

\[ g = \frac{t^2 \text{Var} \ (b)}{b^2} \]

and \( t \) is a \( t \)-distribution deviate for a probability level. Calculations for equation (1.4) and (2.4) provide: (1) non-business travel: \( \text{VOT}_u = 4; \text{VOT}_l = 54 \) drachmas per hour; (2) business travel: \( \text{VOT}_u = 81; \text{VOT}_l = 131 \) drachmas per hour. These upper and lower limits correspond to 90% confidence level (i.e. a value \( t = 1.66 \) was used). It appears that these limits are rather broad, and therefore that the average values of time found should not be accepted uncritically.

It is interesting that the problem of collinearity between time and price differences, which has troubled analysts for some time, is not a problem for the present set of data. Correlations between \( \Delta t \) and \( \Delta p \) are sufficiently low (–0.524 for non-business and –0.589 for business travel). According to Klein’s rule of thumb, collinearity is not necessarily a problem when \( r^2 < R^2 \). This condition is fulfilled here, and the data therefore provide a reasonable base for separating the effects of the two variables. Overall results are more reliable than those reported elsewhere by, e.g., Hensher and Hotchkiss (1974), or Watson (1974).
It is noteworthy that the values of time estimated vary little with various specifications of the distance dummy in the constant term (see Tables 1 and 2). This dummy is statistically significant (at a 95% confidence level) in all equations for the car mode, suggesting that there is an inherent comparative advantage to car over rail for short distances. For bus there appears to be a similar advantage for non-business travel, but not for business. The coefficients are smaller than in business travel, suggesting a weaker advantage. For business travel the distance dummy is statistically insignificant.

The relative sizes of the constant terms in Tables 1 and 2 confirm the view that the advantage to other modes over rail is higher for business travel. Equation (4) was finally restricted to a single constant for non-rail modes, because (1) preliminary estimations of the full equation had shown no significant difference in the mode-specific constants,14 (2) one constant throughout helped to reduce the number of parameters estimated from the data and to avoid multicollinearity problems between the various dummies.

Sensitivity tests

A series of tests was performed in order to find out whether response to price and time differences depends on income. A search method was used in estimation: the elasticity \( r \) was set at some value and \( \rho \) was varied over a range. The process was repeated for various values of \( r \).

Unfortunately, multiplying both \( \Delta p \) and \( \Delta t \) by powers of income created severe problems of collinearity, and it was impossible to differentiate between equations. Note that similar problems were encountered by Train and McFadden (1978). So the full model was abandoned and a simpler one was tried in which only one coefficient, that of \( \Delta p \), was expressed as a power function of income. Again the results suggested that the model fit (measured by \( R^2 \)) changes little with the elasticity \( r \), and “best fit” may not be a satisfactory criterion for discriminating between various \( r \). Moreover, the best value for \( r \) varies according to the method of estimation. For example, for non-business travel the best \( r \) was 0.6–0.7 for OLS, 0.0–0.2 for WLS, and 0.8–1.0 for RML (1 and 2). The difference in \( R^2 \) between the “best” fit equations and the no-income-effect equations (i.e. \( r = 0 \)) was very small, well within the range of statistical error.

These results, though not conclusive, suggest that income may not have a significant effect. It is possible, however, that some other functional relationship may exist, not captured by the elasticity relationships tried. This possibility was tested by an "omitted-variable test". Observations were arranged in ascending order of income, and the Durbin–Watson statistic was estimated. If income belongs to the model, the regression residuals \( u \) will be correlated with income and the D–W statistic will take a small value (see Westley, 1979). However, the results gave almost perfect D–W values, very close to two. This suggests that income does not have any significant effect which has been omitted from the equation.

Overall, the hypothesis that income affects the response to price, and hence the

14 These results are not reported here, but can be obtained from the author on request.
TABLE 4

VOT Sensitivity to Kilometric Distance

<table>
<thead>
<tr>
<th>Purpose of travel</th>
<th>Method of estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
</tr>
<tr>
<td>Non-business</td>
<td>−0.85</td>
</tr>
<tr>
<td>Business</td>
<td>−0.75</td>
</tr>
</tbody>
</table>

VOT, is rejected. Income, or more exactly the auto/person ratio, plays no role at the aggregate level. A possible explanation is that the influence of income has been effectively accounted for by the distinction between business and non-business purposes.

Similar tests were performed with distance. In this case all criteria indicated a significant effect. Table 4 shows the “best fit” estimates obtained for the elasticity coefficient $s$ (i.e., elasticity of the response coefficient $c$ with respect to distance):

Although numerically these coefficients may not be very reliable (because of multicollinearity associated with kilometric distance $D$), they suggest that the value of time decreases with distance. This result confirms the hypothesis that time savings over longer distances are valued less than over short distances. The elasticity is quite close to $-1$, roughly suggesting that when the length of the trip is doubled the value of time savings may fall to half.

CONCLUSIONS

The various results of the multinomial logit analysis of mode shares are sufficiently consistent. Reasonably close estimates were obtained with alternative methods of estimation for different specifications of the distance dummy. The robust maximum-likelihood method of estimation appears somewhat superior to others for the analysis of residuals. This method provided average values of time of 25–26% for non-business travel, and 86–97% for business travel, of the workers’ average wage rate (including overheads).

The estimated values of time lie within, and near to the low end of, the range of values suggested by other studies in developed countries, when the values are expressed in wage rate units. This suggests that the practice of transferring wage proportions from developed to developing countries may give a reasonable approximation. But such comparisons should not be accepted uncritically, because the received proportions span a wide range and because measurements of wage rates differ. In absolute terms the monetary values of time obtained for Greece are substantially lower than for other countries; this is perhaps accounted for by the much lower wage rates in Greece.

Sensitivity tests suggest that the value of time depends on the total length of the journey, decreasing substantially with distance. No significant effect was identified from local variations in income, measured by the auto/person ratio.
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It is felt that the average values of time found in the present study could be very helpful for bodies engaged in policy-making and transport planning, in Greece and internationally. Similar analyses in other developing countries would help in investigating variations in the value of time in different cultural contexts and for different levels of economic development.

The experience derived from this study also suggests a satisfactory performance of the multinomial logit model. Many researchers using logit analysis in different situations have encountered problems with their data. Set against this background, the present study enhances confidence in using these models.

APPENDIX I

Data Sources and Measurement

The data used in this study are described in detail in the Railway Modernisation Study prepared by Transmark (Supplement on Economic and Market Research). Here we will provide only a broad description. In a first stage, the Athens–Thessaloniki corridor, which was the focus of the study, was divided into 44 zones on the basis of local administrative areas. In the areas around Athens and Thessaloniki, where 45% of the population is concentrated, zones were based on the smallest administrative divisions (eparchies). Further disaggregation would have created problems in data collection, and also because traffic flows would be small and less reliable, especially for business travel. In areas outside the corridor zones were larger.

Inter-zonal traffic flows for passengers were determined by combining information from several sources. In particular: (1) The Railways Organisation provided computer tapes giving details of all tickets issued on the rail network, including the origins and destinations of passengers. (2) Bus services are divided between the state-owned Railways Organisation (OSE) and the privately operated co-operatives (KTEL). For OSE buses, data on passenger movements were extracted from OSE computer tapes. For KTEL, data on the number of buses per day and average occupancy factors were obtained from the Ministry of Transport and the KTEL themselves; the origins and destinations of OSE buses were then used to make the KTEL data compatible with OSE data. (3) Information on road vehicle movements and occupancies was obtained from a programme of screen-line and cordon surveys carried out by Louis Berger International for the Ministry of Public Works. (4) Information on flows on domestic air routes was obtained from the internal records of Olympic Airways and was supplemented with an air survey. All data refer to the year 1979.

Data on the purposes of journeys were obtained by mounting special surveys (such data were available only for the car mode). The weighted national average proportions of business travel for different modes are: rail: 7%; bus: 8%; car: 18%; and air: 46%. Given that only three domestic routes for air are included, these proportions show that business travel is comparatively small. Flows of less than 100 passengers per annum (in any mode) were omitted from the estimation.

Information on trip prices and time was collected from various sources. Opportunity for choice of route in Greece is very limited, so the measurement of price
and time variables presented no difficulties. For rail and bus, costs include fares (i.e.,
out-of-pocket costs) and connector costs. Fares were provided by the respective
organisations. Connector costs were based on average taxi fares for business
travellers and on bus fares for other-purpose travellers. For the car mode the costs
included are: fuel and road tolls for non-business travel; fuel, road tolls, lubricant,
 tyres and distance-related maintenance for business travel. It was assumed that these
costs express the costs perceived by travellers, rather than the total economic costs.
This treatment is in line with accepted practice. The perceived car costs reflect running
costs for a standard 1300cc saloon operating under Greek conditions. Various
domestic and international sources were used in estimating these costs.

The time variable for bus and rail includes in-vehicle time plus connector and
waiting time. Connector times were based on the average trip length of centroid
connectors, and waits times were estimated on the basis of the service interval and
survey results. A weight of 1.3 was applied to wait times; this was obtained from the
relative valuation of wait times in the Athens Transportation Study (W. Smith, 1974).
An interchange time penalty was also estimated for routes with interchange points.
These elements of time are significantly intercorrelated, and including them separately
would create considerable problems of multicollinearity. For the car mode, only
in-vehicle time was included. Since roads all extended to the centres of large towns, no
connector times and costs were included for car.

APPENDIX II

Robust Regression based on Maximum Likelihood

Given a sample of \( N \) independent observations of relative market shares, we want to
estimate a linear model of the form:

\[
\ln \{ \text{ODDS} \}_i = a + b x_{ci} + c d t_i + \ldots + u_i \quad (i = 1, \ldots, N)
\]

where \( u_i \) are independent random errors. If we denote with \( y \) the dependent variable, \( x_j \)
the vector of independent variables \( (j = 1, \ldots, p) \) and \( \theta_j \) their respective coefficients,
the model can be written in the standard regression notation:

\[
y_i = \sum_j x_{ji} \theta_j + u_i \quad (j = 1, \ldots, p; i = 1, \ldots, N)
\]  \tag{1}

Classically, the parameters \( \theta \) are estimated by OLS, i.e. by minimising the sum

\[
\sum_i \left( y_i - \sum_j x_{ji} \theta_j \right)^2 = \text{min}
\]  \tag{2}

or, equivalently, by solving the system of \( p \) equations, obtained by differentiating (2):

\[
\sum_i \left( y_i - \sum_j x_{ji} \theta_j \right) x_{ji} = 0
\]  \tag{3a}

or

\[
\sum_i u_i x_{ji} = 0 \quad (j = 1, \ldots, p)
\]  \tag{3b}

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The WLS method assigns weights \( w \) to observations, so that equation (1) becomes:

\[
    w_i y_i = \sum_j (w_j x_{ij}) \theta_j + u_i^* \tag{4}
\]

where \( u_i^* = w_i u_i \). In a similar way (4) can be estimated by OLS. The parameters \( \theta \) would result from conditions similar to (3b):

\[
    \sum_i w_i u_i x_{ij} = 0 \quad (j = 1, \ldots, p) \tag{5}
\]

Next we will assume that the weights \( w \) have already been applied to the original observations for \( y \) and \( x \), so that the notation in (1) can be used.

Recent work by Tukey and others has shown that when the errors \( u_i \) are not normally distributed the conventional least-squares or maximum likelihood methods can give surprisingly unrobust estimates. For instance, when distributions are fat-tailed the efficiency of the estimates is impaired (see Huber, 1972, 1973). Robust alternatives have been developed which are much more efficient for fat-tailed distributions. These methods are generally based on maximum likelihood, and use specific distribution functions of \( u \) that put less weight to outlying values.

If \( f(u) \) is the density function of the random term \( u \) in (1), the joint probability of observations (i.e. likelihood function) conditional on \( \theta \) is given by:

\[
    L = \prod_{i=1}^{N} f(u_i(\theta)) \tag{6}
\]

The method of maximum likelihood chooses \( \theta \) as that vector of parameter values which gives random terms with the highest joint probability \( L \). Usually we maximise the log \( L \) instead of \( L \). Setting the first-order derivatives of the log-likelihood function equal to zero, we get \( p \) optimisation conditions, as follows:

\[
    \sum_i f'(u_i) \left( \frac{\partial u_i}{\partial \theta_j} \right) = 0 \tag{7}
\]

From the linear model (1) we have:

\[
    \frac{\partial u_i}{\partial \theta_j} = -x_{ij},
\]

so the conditions (7) become:

\[
    \sum_i \psi(u_i) x_{ij} = 0 \quad (j = 1, \ldots, p) \tag{8}
\]

where

\[
    \psi(u) = \frac{-f''(u)}{f(u)}. \tag{9}
\]

If \( u \) is normally distributed then \( \psi(u) = -f' / f = u \exp(-u^2/2)/\exp(-u^2/2) = u \).

Thus condition (8) becomes identical to condition (3b), and the maximum likelihood estimates are the same as the OLS estimates. For fat-tailed distributions the ideal \( \psi(u) \) would be close to \( \psi(u) \simeq u \) for small \( u \) (as in the normal distribution), but \( \psi(u) < u \) for large \( u \). Two functions which have such properties are:
(a) \textit{Inverse tangent function}: \( \psi(u) = \tan^{-1}(u) \) \hfill (10)

This corresponds to a density function:

\( f(u) \sim \exp[\frac{1}{2} \log (1 + u^2) - u \tan^{-1}(u)] \)

and has the shape indicated in Figure 1.

(b) \textit{Square root function}: \( \psi(u) = u/\sqrt{1 + u^2} \) \hfill (11)

The corresponding density function is:

\( f(u) = \exp (1 - \sqrt{1 + u^2}) \)

and the shape is similar to (a), with slightly fatter tails (see Figure 1).

With these \( \psi(u) \) distributions, however, the optimisation conditions (8) become non-linear and can be solved by applying a Newton–Raphson technique, as follows. Let \( \theta^0 \) be an initial set of values for the parameters \( \theta \). If we carry out a Taylor series expansion of \( \psi(u) = \psi(u(\theta)) \) about the point \( \theta^0 \) and curtail the expansion at the first derivative, we get:

\[ \psi \simeq \psi^0 + \sum_{j=1}^{p} \delta_j \frac{\partial \psi}{\partial \theta_j} \]

where \( \theta = \theta^0 + \delta^0 \) and \( \psi^0 = \psi(u(\theta^0)) \). Substituting in (8) and taking into account that the linear model (1) implies

\[ \frac{\partial \psi_i}{\partial \theta_j} = -x_{ij} \frac{\partial \psi_i}{\partial u_i} = -x_{ij} \psi_i' \]

we obtain:

\[ \sum_i \left( \psi^0_i - \psi'_j x_{ij} \delta^0_j \right) x_{ii} \simeq 0 \] \hfill (12)

Equation (12) is similar to (3a); this suggests that we can estimate \( \delta^0 \) by least squares. The dependent variable will be \( \psi^0 \) and the independent variables \( \psi'_j x_j \). Let us denote the parameters \( \delta \) which are estimated by least squares with \( \delta^0 \). This would provide a
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new set of parameters $\theta^1 = \theta^0 + \delta^1$. We can now use $\theta^1$ instead of $\theta^0$ and repeat the same procedure. This leads to another set of revised estimates $\theta^2 = \theta^0 + \delta^2$, and so on. In the present study the iterative process started with the WLS estimates. Convergence was fast (four to six iterations were performed in order to reach the results reported in Tables 1 and 2).

REFERENCES


