ALTERNATIVE DEMAND MODELS AND THEIR ELASTICITY ESTIMATES

By Tae Hoon Oum

INTRODUCTION

An understanding of the demand for transport, and accurate forecasting of future demand, are essential for the analysis of serious policy and planning questions. Demand analysis has accordingly had considerable attention in transport literature. One of the most striking features of this literature is the variety of demand and forecasting models put forward. Two important sources of this variety are the choice between disaggregate and aggregate data (level of aggregation) and the choice of functional form. Differences in types of data and in demand models can create significant problems for planners and policy analysts, because empirical results such as elasticities and traffic forecasts are likely to be contingent upon those choices.

The basic unit of observation in the aggregate modelling approach is an aggregate volume or share of a particular mode in a market; in the disaggregate modelling approach it is an individual decision maker’s distinct choice. The advantages and disadvantages of these two approaches are described in detail in two recent survey papers by Winston (1983, 1985). In addition, it needs to be emphasised that the choice between aggregate and disaggregate models depends largely on the purpose of the study and the availability and cost of obtaining the data. When the purpose of the model is to forecast aggregate traffic volumes, it is natural and even preferable to use aggregate data. If disaggregate data were used, the extrapolation of sample survey results to the aggregate market level would widen the confidence interval. On the other hand, if the purpose of the model is to simulate how decision makers such as shippers or travellers would respond to changes in policy or managerial control variables, disaggregate data are more

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1 Westin (1974) describes how the market level predictions can be made by using binary choice models estimated from survey sample data.
attractive. As noted by Winston (1983), a disaggregate model requires an extensive data base. Furthermore, disaggregate data are not always easy to obtain, because of the confidentiality of private information. Even when it is feasible to collect the data, the results are expensive. Therefore, it would seem prudent to use an aggregate model to generate the preliminary results before undertaking a larger study with disaggregate data. In short, despite some criticisms of aggregate models, both the aggregate and disaggregate models have their distinct roles in demand studies, and they are sometimes complementary rather than competing.

A variety of models have been used for analysing and forecasting transport market demand. The choice of model is likely to affect the forecast of future traffic volumes and other empirical results, such as elasticities of demand with respect to price and service quality variables. However, in the past there has been little attempt to measure systematically the effects of alternative models on the empirical results. Nor has there been much research undertaken on procedures for model selection. One exception is Gaudry and Wills (1978), who compared and tested a number of functional forms for travel demand models, using the so-called Box-Tukey transformation of the original variables (Box and Cox, 1964; Tukey, 1957; and Zarembka, 1974). Among other things, they compared linear, log-linear and various special cases of Box-Tukey functional forms for the direct (volume) demand models and for the market share models. We are unaware of any previous work comparing these models with an aggregate logit model or with the demand systems which can be derived from the so-called “flexible” utility or production functions, such as the translog or generalised Leontief functions. The purpose of this paper is, therefore, to fill this gap. Specifically, this paper will compare, theoretically, methodologically and empirically, the four representative forms which have been used most widely for estimating aggregate demand models. Since these four models will be estimated from identical data sets, the differences in empirical results will be due strictly to the different formulations of the models.

This paper concentrates on aggregate freight demand models, because of the particular data set used in the four models to be compared, but the theoretical and methodological discussions are directly applicable to demand studies for any goods and services, including passenger transport modes.

**THE MODELS COMPARED**

The functional forms used to estimate most of the aggregate transport demand models may be classified into the following four categories:

(a) Linear demand model;
(b) Log-linear demand model;
(c) Logit model applied to aggregate market share data;
(d) Translog demand system based on neoclassical demand theory.

Though a model may be specified by mixing some features of two or more of these functional forms, for simplicity we treat only the four distinct cases in this paper. The theoretical and intuitive properties of each are discussed below.
(a) Linear demand model

The linear function has been extensively used in demand and sales forecasting models because it is simple to estimate and the empirical results are easily interpreted. It is an advantage that each elasticity of demand depends on the value of the variable, but for many variables the assumption of a linear effect may not be realistic.

(b) Log-linear demand model

The long-linear (double logarithmic or Cobb-Douglas) model specifies the logarithm of traffic volume as a linear function of the logarithms of potential determinants, such as prices and quality variables. This is the most widely used functional form for transport demand models, because (a) the coefficients themselves are the respective elasticities of the demand; (b) the log-linear function is capable of modelling nonlinear effects; (c) it resembles the demand function obtainable from a Cobb-Douglas utility (production) function; and (d) the variants of the gravity-type models and abstract mode models (Quandt and Baumol, 1966, and Quandt and Young, 1969) which dominated the conceptual framework for transport demand modelling during the 1960s and the early 1970s invariably used a log-linear functional form.

The main drawback of this model is that each elasticity is invariant across all data points, and not dependent on the location of the demand curve. This is an unacceptable restriction when one is interested in estimating variations in elasticities across different cross-sectional routes and over time within a route.

(c) Aggregate logit model

Many regulatory analysts in transport have used the so-called linear logit functional form for modelling market shares of alternative modes of transport. This is chiefly because an intuitive and theoretical rationale is provided by the discrete choice version of the logit model. That is, the logit model can be derived from the utility maximising choice process of a rational decision maker (for example, shipper or traveller) who has a random utility function, defined over price and attribute space, which is separable from the individual's other choice decisions (see Quarmby, 1967, McFadden, 1973, 1974, and Domencich and McFadden, 1975). The second reason for the popularity of the aggregate logit model is that it can be estimated by any regression program. The third reason appears to be that the S-shaped market share curve is intuitively attractive and realistically describes the mode-switching behaviour of decision makers. Furthermore, the S-shaped market share curve constrains the predicted market shares between zero and one, and this is a desirable feature.

Researchers have used aggregate logit models to investigate the sensitivity of aggregate market shares of alternative freight modes to changes in regulatory or managerial control variables, such as relative prices and relative quality attributes. Perle (1964), Morton (1969), Kullman (1973), Turner (1975), Boyer (1977) and Levin (1978) are examples. The following two alternative specifications have been used as aggregate logit models for modal split analysis:
Type 1: ratio form

\[
\ln \left( \frac{VS_i}{VS_m} \right) = a_{i0} + \sum_{k=1}^{K} a_{ik} \left( \frac{X_{ik}}{X_{mk}} \right) + \sum_{n=1}^{N} b_n X_n
\]

Type 2: difference form

\[
\ln \left( \frac{VS_i}{VS_m} \right) = a_{i0} + \sum_{k=1}^{K} a_k (X_{ik} - X_{mk}) + \sum_{n=1}^{N} b_n X_n
\]

where

\[
\frac{VS_i}{VS_m} \quad \text{the ratio of demand (or volume share) of mode } i \text{ to the base mode } m; \\
X_{ik} \quad \text{kth attribute of mode } i; \\
X_n \quad \text{n th variable common to all modes, which attempts to capture the difference among links; } \\
X_{mk} \quad \text{kth attribute variable for the “base” mode; } \\
a_{i0}, a_{ik}, b_n \quad \text{parameters of the logit model.}
\]

Oum (1979a) has shown that there is no consistent differentiable preference (or production technology) structure for the decision maker underlying either form of the logit model. Furthermore, both forms of the linear logit model restrict the cross-elasticities of demand for various alternative modes with respect to an attribute of any given mode (except for those with respect to the base mode in case of the “ratio form” model) to be identical (see Hausman, 1975, Oum, 1979a, and McFadden, 1980).²

Oum also shows, for the case of the “ratio form” model, that the choice of the base mode (the market share of which becomes the denominator of the dependent variable) affects the empirical results, including own and cross elasticities of demand for a given mode. This makes the “difference form” superior to the “ratio form”. Therefore, in this paper we will compare only the “difference form” logit model with other alternative models.

(d) Translog demand system

Since the mid 1970s, economists have begun to use the so-called demand system derived from a “flexible” utility (for consumer demands) or production (for input demands) function. The “flexible” functions capable of providing a quadratic approximation to the unknown true function include the translog (Christensen, Jorgenson and Lau, 1973), generalised Leontief (Diewert, 1971) and generalised

² This restriction on cross elasticities is caused by the independence of irrelevant alternatives (IIA) property of the standard logit model. The Dogit model developed in Gaudry and Dagenais (1979) and applied in Gaudry (1980), and the universal logit model developed by McFadden (1979), are free from this criticism.

The demand system derived from the translog utility or production/cost function is not only consistent with the neoclassical theory of consumption or production, but also allows for free variation of the elasticities of substitution between transport modes and of the own- and cross-elasticities. These advantages, of course, come at the cost of substantially increased computation when compared with models (a), (b) and (c).

For the purpose of this paper, we adopted the following translog system, which Oum (1979b) applied to the study of rail-truck competition in Canada, using 1970 Canadian inter-regional freight flow data.

\[
\ln U_{C_t} = a_0 + a_1 \ln (P_{1t} Z_{11t}^{\beta_1} Z_{12t}^{\gamma_1} D_{t1}^{\delta_1}) + a_2 \ln (P_{2t} Z_{21t}^{\beta_2} Z_{22t}^{\gamma_2} D_{t2}^{\delta_2}) + \frac{1}{2} a_{11} \left[ \ln (P_{1t} Z_{11t}^{\beta_1} Z_{12t}^{\gamma_1} D_{t1}^{\delta_1}) \right]^2 + \frac{1}{2} a_{22} \left[ \ln (P_{2t} Z_{21t}^{\beta_2} Z_{22t}^{\gamma_2} D_{t2}^{\delta_2}) \right]^2 \]

(2a)

\[
+ a_{12} \ln (P_{1t} Z_{11t}^{\beta_1} Z_{12t}^{\gamma_1} D_{t1}^{\delta_1}) \ln (P_{2t} Z_{21t}^{\beta_2} Z_{22t}^{\gamma_2} D_{t2}^{\delta_2}),
\]

where

\[
U_{C_t} = \text{weighted average unit cost to shippers in cents per ton-mile on link } t, t = 1, 2, \ldots, L.
\]

\[
P_{1t} = \text{average railway freight rate in cents per ton-mile on link } t.
\]

\[
P_{2t} = \text{average trucking freight rate in cents per ton-mile on link } t.
\]

\[
Z_{11t} = \text{average speed of railway services in mile per day on link } t.
\]

\[
Z_{21t} = \text{average speed of trucking services in miles per day on link } t.
\]

\[
Z_{12t} = \text{reciprocal of coefficient of variation in transit time distribution of railway services on link } t \text{ (a measure of reliability of transit time).}
\]

\[
Z_{22t} = \text{reciprocal of coefficient of variation in transit time distribution of trucking services on link } t.
\]

\[
D_t = \text{distance of link } t \text{ in miles.}
\]

\((\beta_1, \gamma_1, \delta_1)\) and \((\beta_2, \gamma_2, \delta_2)\) are parameters of the hedonic aggregator functions for rail and truck modes, respectively; and

\((a_1, a_2, a_{11}, a_{22}, a_{12})\) are parameters of the macro translog cost function.

The linear homogeneity conditions of the translog function are

\[
a_1 + a_2 = 1
\]

\[
a_{11} + a_{12} = 0, a_{12} + a_{22} = 0.
\]

After imposing this linear homogeneity condition and applying Shephard's lemma to the cost function in equation (4a), we obtain the following expenditure share functions:
\[ S_{1t} = a_1 + a_{11} \ln \left( \frac{P_{1t}}{P_{2t}} \right) + a_{11} (\beta_1 \ln Z_{11t} - \beta_2 \ln Z_{21t}) + a_{11} (\gamma_1 \ln Z_{12t} - \gamma_2 \ln Z_{22t}) + a_{11} (\delta_1 - \delta_2) \ln D_t \] (2b)

\[ S_{2t} = (1 - a_1) - a_{11} \ln \left( \frac{P_{1t}}{P_{2t}} \right) - a_{11} (\beta_1 \ln Z_{11t} - \beta_2 \ln Z_{21t}) - a_{11} (\gamma_1 \ln Z_{12t} - \gamma_2 \ln Z_{22t}) - a_{11} (\delta_1 - \delta_2) \ln D_t, \] (2c)

where

- \( S_{1t} \) = revenue share of railway mode on link \( t \), and
- \( S_{2t} \) = revenue share of highway mode on link \( t \).

Note that, as in Oum (1979b), we embedded the log-linear hedonic (quality-adjusted) price functions for rail and truck modes in the shippers’ translog unit cost function.

THE DATA

Data for Canadian inter-regional freight flows in 1979 are used for the empirical part of this study. We investigate both the total freight flows of all commodities and the flows of commodity group 14 (fruits, vegetables, and edible foods; henceforth referred to as CFTM 14 — see Table 1). This will help to identify the effect of aggregation over commodities on the relative performance of the four alternative models.

Freight transport demand models for traffic forecasting normally include generation factors in the origin and attraction factors in the destination, as well as impedance factors such as distance, freight rate and transit time, or a combination of these variables. (These variables are justified as the necessary components in a gravity-type model.) Both models for traffic forecasting and models for analysing the nature of demand usually include attributes of the competing modes such as prices, transit times, and reliability of service measures (see, for example, Quandt and Baumol, 1966, and Baumol and Vinod, 1970). The aggregate flow volume of each mode on each of the origin-destination pairs (henceforth referred to as links) is then regressed on the service attribute variables. For the cases of the aggregate logit model (1) and the translog demand system (2), there is no reason to include the attraction and the generation factors, as these models are concerned mainly with competition between modes instead of with traffic forecasting.

In addition to the variables defined in equations (2), the following variables are also required for estimating the four alternative models:

- \( V_{1t} \) = total railway freight tonnage on link \( t \).
- \( V_{2t} \) = total truck freight tonnage on link \( t \).
- \( W_{OUTt} \) = total inter-regional outflow volume emerging from an origin \( t \), to all destinations,

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### TABLE 1

**Commodities included in the CFTM commodity 14**

<table>
<thead>
<tr>
<th>CFTM 14</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>076</td>
<td>Dried and dehydrated fruits</td>
</tr>
<tr>
<td>078</td>
<td>Fruit juices, and fruit juice concentrates, not frozen</td>
</tr>
<tr>
<td>080</td>
<td>Fruit juice concentrates, frozen</td>
</tr>
<tr>
<td>082</td>
<td>Fruits and fruit preparation, n.e.s.</td>
</tr>
<tr>
<td>084</td>
<td>Nuts, except oil nuts</td>
</tr>
<tr>
<td>104</td>
<td>Vegetables, dried</td>
</tr>
<tr>
<td>106</td>
<td>Vegetables and preparations, n.e.s.</td>
</tr>
<tr>
<td>112</td>
<td>Sugar preparations (inc. confectionary), n.e.s.</td>
</tr>
<tr>
<td>114</td>
<td>Coffee</td>
</tr>
<tr>
<td>116</td>
<td>Cocoa and chocolate, tea, spices and vinegar</td>
</tr>
<tr>
<td>118</td>
<td>Margarine and similar products</td>
</tr>
<tr>
<td>120</td>
<td>Shortening and lard</td>
</tr>
<tr>
<td>122</td>
<td>Soups and infant and junior foods</td>
</tr>
<tr>
<td>124</td>
<td>Pre-cooked frozen food preparations</td>
</tr>
<tr>
<td>126</td>
<td>Food preparations and materials for food preparations, n.e.s</td>
</tr>
</tbody>
</table>

\[ WTIN_j = \text{total inter-regional inflow volume going to a destination } j, \]
\[ \text{from all origins,} \]
\[ W_{T,t} = \text{total freight volume on a link } t. \]

Since the empirical implementation has to be done separately for each commodity group, it is necessary to construct these variables for all commodity groups to be studied.

To construct all the relevant variables, it is essential to obtain the following data for each commodity group and for each link: distance of the link, total tons moved by railway mode, total tons moved by trucking mode, average railway freight rate, average trucking freight rate, rail mode’s average transit time, trucking mode’s average transit time, standard deviation of the rail mode’s transit-time distribution, and standard deviation of the trucking mode’s transit-time distribution.

In the remainder of this section we discuss in detail the sources from which these data were obtained and the ways in which they were used to construct the variables included in the models.

(a) **Freight rate and commodity flow data**

In Canada, confidential data on waybill records of actual freight shipments are kept by various government agencies and by certain carrier companies. Most of
the data available in the past were inappropriate for a multimodal demand study such as this, primarily owing to inconsistencies in data between modes with respect to classification of commodities, definition of links, units of measurement, and methods of sampling.

Fortunately, Petersen (1972) has developed the "Canadian Freight Transportation Model (CFTM) Data Base" which uses common systems of classifying commodities and of designating geographical regions of origin and destination. The CFTM data base uses 78 commodity groups (CFTM commodity codes), with cross references to the STCC (Standard Transportation Commodity Code) and SCC (Standard Commodity Classification) systems, and 69 geographic regions (CFTM Canadian regions) with cross references to the Statistics Canada Census Divisions, SGC (Standard Geographic Code) and CN/CP station numbers. This data base is maintained and updated by the Canadian Institute of Guided Ground Transport (CIGGT) of Queen's University.

The CFTM data base for the year 1979 was used to determine, for all commodities and for CFTM commodity 14 (fruits, vegetables and edible foods), the rail car-load and for-hire truck-load freight volumes in tons for each link, and the corresponding average freight rates charged by rail and truck modes on each link. Among the 4692 (= 69 x 68) potential Canadian integrational links, the number of links at which the two modes actually shared the traffic varied from commodity to commodity, and was 1,707 links for the aggregate of all commodities and 201 links for CFTM 14.

(b) Traffic generation and attraction factors

In this study, the total freight originated from the origin region is used as the generation factor, and the total freight attracted to the destination region as the attraction factor.

(c) Distance of the link

A major city was chosen in each CFTM region to be regarded as the centroid of the region. Each possible pair of CFTM regions was treated as a link forming part of the Canadian freight transport network. Both railway and highway distances between the major cities of each pair of regions were measured from handbook sources. The average of railway and highway distances was used as the distance measure of the link ($D_r$).

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3 Mileage information listed in CN/CP Regional Timetables was used to measure railway distances of links. An official Canadian Highway Map was used to measure highway distances.

4 As long as a shipper gets his cargo moved from one location to another, the fact that a certain mode has a longer or shorter distance than the other mode is of no direct significance to him, except for its impact on ton-mile freight rates and the quality attributes which enter directly in the model. For this reason, a common measure of distance ($D_r$) is used in this study. Links with distances that are significantly different between the two modes were eliminated from the data set, since on these links the distance would be the dominant factor determining the modal freight rates and quality attributes, and thus the modal choice decisions.
(d) Transit time and its variability

Since information on transit time is not available in the CFTM data base, we used other sources. The daily car movement records of the Canadian National railways (during October 1970) and Canadian Pacific railways (during March 1971) were used to compute the transit time of each car movement. These transit times were then multiplied by 0.96 for CN and by 1.10 for CP, in order to reflect the average changes in speed of freight trains on the two railways between the base year and 1979. The resulting data were used to compute the average transit time and the standard deviation of transit time for each link. For those links for which there were no car movements recorded during the observation periods, regression equations estimated from the available data were employed to generate average transit time and standard deviation of transit time, as was done in Oum (1979b).

For the data on trucking performance, 1971 survey information on the actual transit times of 1,274 truck-load shipments obtained by Turner (1975) was updated by multiplying transit times by 1.038 in order to reflect the change in the average speed of commercial trucking services between 1971 and 1979. For those links for which the transit time data were unavailable, the regression equations estimated from the available data were used to estimate average transit time and standard deviation of transit time.

(e) Mean values of some important variables

Though use of a model is essential because of potential joint relationships among the independent variables, the data themselves can be used to give intuitive support for the conclusions that are drawn from the model. Table 2 shows the sample means of important variables such as shares of revenue, tonnage and ton-mile, average freight rates per ton-mile, average speed, and average reliability of the two modes for the commodity group CFTM 14 and for the aggregate of all commodities.

The average trucking rates per ton-mile for CFTM 14 were more than double the average railway rates. This is probably due to the concentration of trucking activity on relatively short-haul traffic. The fact that the average trucking rate for all commodities is more than three times that of the average rail rate reflects, at least partially, the differences in composition of commodities and in length of haul on the two modes. Though the average speed of the railway mode was substantially lower than that of the trucking mode, the reliability measure (reciprocal of the coefficient of variation in transit time distribution, that is, mean/standard deviation) has been consistently higher for the railway mode.

5 The changes in average speeds of CN and CP freight trains during the period 1956–81 are described in Freeman, Oum, Tretheway and Waters II (1987), chapter 3.

6 The information on average trucking speeds was compiled from the U.S. Interstate Commerce Commission, Highway Form B Applications Statement Nos. 2C2-71 and 2C18-79. Similar data were not available for Canada.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Modes</th>
<th>Commodity Group</th>
<th>All Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CFTM 14</td>
<td></td>
</tr>
<tr>
<td>Revenue share</td>
<td>Railway</td>
<td>0.36</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Trucking</td>
<td>0.64</td>
<td>0.56</td>
</tr>
<tr>
<td>Tonnage share</td>
<td>Railway</td>
<td>0.31</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>Trucking</td>
<td>0.69</td>
<td>0.44</td>
</tr>
<tr>
<td>Total tonnage</td>
<td>Railway</td>
<td>4080</td>
<td>65,686</td>
</tr>
<tr>
<td></td>
<td>Trucking</td>
<td>8891</td>
<td>52,291</td>
</tr>
<tr>
<td>Average Rate</td>
<td>Railway</td>
<td>6.29</td>
<td>4.99</td>
</tr>
<tr>
<td>(cents per ton-mile)</td>
<td>Trucking</td>
<td>14.32</td>
<td>17.60</td>
</tr>
<tr>
<td>Speed in miles per day</td>
<td>Railway</td>
<td>150</td>
<td>154</td>
</tr>
<tr>
<td></td>
<td>Trucking</td>
<td>248</td>
<td>252</td>
</tr>
<tr>
<td>Reliability</td>
<td>Railway</td>
<td>4.42</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td>Trucking</td>
<td>2.04</td>
<td>2.04</td>
</tr>
<tr>
<td>Weighted average rate (cents per ton-mile)</td>
<td>9.73</td>
<td>8.01</td>
<td></td>
</tr>
</tbody>
</table>

**METHODOLOGIES FOR THE COMPARISON**

There is no general method for selecting the best model from a set of alternative models when these models do not have nesting relationships. Recently several alternative statistical procedures have been developed for testing “non-nested” hypotheses (see Davidson and MacKinnon, 1981, and Pesaran and Deaton, 1978, for two attractive procedures); but these tests are confined to cases where the dependent variables of the alternative models are identical in form. In our case, the four models have totally different dependent variables: volume for the linear model, logarithm of the volume for the log-linear model, logarithm of the ratio of truck volume to rail volume for the logit model (1), and weighted average unit freight cost per ton-mile and rail (or truck) revenue share, respectively, for the cost and revenue share equations of the translog demand system (2). This calls for a different and more general statistical procedure. However, we are not aware of the existence of anything of the kind, except that there is a formal procedure for comparing log-linear with linear models.

In this paper, we first choose the best model from the log-linear and linear models, and compare it with the logit and the translog models. In the absence of
a formal procedure, we will merely evaluate the models, as opposed to choosing the best one, by comparing reasonableness of the signs and magnitudes of the parameters of the model and various elasticity estimates. We admit that this is an arbitrary procedure, but it cannot be avoided in the current state of statistical knowledge.

We use two different approaches for comparing the empirical results across the alternative models. First, we compare the empirical results of the models which include all the price, speed and reliability variables for rail and truck modes, and the distance measure. This will allow us to isolate pure effects of the models. Second, we compare the empirical results of the models which are considered the best in their respective categories. This exercise will allow us to examine the extent to which the empirical results depend on the choice of functional form in a realistic setting, assuming that the researcher does a good job in selecting the variables for the model which is finally chosen. We apply a series of (conventional) nested tests to determine the final set of variables in each of the four demand models for rail and truck modes. We also eliminate the variables with wrong signs.

The test procedure used to compare the log-linear and linear models is based on the concept of the Box-Cox metric (Box and Cox, 1964).

Since the limiting value of the Box-Cox metric is

\[
\lim_{\lambda \to 0} \frac{V^\lambda - 1}{\lambda} = \ln V,
\]

and

\[
\frac{V^\lambda - 1}{\lambda} \quad \text{when } \lambda = 1
\]

both linear and log-linear models are nested in the following Box-Cox transformation of dependent and independent variables (Box and Cox, 1964):

\[
Y_i(\lambda) = \sum_j \beta_j x_{ij}(\lambda) + \sum_j \gamma_j z_{ij} + u_i,
\]

(3)

where \( V(\lambda) \) denotes the Box-Cox metric\(^7\)

\[
V(\lambda) = \begin{cases} 
\frac{V^\lambda - 1}{\lambda} & \text{for } \lambda \neq 0, \\
\ln V & \text{for } \lambda = 0,
\end{cases}
\]

\( X_{ij} \) is the \( i \)th observation on regressor \( j \) which can be sensibly logged, and \( z_{ij} \) is the \( i \)th observation on regressor \( j \) which cannot be sensibly logged, including the constant term, dummy variables, time trends, etc.

Both the linear and log-linear models are nested in the more general Box-Cox model, since the linear and log-linear models can be obtained simply by setting

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\(^7\) The general form of Box-Cox regression would allow a different value of \( \lambda_j \) for each variable. See, for example, Gaudry and Wills (1978) for such applications.
the value of the Box-Cox parameter, $\lambda$, to one and zero, respectively. One can simply choose the model which yields the higher maximum value of the likelihood function. However, since the Box-Cox function is the most general model of the three, it can be identified as the most appropriate functional form if the Box-Cox parameter estimate $\lambda$ is significantly different from one or zero. For this reason, in the remainder of this paper the Box-Cox regression model is also compared with the four alternative models we originally set out to compare.

Assuming normal errors, the log-likelihood function ($L$) corresponding to the Box-Cox model is

$$ L = -(n/2)\ln(2\pi) - n\ln(\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} u_i^2 + (\lambda - 1) \sum_{i=1}^{n} \ln Y_i, $$

where $u_i$ is implicitly defined as a function of $Y_i$ by equation (3) and $n$ is the number of observations. The last term in (4), which vanishes in the case of the linear model, is the log of the Jacobian of the transformation from $Y_i(\lambda)$ to $Y$.

We compare the three models (log-linear, linear and Box-Cox) by applying the asymptotic Chi-square test. The test statistic with one degree of freedom is $-2$: that is $(\ln LR - \ln LU)$ where $LR$ is the likelihood value of the restricted model and $LU$ is the likelihood value of the unrestricted model (see Theil, 1971, page 396, for the derivation).

The formulae for computing various elasticities from the five alternative models are presented below.

(a) Linear models

$$ F_{yk} = \frac{\partial Y}{\partial X_k} \frac{X_k}{Y} $$

where $F_{yk} = \text{elasticity of the ordinary (Marshallian) demand, } Y, \text{ with respect to the explanatory variable, } X_k$.

(b) Log-linear models

$$ F_{yk} = b_k, $$

where $b_k$ is the coefficient of variable $X_k$.

---

8 Like most previous studies, we assume that $u_i$ is normally distributed with mean zero and constant variance. Strictly speaking, this assumption may be untenable, since, except for certain values of $\lambda$, $Y_i(\lambda)$ cannot take on values less than $-1/\lambda$, while with normal errors there is always the possibility that the right-hand side of (5) may be less than $-1/\lambda$. This difficulty can be circumvented by truncating $u_i$ in some fashion (see Poirier and Ruud, 1979). For our case it seems reasonable to ignore the problem, because $E(Y_i)$ are expected to be large relative to 0. The possibility that $u_i$ will be negative and so large in absolute value as to make the right-hand side of (5) unacceptably small can be safely ignored.
ALTERNATIVE MODELS AND THEIR ELASTICITY ESTIMATES
T. H. Oum

(c) Box-Cox models

\[ F_{yk} = b_k \left( \frac{Y}{X_k} \right)^\lambda, \]  

(7)

where \( b_k \) is the coefficient of variables \( X_k \), and \( \lambda \) is the Box-Cox parameter.

(d) Logit models

Since logit models analyse modal market shares, we first compute the elasticities of market shares, using the “difference-form” logit model (1b) as follows:

\[ M(i, X_{ik}) = a_k \cdot X_{ik} (1 - VS_i), \quad i = \text{rail or truck} \]
\[ M(i, X_{jk}) = -a_k \cdot X_{jk} VS_j, \quad i \neq j \]  

(8)

where \( M(i, X_{jk}) \) is the elasticity of market share for mode \( i \) with respect to the \( X_{jk} \) attribute of mode \( j \), and \( VS_j \) represents volume share of mode \( j \).

Note that, at least for the case of binary choice (two modes), the logit models in both (1a) and (1b) satisfy the following internal consistency requirement advocated by Taplin (1982):

\[ \sum_{k=1}^{n} VS_k M_{kj} = 0 \]

This means that the share of trips diverted from the \( j \)th mode by a rise in the \( j \)th price will be exactly equal to the increase in the shares of the other modes.

Unless the total demand for freight services is fixed (that is, independent of prices and service levels), the above mode-split elasticities differ from the ordinary elasticities. Quandt (1968) has shown the relation between mode-split and ordinary elasticities for the two-mode case. Taplin (1982) extended it to the general \( N \)-mode case, and obtained the following relationship:

\[ F_{ij} = M_{ij} + \eta_j \]  

(9)

where

\[ F_{ij} = \text{ordinary elasticity of demand for mode } i \text{ with respect to price of mode } j, \]
\[ M_{ij} = \text{mode split elasticity of mode } i \text{ with respect to price of mode } j, \]
\[ \eta_j = \text{elasticity of demand for aggregate freight with respect to the price of mode } j. \]

Therefore, the computation of the \( F_{ij} \)'s requires a second stage elasticity of demand for aggregate freight with respect to the price of each mode. These elasticities were obtained by estimating a log-linear regression of the total flow volume (rail-truck combined) on the truck and rail freights in addition to other determinants of the total volume. The elasticity estimates for the aggregate freight were \(-0.736\) and \(-0.506\), respectively, for rail and truck freight rates, while those for the commodity group 14 were estimated at \(-0.331\) for rail and \(-0.814\) for truck freight rates.
Translog demand systems

In the context of our two-mode translog demand systems, Oum (1979b) derives the following expressions for elasticities:

\[
\sigma_{12} = \frac{a_{12}(S_1 \cdot \hat{S}_2) + 1}{(a_{11} - S_1 \cdot \hat{S}_1)S_1} \quad E_{ij} = \frac{(a_{ij} - \hat{S}_i \cdot \hat{S}_j)S_j}{a_{ij}S_j} \\
F_{ij} = \{\sigma_{ij} + \eta \cdot E(P_y, P_f)\}S_j \\
E_{ij}^1 = E_{ij}\beta_j \\
E_{ij}^2 = E_{ij}\gamma_j
\]

(10)

where

- \(\sigma_{12}\) = elasticity of rail-truck substitution;
- \(S_i\) = fitted expenditure share of the \(i\)th mode;
- \(E_{ij}\) = elasticity of Hicksian (compensated) demand for the \(i\)th mode with respect to price of \(j\)th mode;
- \(F_{ij}\) = elasticity of Marshallian (ordinary) demand for the \(i\)th mode with respect to price of the \(j\)th mode;
- \(E_{ij}\) = elasticity of demand for the \(i\)th mode with respect to the speed of the \(j\)th mode;
- \(E_{ij}\) = elasticity of demand for the \(i\)th mode with respect to the reliability of \(j\)th mode;
- \(a_{ij}\) = second order translog parameter; and
- \(\beta_j, \sigma_j\) = parameter estimates associated with speed and reliability variables, respectively, in the \(j\)th mode's hedonic aggregator,
- \(\eta\) = ordinary price-elasticity of shipper's output (assumed as \(-1.0\)),
- \(E(P_y, P_f)\) = elasticity of output price \((P_y)\) with respect to aggregate freight rate \((P_f)\), which is assumed at 0.1 for "all commodities", and 0.3 for "commodity 14".

**EMPIRICAL RESULTS**

This section presents the empirical results on model estimation in the following order. First, we outline the models for aggregate freight. This is followed by a discussion of the models for CFTM 14 (fruits, vegetables and edible foods). The relative performance of the five models will be compared both for the commodity group CFTM 14 and for aggregate freight, in order to observe effects of the aggregation level.

(a) Results on model estimation for aggregate freight

All the model results reported in this sub-section were obtained with the data for 1,707 links. Table 3 reports the demand results of the linear models for both truck and rail modes. The first and third columns are the truck and rail demand models, respectively; these include the price, speed, reliability and distance variables for both modes, regardless of their statistical significance or the signs
of the coefficients. Note that most of the coefficients of the speed and reliability variables have incorrect signs and/or are statistically insignificant. The second and fourth columns are the best linear models; they include only the variables with correct signs.

The log-linear models are reported in Table 4. All the speed and reliability variables in column 1 have incorrect signs. They are therefore eliminated from the best truck model in column 2. Similarly the rail demand model in column 3 has the wrong sign for the trucking rate variable; so this variable too is eliminated from the best model in column 4.

The Box-Cox models presented in Table 5 have the same format as in Tables 3 and 4. The results of these models are very similar to those of the log-linear models. This is to be expected, since the estimates of the Box-Cox parameter are very close to zero in all cases. In addition, the log-linear models have a consistently higher log-likelihood value than the linear model (see Table 6). The result of the likelihood ratio tests for model selection conducted in Table 6 is that, for both rail and truck demands, the log-linear models are rejected in favour of Box-Cox models at any reasonable level of significance.

The “difference-form” logit models are reported in Table 7. Again the signs of the coefficients of the relative speed and reliability variables in column 1 are incorrect, and thus they are excluded in column 2. In contrast, all the coefficients of the translog demand system reported in Table 8 are statistically significant and
have correct signs (see the signs of the elasticities reported in Table 9). The translog results are also very stable.

In short, for the purpose of analysing aggregate freight demand, the translog demand system appears to be the best of the models examined. If we were to rank the performance of the models, the Box-Cox form would take second place and the log-linear form would be third. The linear and linear logit models perform rather poorly in this application. In particular, the coefficients of all the quality-of-service variables have incorrect signs when these variables are included in the linear and logit models.

In order to compare the reasonableness of the demand elasticities obtained from the various models, Table 9 presents the demand elasticities evaluated at the mean data points, using each of the five models studied (including the Box-Cox model). The ordinary cross-price elasticities obtained from the logit model are negative. This counter-intuitive result occurs even though the logit model in Table 7 has the correct (negative) sign for the price coefficient. This is because $\eta_j$'s (elasticity of demand for aggregate freight with respect to mode $j$'s price) in equation (9) have negative values: $-0.736$ and $-0.506$, respectively, for rail and truck freight rates. The own-price and own-quality elasticities for ordinary demand obtained from the Box-Cox and the log-linear models appear to be on the high side. The elasticities associated with the translog and linear models have more reasonable magnitudes. Since the linear model excluded all quality of service variables, the empirical results on prices are likely to be biased. An inspection of
TABLE 5

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent = Truck Volume</th>
<th>Dependent = Rail Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Variables Included</td>
<td>Chosen Variables</td>
</tr>
<tr>
<td>[( \lambda = 0.05 )]</td>
<td>0.536 (21.92)</td>
<td>0.575 (21.57)</td>
</tr>
<tr>
<td>[( \lambda = 0.04 )]</td>
<td>0.435 (19.21)</td>
<td>0.474 (19.35)</td>
</tr>
<tr>
<td>Distance</td>
<td>-3.020 (0.97)</td>
<td>-2.120 (43.32)</td>
</tr>
<tr>
<td>Truck rate</td>
<td>-1.689 (31.16)</td>
<td>-1.570 (30.24)</td>
</tr>
<tr>
<td>Rail rate</td>
<td>0.553 (5.55)</td>
<td>0.584 (6.23)</td>
</tr>
<tr>
<td>Truck speed</td>
<td>2.173 (0.60)</td>
<td>2.113 (4.95)</td>
</tr>
<tr>
<td>Rail speed</td>
<td>-6.041 (4.02)</td>
<td>-5.631 (3.97)</td>
</tr>
<tr>
<td>Truck reliability</td>
<td>27.058 (1.06)</td>
<td>27.058 (1.06)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-7.429 (0.28)</td>
<td>6.70 (7.45)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-16491</td>
<td>-16523.7</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1707</td>
<td>1707</td>
</tr>
</tbody>
</table>

TABLE 6

Results of Box-Cox Metric Test of Linear, Log-linear and Box-Cox Models for All Commodities

<table>
<thead>
<tr>
<th>Modes</th>
<th>Box-Cox Parameter ( \lambda )</th>
<th>Log-likelihood</th>
<th>Significance level of ( \chi^2 ) statistic for log-linear vs. Box-Cox comparison</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>0 (log-linear)</td>
<td>-16542.0</td>
<td>(( \chi^2 = 36.6 )) less than</td>
<td>Favours log-linear model but rejects log-linear in favour of the general Box-Cox model</td>
</tr>
<tr>
<td></td>
<td>1 (linear)</td>
<td>-23501.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04 (Box-Cox)</td>
<td>-16523.7</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>0 (log-linear)</td>
<td>-17633.7</td>
<td>(( \chi^2 = 19.8 )) less than</td>
<td>Favours log-linear over linear model but rejects log-linear in favour of the general Box-Cox model</td>
</tr>
<tr>
<td></td>
<td>1 (linear)</td>
<td>-24135.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04 (Box-Cox)</td>
<td>-17623.8</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

\( \chi^2 \) statistics are computed as follows:
\(-2 \left[ \log \text{likelihood (} \lambda = 0; \text{log-linear}) - \log \text{likelihood (} \lambda \neq 0; \text{Box-Cox}) \right]\)
### TABLE 7

"Difference Form" Logit Model for All Commodities  
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent: Log of Truck/Rail Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Variables Included</td>
</tr>
<tr>
<td>Freight rate*</td>
<td>-0.04283 (18.69)</td>
</tr>
<tr>
<td>Speed**</td>
<td>-0.01121 (8.12)</td>
</tr>
<tr>
<td>Reliability***</td>
<td>-4.9415 (13.24)</td>
</tr>
<tr>
<td>Distance</td>
<td>-0.0011 (11.47)</td>
</tr>
<tr>
<td>Total tonnage</td>
<td>0.000001 (8.87)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-9.292</td>
</tr>
</tbody>
</table>

R-squared 0.35  
Number of observations 1707

* Difference between truck and rail freight rate.  
** Difference between truck and rail speed.  
*** Difference between truck and rail reliability.

### TABLE 8

Translog Model for All Commodities  
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail rate</td>
<td>0.543</td>
</tr>
<tr>
<td>Truck rate</td>
<td>0.457</td>
</tr>
<tr>
<td>Rail speed</td>
<td>-13.262</td>
</tr>
<tr>
<td>Truck speed</td>
<td>-1.400</td>
</tr>
<tr>
<td>Rail reliability</td>
<td>-39.509</td>
</tr>
<tr>
<td>Truck reliability</td>
<td>-2.017</td>
</tr>
<tr>
<td>Rail distance</td>
<td>11.008</td>
</tr>
<tr>
<td>Truck distance</td>
<td>1.988</td>
</tr>
<tr>
<td>(Rail freight)²</td>
<td>-0.047</td>
</tr>
<tr>
<td>(Truck freight)²</td>
<td>-0.047</td>
</tr>
<tr>
<td>Rail freight x truck freight</td>
<td>0.047</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.863</td>
</tr>
</tbody>
</table>

Log-likelihood -893.013  
Number of observations 1707
### TABLE 9

Elasticities for 1979 All Commodities  
(Evaluated at means of Variables: t-statistics in parentheses)

<table>
<thead>
<tr>
<th>Elasticities</th>
<th>Translog</th>
<th>Log-linear</th>
<th>Linear</th>
<th>Box-Cox</th>
<th>Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRH</td>
<td>1.190 (59.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERR</td>
<td>-0.544 (60.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH</td>
<td>-0.646 (43.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR</td>
<td>-0.598 (74.8)</td>
<td>-1.517 (18.7)</td>
<td>-0.638</td>
<td>-1.384</td>
<td>-0.830</td>
</tr>
<tr>
<td>FRH</td>
<td>0.498 (124.5)</td>
<td>0.059</td>
<td></td>
<td></td>
<td>-0.175</td>
</tr>
<tr>
<td>FHH</td>
<td>-0.692 (46.1)</td>
<td>-1.341 (31.9)</td>
<td>-0.048</td>
<td>-1.140</td>
<td>-0.928</td>
</tr>
<tr>
<td>FHR</td>
<td>0.592 (84.6)</td>
<td>0.453 (6.2)</td>
<td>0.838</td>
<td>0.403</td>
<td>-0.616</td>
</tr>
<tr>
<td>ERR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>7.210 (7.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-0.761 (23.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>0.904 (5.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-8.566 (36.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>7.930 (7.5)</td>
<td>24.365 (5.3)</td>
<td></td>
<td></td>
<td>16.007</td>
</tr>
<tr>
<td>FRH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-0.697 (34.9)</td>
<td>-4.076 (4.2)</td>
<td></td>
<td></td>
<td>-3.822</td>
</tr>
<tr>
<td>FHH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>0.968 (5.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-7.846 (54.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>21.478 (5.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-1.096 (26.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>1.303 (6.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-25.520 (24.8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>23.624 (5.2)</td>
<td>127.720 (5.0)</td>
<td></td>
<td></td>
<td>90.964</td>
</tr>
<tr>
<td>FRH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-1.004 (40.2)</td>
<td>-3.850 (3.7)</td>
<td></td>
<td></td>
<td>-3.945</td>
</tr>
<tr>
<td>FHH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>1.395 (5.9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-23.375 (38.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*SRH* = elasticity of rail-truck substitution  
*E<sub>ij</sub>* = compensated elasticity of demand for *i*th mode with respect to freight rate of *j*th mode  
*F<sub>ij</sub>* = ordinary elasticity of demand for *i*th mode with respect to freight rate of *j*th mode  
*E<sub>ij1</sub>* = compensated elasticity of demand for *i*th mode with respect to speed of *j*th mode  
*E<sub>ij2</sub>* = compensated elasticity of demand for *i*th mode with respect to reliability of speed of *j*th mode  
*F<sub>ij1</sub>* = ordinary elasticity of demand for *i*th mode with respect to the *j*th mode's speed  
*F<sub>ij2</sub>* = ordinary elasticity of demand for *i*th mode with respect to the *j*th mode's reliability  
*R* = rail mode  
*H* = truck mode
the elasticities obtained from the translog function at all the links suggests that the
theory-based model is highly robust. Overall, the elasticity estimates also seem
to favour the translog model over the other four models.

The elasticity of substitution between rail and truck modes of 1.19 for the
aggregate total commodity indicates that the two modes are highly substitutable
on an average link (O-D pair). The magnitudes of all the own and cross-price
elasticities for the ordinary and compensated demands of the two modes com-
puted from the translog demand system appear to be reasonable. For example,
the ordinary rail demand has an own-price elasticity (FRR) of −0.598 and a cross-
price elasticity (FRH) of 0.498, while for the ordinary demand for trucking the
corresponding figures are −0.692 and 0.592. The speed elasticities of both
compensated and ordinary demands for rail and truck indicate that the speed of
the rail mode is a far more important determinant of both rail and truck volumes
than the truck speed. In other words, change in rail speed has far more impact on
the demands for both modes than a similar change in trucking speed. The reli-
ability elasticities also reveal that the demands for both modes are far more
affected by a change in reliability of rail service than by a similar change in the
reliability of trucking service. In sum, the quality of service associated with the
rail mode is a key determinant of traffic allocation between the two modes.

(b) Results of model estimation for commodity group 14

The data for 201 links were used for estimating the CFTM 14 models. For the
sake of saving space, the main results are described without presenting the co-
efficient tables. With the exception of the rail reliability variable in the trucking
demand equation, all the service quality variables were dropped from the linear
model because they had incorrect signs and/or were not statistically significant.
Furthermore, coefficients of the price for the competing mode and distance
variables were not statistically significant.

As in the log-linear model, we found that in the truck demand equation the
coefficients of the rail freight rate, rail speed, truck reliability, and distance
were not statistically significant, but the reliability of delivery time of rail was
an important variable. It is consistent with this result that the magnitude and
reliability of both rail and trucking speed were important determinants of rail
freight demand, while trucking rate and reliability were not statistically significant
in that equation.

The same set of variables as in the log-linear models was left in the finally
chosen Box-Cox model. In both rail and truck demand models the price of the
competing mode was statistically significant. The truck demand equation included
its own freight rate and the reliability of service of rail, while the speed of
trucking service was only marginally significant. The rail rate, rail speed, the
reliability of truck transit time, and distance were not statistically significant
in the trucking demand equation. The rail demand was dependent upon the
distance and its own freight rate, speed and reliability. The Box-Cox parameter
estimates were 0.07 for the truck model and 0.00 for the rail model.

The log-likelihood values indicated superiority of the log-linear models over
their linear counterparts. The likelihood ratio test indicated that the Box-Cox
model was better than the log-linear model for the trucking demand. For rail demand, however, the log-linear model could not be rejected in favour of the Box-Cox model. In fact, the two models were identical, because the Box-Cox parameter was exactly zero.

As in the "difference-form" logit model (1b), both reliability and distance variables were excluded because the signs of their coefficients were found to be incorrect. Differences in freight rates and speed were statistically significant in the logit model.

As in the case of the analysis of aggregate freight demand, all coefficients of the translog demand system had correct signs and were at least marginally significant.

In sum, the translog demand system has again produced the most reasonable results, and the linear model appears to be the poorest (that is, all the quality-of-service variables are insignificant and/or have the wrong signs). The Box-Cox model appears to be the second best, while the logit and the log-linear models share third place. These three models include some of the quality-of-service variables in their final equations.

Table 10 reports various elasticities of demand for rail and truck, evaluated at the mean data points. A comparison of the elasticities computed from the five different models reveals the following:

The estimates of cross-price elasticities for ordinary demands obtained from the logit model are negative (a counter-intuitive result) because of the large negative values for η_j's in equation (9): −0.331 for rail mode and −0.814 for truck mode.

The Box-Cox and log-linear models yielded elasticities with similar magnitudes. All these elasticities are larger, and some are significantly larger, than those obtained from the translog system. Perhaps some are too large to be realistic (see, for example, rail mode's own reliability elasticity of 243).

Overall, the translog system generates what appear to be realistic elasticities for all variables considered. This conclusion is strengthened by the fact that the elasticities for all observations exhibit stability and predictability, and their standard errors are significantly smaller than those of the log-linear models.

All four models which included the reliability measures produced very high elasticities; this implies that reliability is the key for determining the traffic shares for the two modes. Furthermore, the demands for both modes are far more sensitive to rail speed and reliability than to the speed and reliability of the truck mode.

In general, for the CFTM 14 commodity group (fruits, vegetables and other edible foods) the speed and reliability variables are far more effective competitive tools than the freight rates. Again the overall performance of the translog function appears to be the best in terms of its ability to generate all the elasticities that it is necessary to measure.
## TABLE 10

**Elasticities for Commodity 14 (Fruits, Vegetables and Other Edible Foods)**
(Evaluated at means of Variables: *t*-statistics in parentheses)

<table>
<thead>
<tr>
<th>Elasticities</th>
<th>Translog</th>
<th>Log-linear</th>
<th>Linear</th>
<th>Box-Cox</th>
<th>Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRH</td>
<td>1.147 (16.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERR</td>
<td>-0.688 (16.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH</td>
<td>-0.459 (12.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR</td>
<td>-0.796 (18.9)</td>
<td>-0.795 (2.8)</td>
<td>-0.391</td>
<td>-0.795</td>
<td>-0.484</td>
</tr>
<tr>
<td>FRR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>0.495 (45.0)</td>
<td></td>
<td></td>
<td></td>
<td>-0.466</td>
</tr>
<tr>
<td>FHH</td>
<td>-0.652 (18.6)</td>
<td>-1.542 (9.0)</td>
<td>-0.318</td>
<td>-1.248</td>
<td>-0.970</td>
</tr>
<tr>
<td>FHR</td>
<td>0.351 (39.0)</td>
<td></td>
<td></td>
<td></td>
<td>-0.262</td>
</tr>
<tr>
<td>ERR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>15.914 (2.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-2.285 (6.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>1.523 (2.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-10.607 (5.9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>18.413 (2.3)</td>
<td>26.559 (2.3)</td>
<td></td>
<td>26.561</td>
<td>2.52*</td>
</tr>
<tr>
<td>FRR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-1.644 (5.8)</td>
<td>-8.795 (1.9)</td>
<td></td>
<td>-8.776</td>
<td>-4.15*</td>
</tr>
<tr>
<td>FHH&lt;sup&gt;1&lt;/sup&gt;</td>
<td>2.166 (2.8)</td>
<td>3.892 (1.8)</td>
<td></td>
<td>2.808</td>
<td>2.34*</td>
</tr>
<tr>
<td>FHR&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-8.119 (6.2)</td>
<td></td>
<td></td>
<td></td>
<td>-1.41*</td>
</tr>
<tr>
<td>ERR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>44.589 (1.9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-4.127 (6.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>2.751 (2.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EHH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-29.720 (5.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>51.592 (2.0)</td>
<td>243.388 (2.2)</td>
<td></td>
<td>243.41</td>
<td></td>
</tr>
<tr>
<td>FRR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-2.969 (6.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHH&lt;sup&gt;2&lt;/sup&gt;</td>
<td>3.911 (3.0)</td>
<td></td>
<td></td>
<td></td>
<td>-30.269</td>
</tr>
<tr>
<td>FHR&lt;sup&gt;2&lt;/sup&gt;</td>
<td>-22.750 (5.2)</td>
<td>-48.563 (5.4)</td>
<td></td>
<td>-40.324</td>
<td></td>
</tr>
</tbody>
</table>

The elasticity notations are defined as in Table 9.
* Since the modal speed variables were not statistically significant in the total volume (rail and truck combined) equation, these ordinary demand elasticities for speed variables are in fact the same as the share elasticities.

## CONCLUSIONS

The theoretical investigations, statistical tests and empirical results on elasticity estimates carried out in this paper lead us to conclude that:

For both the aggregate freight and the reasonably homogeneous commodity group (fruits, vegetables and edible foods), the variables included in the model finally chosen and the elasticity estimates depend crucially on the...
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T. H. Oum

functional form being used for estimation. This implies that choice of
functional form for the demand (forecasting) model is very important for
empirical research.

Of the five alternative models (including the Box-Cox model) examined in
this paper, the translog demand system with theoretical constraints imposed
on the parameters clearly performs the best in all respects. The Box-Cox
model is placed second in this overall ranking, and the log-linear model is
a close third. The linear logit and linear regression models perform rather
poorly in this application. This conclusion is based on the variables included
in the final models, the ability to measure various elasticities, the reasonableness
of the elasticity estimates, threoretical consistency and some statistical
tests.

Obviously these conclusions are tentative pending further investigation. Further
investigation is necessary, mainly for two reasons: (i) explicit empirical tests were
not used to compare the five models, except for comparing the linear, log-linear
and Box-Cox models, and (ii) use of more general forms of the Box-Cox model
and of the logit model (the generalised logit and universal logit) would be likely
to improve their relative performance.

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