WELFARE IMPLICATIONS OF THE OMISSION OF INCOME EFFECT IN MODE CHOICE MODELS

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INTRODUCTION

In their general specification, demand models are intended to represent analytically the effect of different variables on the quantities of goods or services requested by an individual or by groups of individuals. In applications, those variables are usually the price of the good, prices of related goods (substitutes or complements), and income. In a model of the choice of discrete alternatives within a class of goods or services, qualitative characteristics are also included as independent variables. In particular, usual specifications of mode choice models do include both prices and qualities of all modes involved. Income also appears frequently in these models, but it is included there only as a proxy for other variables, such as taste (after McFadden, 1981) or a wage rate (after Train and McFadden, 1978). Thus, income normally does not represent purchasing power in mode choice models; therefore the income effect is not accounted for. This has been shown to be an incorrect approach, at least in the context of developing countries, and some reformulations have already been suggested (Jara-Díaz and Farah, 1987).

The failure of income to reflect the constraint associated with purchasing power in mode choice models does not necessarily cause an inadequate replication of observed choice; nor will it generate particularly bad forecasts in the presence of income effects. After all, the variable is there. However, the failure to recognise income as such has two implications which may be of some importance in the following contexts:

(i) the specification of the model may be microeconomically inconsistent; that is, it may not fulfil theoretical properties such as Roy's identity (see Viton, 1985);

(ii) the welfare analysis will be incorrect; that is, money measures of variation of utility are different when income influences choice.

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In this paper we shall deal with the qualitative impact on welfare measures of the omission of income effect in mode choice models. To do this, in the next section we present a method of calculating the Hicksian measures of welfare (compensating or equivalent variation) directly from market demands. Then we shall show that those measures can be viewed as comprising two parts, one representing an approximation of the Marshallian consumers' surplus variation, and one representing the effect of income on welfare. This result is applied to the analysis of the true benefits of modal price variations from mode choice models. The conditions under which users' benefits are underestimated at a mode choice level are established, with particular emphasis on the role of the (mode dependent) marginal utility of income. This is illustrated with an example using previously estimated figures. The role of income in the different levels of transport-related decisions is briefly discussed, and directions for future research are suggested.

**APPROXIMATION OF HICKSIAN MEASURES FROM MARKET DEMANDS WITH AN INCOME EFFECT**

Both Hicksian measures, the compensating and equivalent variations of income, are defined as income equivalents of price changes. In particular, the compensating variation \( CV \) corresponds to that variation in income which neutralises the effect of a price change, in the sense of leaving utility at the original level, \( U_0 \). Formally, if prices drop from \( P^0 \) to \( P^1 \)

\[
U_0 = V(P^0, I_0) \equiv V(P^1, I_0 - CV)
\]

(1)

where \( V \) is the indirect utility function, \( I_0 \) is income, \( P \) is the price vector, 0 and 1 denote two states (before and after the price change).\(^1\) Since the expenditure function \( e(P, U) \) is simply the inverse of \( V(P, I) \) in \( I \), it can easily be shown (Jara-Diaz and Farah, 1988) that

\[
CV = e(P^0, U_0) - e(P^1, U_0).
\]

(2)

The expenditure function \( e(P^1, U_0) \) can be approximated through a second order Taylor expansion from \( e(P^0, U_0) \), that is,

\[
e(P^1, U_0) \approx e(P^0, U_0) + \sum \left( \frac{\partial e(P, U_0)}{\partial P_i} \right)_{P^0} \Delta P_i + \sum \frac{\partial^2 e(P, U_0)}{\partial P_i \partial P_j} \left. \right|_{P^0} \Delta P_i \Delta P_j
\]

(3)

where \( \Delta P_i = P_i^1 - P_i^0 \). Using the derivative property \( \frac{\partial e}{\partial P_i} = H_i \), where \( H_i \) is the compensated (Hicksian) demand, we get from equations (2) and (3)

\[
CV = - \sum H_i(P^0, U_0) \Delta P_i - \sum \frac{\partial H_i(P, U_0)}{\partial P_i} \left. \right|_{P^0} \Delta P_i \Delta P_j.
\]

(4)

\(^1\) As we want \( CV \) to be a representation of utility variation, we have defined it in such a way that decreasing (increasing) prices makes it positive (negative), since utility increases (decreases). This is not a trivial point, since the Marshallian consumers' surplus, which will later appear in our derivation, works in the same direction. The sign \( CV \), then, gives the sign of utility variation.
We know that the price derivative of a market demand fulfills the Slutsky equation, that is,

\[
\frac{\partial X_i(P, I)}{\partial P_i} = \frac{\partial H(P, U)}{\partial P_i} - \frac{\partial X_i(P, I)}{\partial I} \cdot X_i(P, I),
\]

where \( U = V(P, I) \) is the utility level which can be reached at prices \( P \) with income \( I \). Solving for \( \frac{\partial H_i}{\partial P_i} \), replacing in equation (4) and noting that \( H_i(P^0, U_0) = X_i(P^0, I_0) \), we get

\[
CV = - \sum_i X_i(P^0, I_0) \Delta P_i - \frac{1}{2} \sum_{i,j} \frac{\partial X_i(P^0, I_0)}{\partial I} \left| \begin{array}{c} \Delta P_i \\ \Delta P_j \end{array} \right| \left. \Delta P_i \Delta P_j \right|_{I^0} - \frac{1}{2} \sum_{i,j} \left. \frac{\partial X_i(P^0, I_0)}{\partial I} \right| \left. X_i(P^0, I_0) \Delta P_i \Delta P_j \right|_{I^0},
\]

Equation (6) then is proposed as an approximation of the compensating variation after a price change, expressed only in terms of market demands, including income effect. In a sense, the result obtained by Glaister (1979), in his approximation of the measure of the welfare loss due to distorted prices, can be looked at as a particular case of ours, since he only accounted for the price effect. In our derivation the Slutsky equation plays a key role, since it provides the link between the (unobserved) compensated demands and the (observed) market demands.

Now we shall show that the expression obtained for \( CV \) can be readily interpreted in terms of more traditional measures. To see this, let us recall that Hotelling's (1938) generalisation of the Marshallian consumers' surplus variation (\( \Delta MCS \)) is given by the line integral

\[
\Delta MCS = - \int_{P^0}^{P^0} \sum_i X_i dP_i,
\]

which would yield an exact measure of welfare if there were no income effect. It can be shown, by a simple expansion of the demand about the initial price vector, that the first two terms of equation (6) represent an approximation of \( \Delta MCS \), provided that second order effects of prices on demand are negligible and the Jacobian matrix of the vector of market demands, evaluated at \( P^0 \), is symmetrical (see Appendix). Thus, the approximated measure of \( CV \) has two components: the traditional welfare measure that would be used if income effect were not taken into account, and an income-induced welfare impact (IWI) given by the last term in equation (6), that is,

\[
CV = \Delta MCS - \frac{1}{2} \sum_{i,j} \left. \frac{\partial X_i(P^0, I_0)}{\partial I} \right| \left. X_i(P^0, I_0) \Delta P_i \Delta P_j \right|_{I_0} = \Delta MCS + IWI
\]
Finally, it should be noted that a similar result can be obtained for the equivalent variation $EV$, by simply expanding the expenditure function around $(P^1, U_1)$.

**BENEFITS FROM MODAL CHOICE CHANGES**

Changes in modal split due to price or quality variations in one or more modes do provide benefits to users, even if total trips remain constant (that is, if distribution is unchanged). Many authors have analysed this problem from different viewpoints. The best known result is the log-sum measure derived by Williams (1977), McFadden (1981) and Small and Rosen (1981), within the framework provided by the random utility approach and the theory of discrete choices; that measure corresponds to the expected maximum utility under the logit specification, divided by the marginal utility of income $\lambda$ implicitly or explicitly assumed constant (independent of prices and qualities of modes). In this section we shall derive an expression for the benefits to users from a logit mode choice model, based upon our previous derivation. The $IWF$ part will then be analysed with particular emphasis on the role of $\lambda$. First, though, we shall show that the result obtained in the previous section remains valid within the discrete choice framework. We can do this by using the link between the analyses of continuous and of discrete consumption provided by the work of Small and Rosen (1981).

Small and Rosen view the discrete choice problem in its entirety: that is, which good $i$ to choose among a discrete set of alternatives, and how much of this good to buy ($X_i$). Under this approach, they extend the relation between the price effect on market and compensated demands, given by the Slutsky equation, to account for the probability of choosing discrete alternative $i$, $\pi_i$. They derive what they call “a Slutsky-like equation for the aggregate demand functions” (p. 117); they do this for the own price case, but replication of their procedure allows for a generalisation which yields

$$\frac{\partial H_i}{\partial P_i} = \frac{\partial X_i}{\partial P_i} + \frac{X_i}{\pi_i} \left[ \frac{\partial X_i}{\partial I} - \frac{\partial \pi_i}{\partial I} \frac{X_i}{\pi_i} (1 - \pi_i) \right]. \tag{9}$$

In the case of mode choice with distribution assumed fixed (that is, number of total individual trips $N$ is constant, but mode choices are allowed to change), we can use $X_i = N\pi_i$, which reduces equation (9) to

$$\frac{\partial H_i}{\partial P_i} = N \left[ \frac{\partial \pi_i}{\partial P_i} + N\pi_i \frac{\partial \pi_i}{\partial I} \right]. \tag{10}$$

This makes equation (6) valid in the discrete framework, at an aggregate level, if $X_i$ is understood as $N\pi_i$. The modified expression is

$$CV \overset{N}{=} -\sum_i \pi_i^0 \Delta P_i - \frac{N}{2} \sum_{i \neq j} \frac{\partial \pi_i^0}{\partial P_i} \Delta P_i \Delta P_j - \sum_i \frac{N}{2} \sum_{j \neq i} \pi_i^0 \Delta P_i \Delta P_j, \tag{11}$$
where $\partial \pi_i^0 / \partial \theta$ means $\partial \pi_i / \partial \theta$ evaluated at $P^0, I$.

It should be made clear, though, that the discrete case represented by equation (11) (one good) differs from the general case in equation (6) (all goods) in an important aspect: all terms $\partial X_i / \partial I$ can be positive, but necessarily some terms $\partial \pi_i / \partial I$ will be positive and some negative, since $\Sigma \pi_i = 1$ always. Secondly, $N$ is the number of trips during the same reference period in which $I$ is earned; this will be important later, when we introduce the marginal utility of income.

Let us apply the result represented by equation (11) to the (popular) logit mode choice model. As we know, the logit specification comes from the random utility theory, assuming modal utilities to be centred on an observed function $V_i$ and randomly distributed Weibull. This yields mode choice probabilities $\pi_i$ given by

$$\pi_i = e^{V_i / \Sigma_j e^{V_j}}$$

(12)

Since $V_i$ is potentially a function of mode price $P_i$, mode qualities $q_i$, and income, $\pi_i$ is a function of all prices $P$, all qualities $Q$ and income; that is, $\pi_i(P, Q, I)$. From equation (12) it is a fairly simple matter to show that

$$\frac{\partial \pi_i^0}{\partial P_i} = \begin{cases} -\pi_i^0 \pi_j^0 \frac{\partial V_i^0}{\partial P_i} & \text{for } j \neq i \\ \pi_i^0 (1 - \pi_i^0) \frac{\partial V_i^0}{\partial P_i} & \text{for } j = i \end{cases}$$

(13)

$$\frac{\partial \pi_i}{\partial I} = \pi_i (\lambda_i - \bar{\lambda}),$$

(14)

where $\lambda_i$ is mode $i$’s marginal utility of income ($\partial V_i / \partial I$) and $\bar{\lambda}$ is the expected marginal utility ($\Sigma \pi_i \partial V_i / \partial I$), both evaluated at the initial state.

Equation (14) clearly shows that a mode-independent marginal utility of income makes $NWIF$ equal to zero. This is just the welfare counterpart of the relation between the absence of income effect and constancy of $\lambda$ in the context of mode choice (Jara-Diaz and Videla, 1989a).

If the conditional indirect utility function $V_i$ is well defined, it should fulfil Roy’s identity at a conditional level; that is,

$$- \frac{\partial V_i}{\partial P_i} / \frac{\partial V_i}{\partial I} = 1 \quad \text{or} \quad \frac{\partial V_i}{\partial P_i} = -\lambda_i.$$

(15)

This is indeed guaranteed by construction, since the conditional utility-maximising problem is stated in terms of a conditional budget constraint $I - P_i$. However (and this is an important point overlooked in mode choice theory), income $I$ in equation (15) should refer to the same period as the market demand for the relevant (discrete) good, that is, one trip. In other words, equation (15) holds for $N = 1$ in equation (11). Replacing equations (13), (14) and (15) in equation (11) yields

$$CV = - \Delta P (1 + \lambda \Delta P - \frac{1}{2} \lambda \Delta P^2) + \frac{1}{2} \lambda \Delta P^2,$$

(16)
where the bar denotes expected value of a variable or product of variables (for example, $\bar{\lambda} \Delta P = \sum \pi_i \lambda_i \Delta P_i$), evaluated at the initial state.

Alternatively, accounting for the random distribution of utility, a stochastic version of Roy's identity can be used, namely (Hau, 1987)

$$-rac{\partial V_i}{\partial P_i} / \sum \pi_i \frac{\partial V_i}{\partial I} = 1 \vee \frac{\partial V_i}{\partial P_i} = -\bar{\lambda}.$$  \hspace{1cm} (17)

If this is replaced along with equations (13) and (14) in expression (11), we obtain

$$CV' = -\bar{\Delta P}(1 + \frac{1}{2} \bar{\Delta P}) + \frac{1}{2} \bar{\Delta P} \bar{\Delta P}.$$  \hspace{1cm} (18)

Equations (16) and (18) are operative expressions which can be used to evaluate the welfare gain (loss) following a price decrease (increment). The validity of each will depend upon the type of Roy's identity imposed. The information required consists of the initial mode shares, the price variations, and estimates of the (mode-dependent) marginal utilities of income; note that if $\lambda_i$ were constant across modes both measures would coincide (as expected).

The impact of the income effect on welfare does not depend on which formula is used. In either case $IW$ is given by

$$IW = -\frac{1}{2} \sum \pi_i \lambda_i (\lambda - \bar{\lambda}) \Delta P_i \Delta P_j = -\frac{\Delta P}{2} (\bar{\lambda} \Delta P - \bar{\lambda} \Delta P).$$  \hspace{1cm} (19)

The evident relationship between the value of $IW$ and the set of values $\lambda_i$ can be used to analyse its relative importance in different income groups. For high income levels, we expect $\lambda_i$ to be small and constant across modes, as the differences $I - P_i$ are dominated by the large value of $I$, so that the influence of modal price is negligible. As stated earlier, this causes $IW$ to be close to zero. Departures from zero are caused by a set of different values $\lambda_i$, that is, truly mode-dependent marginal utilities of income. However, we have to recall that we are dealing with welfare analysis at a mode choice level; equation (19) shows that the effect of income on welfare at this level requires the existence of a reasonably meaningful set of alternatives. To see this, assume one mode (say $k$) was dominant (for example, captive users), then $\bar{\lambda}$ would approximate $\lambda_k$ and $\pi_j$ would be close to zero $V_j \neq k$, which tend to make $IW$ very small or null for price variations in any mode. And this phenomenon is independent of the income level; however, in the case of reduced mode choice sets in low income environments, we would expect the income effect to emerge at another choice level, be it destination or number of trips.\(^2\) Note that $IW$ would also vanish if all modal costs varied by the same amount. Thus, though the percentage of income spent in transport (or the importance of $P_i$ in the difference $I - P_i$) is indicative of the presence of an income

\(^2\) This has indeed occurred in the experiences documented by Raczynski and Serrano (1985), which we have cited elsewhere: real increments in public transport fares have caused poor families in Santiago, Chile, to start buying from the (more expensive) nearby retailers and to stop sending their children to school.
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effect, it is by no means unimportant to establish at which level it is influencing individual behaviour (that is, in generation, distribution or mode choice). In this sense, our method to detect the presence of an income effect specifically in mode choice (Jara-Díaz and Videla, 1989a) has generated some figures which will be particularly useful in the context of our analysis here.

In order to have an idea how relevant the \textit{IWI} can be, let us assume a price change in a single mode \(i\). In this case, the incidence of \textit{IWI} on total welfare variation can be easily shown to be given by either

\[
e = \frac{\text{IWI}}{CV} = \frac{\frac{1}{2} \pi_1 \Delta P_i (\lambda_i - \bar{\lambda})}{1 + \Delta P_i (\pi_i \lambda_i - \frac{1}{2} \lambda_i - \frac{1}{2} \lambda \pi_i)}
\]  

(20)

or

\[
e' = \frac{\text{IWI}}{CV'} = \frac{\frac{1}{2} \pi_1 \Delta P_i (\lambda_i - \bar{\lambda})}{1 + \frac{1}{2} \Delta P_i (\pi_i \lambda_i - \bar{\lambda})}.
\]  

(21)

As estimates for \(\lambda_i\) are required we will use figures and relations from Jara-Díaz and Videla (1989a). They studied the population of a southern zone of Santiago (Chile) in 1985; about one third of the sample had family incomes below the national average (but above the poorest 20 per cent). For that subsample, the marginal utility of income was estimated to be

\[
\lambda_i = 0.374 - 0.00482 P_i.
\]  

(22)

Using this empirical relation and equations (20) and (21), we can construct a simplified example, assuming two modes (public, private) with prices equal to Chilean $30 and $70, respectively, and the aggregate modal split for Santiago 85 per cent and 15 per cent. Although these are rough figures for 1985, they represent a fair picture and yield a value of \(\bar{X} = 2.050\), very close to the 0.1979 value reported in our paper.\(^3\) Assuming a reduction of $10 in the public transport fare, these figures would yield 11.7 and 12 per cent for \(e\) and \(e'\) respectively (9.6 and 6.2 per cent if only the private transport cost were reduced by the same amount). Unfortunately, we cannot analyse here the order of magnitude of \textit{IWI} with respect to the erroneous welfare measure that would be obtained by applying the Marshallian measure to an estimated mode choice model traditionally specified. The reason for this is simply that the resulting log-sum measure rests upon the estimated marginal utility of income (minus the coefficient of cost) whose bias is as yet unknown. In assessing the importance of \textit{IWI} in other realities, one should recall that Chile is not among the poorest countries of the world, and that the population in the example is not particularly poor within the Chilean scene.

\(^3\) Though this might not seem too important, consistency in the numbers is required. Modal probabilities, prices and marginal utilities are indeed linked, so that other combinations simply might not occur.
SYNTHESIS, FURTHER ANALYSIS AND CONCLUSIONS

We have derived an approximation of the Compensating Variation (CV) in terms of market demands, where careful use of the Slutsky equation plays an important role. Viewed in this way, the CV has been interpreted as a sum of the traditional Marshallian welfare measure and an income-induced welfare impact (IWI), which represents the contribution of the income effect to the valuation of users' benefits. This result has been extended to the case of discrete choices, by using the generalisation of a result obtained by Small and Rosen (1981). We have applied this extension to the popular logit mode choice model, showing that the impact of a variation of mode fares on the welfare of low-income users will normally be underestimated if the IWI is not taken into account.

As stated at the beginning of this paper, though income frequently appears in the specification of mode choice models, it is there as a proxy for other variables. Once the presence of income is recognised as reflecting purchasing power, it is a mistake not to take it into account in valuing users' benefits. This makes it a relevant task to search for well defined models which can capture the income effect; and this is important because of the welfare analysis not less than because of the need for good forecasts of demand. Our equation (11) can be used as a general approximation to evaluate benefits on the basis of our general result represented by equation (6), not previously reported in the literature. We have shown that, on the assumption of a logit model structure, this method requires only estimates of mode-specific marginal utilities of income. The method we developed previously to detect income effect in mode choice (Jara-Díaz and Videla, 1989a) can be used for this purpose, as is shown here.

The development of income-sensitive specifications for mode choice models is indeed one of the next tasks for this area of research. The expenditure rate models suggested by Jara-Díaz and Farah (1987) and the suggestions of Viton (1985) and Hau (1987) are steps in this direction. Once a specific income-sensitive model is available, duality can be used to calculate correct measures of welfare in an ad hoc manner. Within this line, the methods of numerical inversion of an aggregate representative utility function (Hau, 1987; Jara-Díaz and Videla, 1987) and the construction of an expected minimum expenditure function (Jara-Díaz and Videla, 1990) can be useful. As we have said, analytically explicit specifications of utility and choice models are required for the application of these methods.

The general expression represented by equation (11) suggests yet another relevant task, which is the analysis of bias in the estimated price elasticity in mode choice or, ultimately, in the coefficient of mode cost in the (indirect conditional) modal utility. To see this, imagine a situation in which the income effect is present in mode choice but the modeller either ignores or decides to ignore it; then the welfare measure after a price variation will be essentially determined by the (incorrectly) estimated sensitivity to price $\frac{\partial \pi_i}{\partial P_i}$. How this relates to the addition of a correct estimation of $\frac{\partial \pi_i}{\partial P_i}$ and the income term from a correctly specified (income-sensitive) model is a matter for econometrical experimentation and comparison.

In synthesis, we have unveiled the essential elements behind the role of income
in the welfare analysis from mode choice models, where the mode-dependent marginal utility of income becomes a key variable. In this sense, it is important to distinguish between the problem presented and discussed here and the impact of income at other levels of choice (such as generation or distribution). We suggest the development of income-sensitive mode choice models, and the assessment of the quantitative influence of \( I/W \) in different socioeconomic environments, as the immediate next steps in this field of research.\(^4\)

**APPENDIX**

**Approximation of the Marshallian Consumers’ Surplus Variation, \( \Delta MCS \)**

Hotelling’s generalisation of the Marshallian consumers’ surplus is given by the line integral

\[
\Delta MCS = - \int \sum_{P^o_i} \frac{\partial X_i}{\partial P_j} dP_l
\]

Expanding \( X_i \) to the first order in prices around \( P^o \) yields

\[
X_i \approx X_i (P^o, I) + \frac{\partial X_i}{\partial P_j} \bigg|_{P^o} (P_j - P^o_j)
\]

such that

\[
\Delta MCS \approx - \int \sum_{P^o_i} \frac{\partial X_i}{\partial P_j} dP_l - \int \sum_{P^o_i} \left[ \frac{\partial X_i}{\partial P_j} \bigg|_{P^o} (P_j - P^o_j) \right] dP_l.
\]

The first line integral in (c) is exact, and given by \( \sum X_i \Delta P_l \).

The second integral \( IN_j \), however, has a unique value iff (Green’s Theorem)

\[
\frac{\partial}{\partial P_k} \left( \frac{\partial X_h}{\partial P_j} \right) |_{P^o_j} (P_j - P^o_j) = \frac{\partial}{\partial P_k} \left( \frac{\partial X_k}{\partial P_j} \right) |_{P^o_j} (P_j - P^o_j)
\]

Since the only variables on each side are the prices (the price derivatives of demand are evaluated at \( P^o \)), only one term of each summation will survive. Thus, equation (d) is equivalent to

\[
\left. \frac{\partial X_h}{\partial P_k} \right|_{P^o} = \left. \frac{\partial X_k}{\partial P_h} \right|_{P^o}
\]

which is the condition for the second line integral in (c) to be unique. Note that

\(^4\) We have considered some of these issues in two recent articles (Jara-Díaz and Videla, 1989b; Jara-Díaz and Ortúzar, 1989).
this resembles the condition for $\Delta MCS$ in (a) to have a unique value; but (e) is less restrictive, since in (a) the condition has to be fulfilled in the whole range $P^0 \rightarrow P^1$, while in (e) it should hold only at $P^0$.

In this case one can choose a seriatim path of integration, moving one price at a time, with the other prices held constant at either the initial or the final level, depending on the order of integration. Thus,

$$IN_2 = \sum_i \int_{P^1}^{P^0} \left[ \sum_j \frac{\partial X_i}{\partial P_j} (P_j - P^0_j) \right] dP_i$$

$$P_j = P^0_j \quad \forall j > i$$

$$P_j = P^1_j \quad \forall j < i$$

When we work out the right hand side of equation (f), the line integral happens to be given by

$$IN_2 = \frac{1}{2} \sum_i \left[ \frac{\partial X_i}{\partial P_i} (P^1_i - P^0_i) + \sum_{j < i} \frac{\partial X_i}{\partial P_j} (P^1_j - P^0_j)(P^1_i - P^0_i) \right]$$

Finally, consistently applying equation (e),

$$IN_2 = \frac{1}{2} \sum_i \left[ \frac{\partial X_i}{\partial P_i} (P^1_i - P^0_i)(P^1_i - P^0_i) \right].$$

Therefore

$$\Delta MCS = \sum_i X_i \Delta P_i + \frac{1}{2} \sum_i \left[ \frac{\partial X_i}{\partial P_i} (P^1_i - P^0_i)(P^1_i - P^0_i) \right].$$

The approximation rests upon negligible second order effects of prices on market demands (equation b) and local symmetry of the Jacobian $X = \{X_i\}$, at $P^0$. Note that if line integral (a) is unique, only the first assumption is additionally required.

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