A THEORETICAL COMPARISON OF
COMPETITION WITH OTHER ECONOMIC
REGIMES FOR BUS SERVICES

By Andrew Evans*

1. INTRODUCTION

If a bus route is open to any number of operators, and they must operate without
any form of collusion or co-operation, will there be a stable pattern of services?
If so, what sort of pattern will it be, and how will it compare with services
operated under other regimes?

These are important questions for Britain, which is about to introduce free
competition in the provision of bus services after 55 years of regulation. In due
course empirical evidence will become available about its effects, but economic
models will still be needed to interpret and evaluate this evidence. In this paper
we consider a basic theoretical model to deal with these questions, and make
some theoretical estimates of the order of magnitude of important features.

We suppose that all operators have equal costs, and that demand and costs
are the same under all regimes. This may not be true in practice; indeed, one of
the Government's principal arguments in favour of deregulation was that costs
would fall (Department of Transport, 1984). However, the effect of deregulation
on costs is a separate question from that of the operation of regimes discussed
here, and it seems most useful for comparative purposes to make the neutral
assumption of common costs under all regimes.

We suppose also that all operators and potential passengers have complete
information about the bus service and fares, and that operators have information
about demand and (common) costs.

The paper continues as follows. Section 2 describes the basic model and
assumptions. Section 3 presents the analysis of a time segment from an infinitely
long day for each of the four economic regimes. Section 4 presents numerical
results based on the analysis of Section 3. Section 5 extends the model and
numerical results to finite time cycles. Section 6 presents the conclusions. There
is one appendix.

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2. BASIC MODEL

We consider a bus route carrying people in one direction between two points. Each potential passenger has a preferred time of departure; preferred departure times have a uniform density throughout the day, so that the number of passengers whose preferred departure times fall in any period of length $\delta t$ is $L\delta t$. Buses depart at discrete moments, so that most passengers have to travel at different times from those which they prefer; we suppose that the cost to them of this rescheduling is proportional to the difference between the preferred and actual times, with constant of proportionality $c$.

We suppose initially that the day is infinitely long, and consider a segment of it. This avoids specific issues concerning the first and last buses; it also avoids issues that otherwise arise from the fact that the number of buses in the day must be an integer. We later consider a cycle of finite length, with the end joined to the beginning. This means that we still avoid the former issues, relating to the first and last buses, but we do consider the integral frequency.

We suppose that in the absence of rescheduling there is an exponential demand curve for travel: the proportion of the $L\delta t$ potential passengers whose preferred departure time is between $t$ and $t + \delta t$ who would actually travel on a bus departing at $t$ is $\exp(-f/v)$, where $f$ is the fare and $v$ is a parameter. Another way of regarding the demand curve is as specifying the distribution of the gross valuations which potential passengers place on their journeys. The proportion of potential passengers who value their journeys at between $w$ and $w + \delta w$ is $\{\exp(-w/v)\delta w\}/v$, and the proportion who value their journeys at $w$ or more is $\exp(-w/v)$. The mean valuation is $v$. The reason for adopting an exponential distribution is that this is commonly used for describing bus demand (for example, Glaister, 1984; Evans, 1985). In addition, the exponential demand curve has some nice mathematical properties, though the equations resulting from it still require numerical solution.

We suppose that the cost to an operator of running a bus on the journey comprises a fixed component $F$ together with an additional marginal cost of $m$ per passenger. There are no costs of entry to or exit from the industry.

We suppose that all buses have enough capacity to carry all passengers who wish to travel at the prevailing fares and times. This again may not be so in practice, and restrictions on capacity could be important in some circumstances. It is likely that the models discussed here could be adapted to include them, though this would be at the cost of added complication.

This model has the same structure as some specifications of models of spatial competition, on which there is a useful literature which can be applied here. Spatial competition is concerned with competition among suppliers (for example, shops) of a homogeneous product, who are located at specific points in a continuum of potential demand (for example, a street). Customers must pay not only the price of the product but also the cost of transporting it from the supplier to their own location. This cost is usually taken as proportional to distance. Suppliers may enter or leave the market, may choose their locations, and may choose the price at which they sell the product. The general questions on spatial competition are whether stable patterns of locations and prices exist, and if so
what are their properties. Among the conclusions of spatial competition models is that stable equilibria may not exist in some circumstances, and that whether they exist or not often turns on the precise details of the choices presumed to be available to the participants in the market and the way in which they interact.

The parallel between bus and spatial competition is that in bus competition time corresponds to one-dimensional location; bus departure times correspond to the location of suppliers; passenger rescheduling costs correspond to transport costs. I am indebted to Foster and Golay (1986) for pointing out this connection. The papers in spatial competition I have found most relevant are those of Salop (1979) and Novshek (1980). Spatial competition is part of oligopoly theory, which is reviewed in the books by Friedman (1983) and Waterson (1984).

**Economic regimes to be compared**

We compare the following economic regimes for a single route.

(a) *Competition.* In competition it is the operators or potential operators who make all the decisions. They decide whether to enter the market at all, at what time(s) to run buses, and what fares to charge. Each operator makes his or her decisions independently. Each operator seeks to maximise profits.

(b) *Breakeven maximum net economic benefit* is a planned service giving maximum net economic benefit subject to the constraint that profit and subsidy are zero. In this regime the decisions are made by a public authority.

(c) *Monopoly* allows only one operator to provide a service. The decisions are made by the operator, who maximises profits.

(d) *Unconstrained maximum net economic benefit* is a planned service giving maximum net economic benefit without restriction on the subsidy level. The decisions are made by a public authority.

We first consider these regimes applied to a representative time segment of a day which is infinitely long, and then, as mentioned above, to a time cycle of finite length with the end joined to the beginning. The difference is that, with an infinitely long day, headways may be of any length, but with a cycle of finite length only discrete headways are permissible.

Our main output measure for the regimes is net economic benefit; that is, consumer surplus plus producer surplus (or minus producer loss), or, in other words, net benefit of the service, as valued by participants, without regard to who are the beneficiaries. Consumer surplus is the gross valuation of their journeys by consumers, less their fares and rescheduling costs. Producer surplus is the operators' profit (if any) or, with a minus sign, subsidy. It should be stated at this point that we adopt the usual convention in this context of including "normal profit" as part of the operators' costs, so that zero profit means "normal profit". Any positive profit is "superprofit", additional to normal profit.

In addition to net economic benefit, we are interested in its separate components: consumer surplus, measuring what the travelling public get out of the service, and producer surplus, measuring superprofits, or, if negative, subsidy.
We are also interested in the fares and headways produced by the different regimes, though these are of interest mainly as descriptive measures, to help us to understand what is happening, rather than because they are important in themselves.

Assumptions about the behaviour of participants

As mentioned above, assumptions about how participants make their decisions are important in determining what bus services are provided. We consider potential passengers first.

Potential passengers

Our principal assumption is that potential passengers know the departure times and fares of all the buses. In principle, each potential passenger calculates the sum of the rescheduling cost and the fare for all buses, identifies the bus for which the sum is smallest, and travels if, and only if, this smallest sum is less than the value placed on the journey. In practice, this is not such a demanding computational task as it seems, because the only buses in the running are those departing immediately before and immediately after a passenger's preferred time. Also, under the assumptions of our model, in each regime all buses have the same fare.

This model of passenger behaviour assumes that passengers are indifferent between forward and backward rescheduling. An alternative model is that passengers do not know the timetable; they arrive at the bus stop at their preferred travel time, or perhaps at a fixed interval ahead of their preferred travel time, and travel on the first subsequent bus. This might apply to high frequency services. It seems a reasonable alternative assumption, and is often implicitly or explicitly assumed in public transport models. We do not pursue it in this paper, largely because it takes us away from the most basic model. It is a model in which some participants (the passengers) have incomplete information, but it is unreasonable to suppose that passengers know nothing, and the analysis of this kind of model depends on just what information we do impute to passengers. Another feature of this model is that under competition at a simple-minded level there is clearly no equilibrium, because operators can always unilaterally increase their profits by retiming their buses to run just ahead of their competitors' buses. However, they cannot expect their competitors not to respond to this, and there may be an equilibrium if we impute to operators an expectation of a response.

Operators

This brings us to the operators. We again suppose that they have full information about demand, their own costs, and other operators' costs. Operators are decision-makers in two of the four regimes — competition and monopoly. They have to make three decisions: whether to enter the market at all (or whether to leave it); at what time(s) to operate their bus(es); and what fares to charge. Under monopoly these decisions are straightforward. Only one operator is allowed to operate, and we suppose that the monopolist chooses fares and headways so as to maximise profit. We suppose the monopolist enters the market and provides a service if, and only if, the maximised profit is at least zero.
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It is under competition that assumptions about the behaviour of operators become more interesting and important, because, though we make the general assumption that operators seek to maximise profit, there are many different decision rules or strategies that they could adopt. This is because the market is an oligopoly, in which the decisions of one operator may directly affect the fortunes of other operators. Each operator therefore has to have a presumption about how affected operators will react to his or her own decisions. Different presumptions are possible, and these may lead to different outcomes. However, rather than start with the presumptions of operators, we adopt the common approach in this context of asking whether a non-co-operative equilibrium exists in the competitive regime. That is, we ask whether there exists a set of bus departure times and fares, which has the property that no operator or potential operator is able to increase profit unilaterally by entering or leaving the market, by changing his or her departure time, or by changing the fares. If there is such an equilibrium, we may expect it to persist once it is achieved, since no profit-maximising operator has reason to make a decision to change it. In considering what unilateral decisions would be profitable, each operator still has to make presumptions about the reactions of other operators, so that the nature and existence of an equilibrium still depends on these presumptions.

The fact that some presumptions by operators may lead to the attainment and maintenance of equilibrium, while others do not, does not necessarily mean that operators adopt the presumptions which lead to equilibrium. That would be a non sequitur. The plausibility of the presumptions of operators must be judged on their merits. Nevertheless, it is plausible that the possibility of equilibrium does influence operators' reactions, and therefore their presumptions about other operators' reactions. Disequilibrium has costs for operators, in the form of uncertainty and possible short-term losses, and because change itself brings costs, such as those of rescheduling staff and vehicles and of distributing new timetables to customers. Therefore, if there is an equilibrium, operators may have an independent mutual interest in adopting strategies which lead to it, and may suppose that other operators will do so too.

Novshek (1980) and others have shown that in the spatial context there is an equilibrium in our model if operators have the following decision rules. First, once operators are in the market, they choose their departure times and fares so as to maximise profit, taking the departure times and fares of other operators as fixed. This is sometimes called "zero conjectural variation". The one exception to this is that operators presume that if they were to decide to adopt the same departure time as that of another operator and charge a slightly lower fare, thereby capturing the second operator’s entire market, the second operator would respond with a fare cut to recapture the market. Novshek calls this "modified zero conjectural variation". Without this modification there is no equilibrium, but with it, and with a fixed number of operators in the market, these decision rules give a unique equilibrium, in which no operator could unilaterally increase profit by changing either departure time or fare. The equilibrium is symmetric, which means that it has equal headways between buses and equal fares on all buses. The maximised profit in this equilibrium may be less than, equal to, or more than zero, according to the number of operators in the market.
The second decision rule concerns entry to or exit from the market. Some existing operators leave the market if the maximised profit discussed above is negative. The remaining operators then adjust fares and headways till they are in equilibrium, and in the segment from the infinitely long day the process continues till the maximised profit is zero, or, in other words, till all the survivors earn the normal profit. Potential entrants enter the market if by doing so they can earn at least the normal profit. In the infinitely long day the signal that they could do so is that the existing operators are earning positive profit (that is, superprofit). In responding to this signal, potential entrants have to be rather more farsighted than just taking the existing departure times and fares as given. This is because a new bus has to be inserted into the time interval between two existing buses, and its insertion creates two shorter-than-normal headways. These short headways will make the new bus unprofitable at first, and it is only after the existing operators have adjusted their fares and timetables that the new bus will begin to earn the normal profit. The decision rule is that potential entrants base their decision to enter on the equilibrium headway, and not on the headways which their own bus will have immediately after entry. With the finite time cycle to be discussed in section 5, potential entrants have the same decision rule, to enter if, and only if, they would earn at least the normal profit after the post-entry fares and headways have reached equilibrium; but the decision is more complicated, because the earning of superprofits by existing operators is no longer a signal that a new operator could do so too.

These decision rules lead to a unique symmetric equilibrium both in the segment from the infinitely long day and in the finite time cycle. It is a separate question whether operators would actually adopt these rules in practice. In this paper, by concentrating on the properties of the equilibrium and comparing them with those of the other regimes, we implicitly assume that the operators would indeed adopt these rules, or something similar. The justification for this assumption is, first, that the rules themselves are not unreasonable, and secondly, as explained above, the fact that they do lead to equilibrium might itself be a reason for their adoption.

The public authority
The final participant is the public authority, which determines fares and headways in the two regimes seeking net economic benefit. We assume that, like the other participants, the public authority has full information about demand and costs, and is able to use this information to specify and implement the optimal services.

Symmetry
As mentioned above, the bus service in competitive equilibrium is symmetric; that is, it has equal headways between all buses and equal fares on all buses. In other regimes the bus services resulting from maximisation of net economic benefit or profit are likewise symmetric. Symmetric services are characterised by just two variables—headway (or frequency) and fare. For each regime we shall therefore calculate the headway and the fare. From these we calculate the
quantities of other aspects of concern — patronage, revenue, cost, profit or subsidy, consumer surplus and net economic benefit.

3. ECONOMIC ANALYSIS OF A SEGMENT OF AN INFINITELY LONG DAY

Consider a time segment from an infinitely long day covering the departure times of three successive buses. Let each of the intervals between the buses be \( h \); let the fare charged by the first and last of the three buses be \( f' \); let the fare charged by the middle bus, which we take as the representative bus, be \( f \). We shall eventually want \( f' \) and \( f \) to be equal (symmetry), but for the moment we want to allow that they may be different. The position is illustrated in Figure 1. For all regimes we are interested in \( f \) and \( h \).

**Demand**

We now consider demand for travel on the representative bus as a function of its own fare, \( f \), taking the fare on the neighbouring buses as fixed at \( f' \). Potential passengers choose to travel, if they travel at all, on that bus for which the total generalised cost (fare plus rescheduling cost) is lowest. Consider the potential passengers whose preferred departure time is \( u \) minutes later than the departure time of the representative bus, where \( 0 \leq u \leq h \). The generalised cost to them of travelling on the representative bus is \( f + cu \), and on the following bus is \( f' + c(h-u) \). The boundary between the markets for the two buses will be the
time at which these generalised costs are equal: that is, that value of \( u \) for which
\[
f + cu = f' + c(h - u)
\]
giving
\[
u = (f' - f + ch)/2c
\]
(1)

Because rescheduling costs affect all potential passengers in the same way, all those whose preferred departure time is up to \( u \) minutes later than that of the representative bus will travel on it, if they travel at all; all those whose preferred departure time is later than this will travel on a later bus.

This argument overlooks the possibility that \( f \) could be less than \( f' - ch \). In that case the representative bus would not only take all passengers whose preferred departure time was between its own departure time and that of the following bus, but also capture the entire market of the following bus, which would be left without any market. This could not happen in equilibrium, because the following bus would have negative profits, but it might be a source of instability. However, a fare as low as that is always unprofitable in this model, so the possibility does not have to be pursued.

Now consider potential passengers with preferred departure time between \( z \) and \( z + \delta z \) minutes later than that of the representative bus, where \( z \leq u \). There are \( L \delta z \) of them. Their generalised cost of travel (on the representative bus) is \( f + cz \). They will travel if, and only if, they value their journeys at more than this generalised cost. From the exponential demand model discussed in section 2, the proportion who do so is \( \exp[-(f + cz)/v] \), and therefore the number of actual passengers on the representative bus from this preferred time band is
\[
L \exp[-(f + cz)/v] \delta z, \quad (2)
\]

To find the total number of passengers, \( q \), we integrate this expression over all time bands \( z \) from 0 to \( u \) (where \( u \) is given by (1)), and then multiply by 2 to include the corresponding passengers with preferred departure times earlier than that of the representative bus. Thus
\[
q = 2L \int_0^{(f' - f + ch)/2c} \exp[-(f + cz)/v] \, dz. \quad (3)
\]

This is a tractable integral. The result is
\[
q = \frac{2Lv}{c} e^{-f/v} \left(1 - e^{-(f' - f + ch)/2v \right)} \quad (4)
\]

This is the key demand model, which plays a crucial role in what follows. It gives the patronage of a representative bus in terms of the fare on the representative bus, the fares on other buses, and the headway.
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Consumer surplus

One of the nice properties of the basic exponential demand model is that consumer surplus is proportional to patronage, with constant of proportionality \( v \). Therefore, the consumer surplus of the passengers from preferred time band \( z \) to \( z + \delta z \) is the patronage from this time band, given by (2), multiplied by \( v \).

The consumer surplus, \( s \), of all the passengers on the representative bus is again got by integrating over time bands. The integral is the same as (3) with a multiplying constant \( v \), and therefore the answer is

\[
s = vq = \frac{2Lv^2}{c} e^{-f/v} \left( 1 - e^{-\left( f - f' + ch \right)/2v} \right)
\]  

(5)

Cost

As stated in section 2, we assume that all operators have the same costs, \( g \), comprising a fixed component \( F \) per journey, which accounts for the greater part of the total, and a small marginal cost of \( m \) per passenger. Thus

\[
g = F + mq
\]  

(6)

Revenue, profit, net economic benefit

Revenue per bus, \( r \), is \( fq \). Profit per bus, \( p \), is the difference between revenue and cost. Thus

\[
p = r - g = (f - m)q - F
\]  

(7)

Net economic benefit per bus, \( b \), is consumer surplus plus profit. Thus

\[
b = s + p = (v + f - m)q - F
\]  

(8)

With symmetric services, we can also express patronage, consumer surplus, revenue, cost, profit, and net economic benefit per minute by dividing the corresponding quantities per bus by the headway, \( h \). We can then also scale by dividing by \( L \) to get the quantities per potential passenger; this is useful for comparative purposes.

Competitive equilibrium

We are now ready to work out the equations to be satisfied by the \( f \)'s and \( h \)'s under the various economic regimes. We begin with the competitive equilibrium, which is the most complicated and least familiar.

We assume that the operator of the representative bus chooses its departure time and fare so as to maximise profit, taking the departure times and fares of other buses as given. The decision about departure time is straightforward: the departure time should be the mid point of the interval of length \( 2h \) (given) between the neighbouring buses. Shifting from the mid point loses more passengers at one end of the market than it gains at the other. The headway between this bus and each of the neighbouring buses is therefore \( h \), and the demand is given by (4). Profit is given by (7) as \((f - m)q - F\). This is to be maximised by
choice of \( f \). The first-order condition for the maximum is
\[
\frac{dp}{df} = q + (f - m) \frac{dq}{df} = 0
\]
By differentiating the demand model (4) with respect to \( f \), we have
\[
\frac{dq}{df} = -\frac{2L}{c} e^{-f/v}(1 - \frac{1}{2}e^{-\left(f' - f + ch\right)/2v})
\]
Substituting for \( dq/df \) gives
\[
q = \frac{2L}{c}(f - m) e^{-f/v}(1 - \frac{1}{2}e^{-\left(f' - f + ch\right)/2v}) = 0
\]
Having obtained this equation by keeping \( f \), the fare with respect to which profit is maximised, separate from \( f' \), taken as fixed, we now say that in symmetric equilibrium \( f \) and \( f' \) will be equal. We therefore set \( f' = f \) in both the demand equation and the condition above, to get respectively
\[
q = \frac{2L}{c} e^{-f/v}(1 - e^{-ch/2v}) \tag{9}
\]
and
\[
q - \frac{2L}{c}(f - m) e^{-f/v}(1 - \frac{1}{2}e^{-ch/2v}) = 0 \tag{10}
\]
The remaining condition for competitive equilibrium is that, when maximised, the profit under competition is zero. As explained in Section 2, we assume that if the maximised profit were more than zero new operators would enter the market till the profit was reduced to zero, and if the maximised profit were less than zero some operators would leave. The equation for zero profit is
\[
(f - m)q - F = 0 \tag{11}
\]
Equations (9), (10) and (11) must all be satisfied in competitive equilibrium, and together determine the values of \( q \), \( f \), and \( h \). There is a unique solution for all parameter values for which a bus service can be operated without subsidy. These equations can be simplified by changing the variables, as is done in the Appendix, but they still require numerical solution. Therefore in the next section we take forward the comparison of regimes numerically. But first we consider the equations to be satisfied by the other economic regimes.

**Breakeven maximum net economic benefit**

In all the other economic regimes we assume that the fares and headways are controlled by a single body and set together. We therefore set \( f' = f \) in the demand equation before any maximisation. Demand for each bus is now given by (9). We want to find the \( f \) and \( h \) which maximise net economic benefit per minute, subject to the breakeven constraint (11). This is the familiar process of balancing fares and service levels, similar to that discussed in Nash (1978) and Glaister (1984). We form a Lagrangian, \( L \), of net economic benefit per minute minus the
constraint (11) multiplied by a Lagrange multiplier, which without loss of generality we can call \( \lambda / h \). That is

\[
\mathcal{L} = \frac{1}{h} [vq + (f - m)(1 - \lambda)q - (1 - \lambda)F]
\]  

(12)

The first order conditions for the constrained maximum are that the differentials of \( \mathcal{L} \) with respect to \( f \) and \( h \) are zero. These give

\[
\frac{\partial \mathcal{L}}{\partial f} = \frac{1}{h} [v \frac{\partial q}{\partial f} + (f - m)(1 - \lambda) \frac{\partial q}{\partial f} + (1 - \lambda)q] = 0
\]  

(13)

and

\[
\frac{\partial \mathcal{L}}{\partial h} = -\frac{1}{h^2} [vq + (f - m)(1 - \lambda)q - (1 - \lambda)F] + \frac{1}{h} \frac{\partial q}{\partial h} [v + (f - m)(1 - \lambda)] = 0
\]  

(14)

By differentiating the logarithm of (9) we have that

\[
dq/df = -q/v
\]  

(15)

Substituting for \( dq/df \) in (13) and simplifying gives

\[
f = m + \frac{\lambda v}{\lambda - 1}
\]  

(16)

By differentiating (9) with respect to \( h \) we have

\[
\frac{\partial q}{\partial h} = Le^{-f/v}e^{-ch/2v}
\]  

(17)

Substituting for \( f \) from (16) and \( dq/dh \) from (17) into (14) leads after some simplification to

\[
hvLe^{-f/v}e^{-ch/2v} - vq + F = 0
\]  

(18)

This equation, the demand equation (9), and the breakeven constraint (11) are the three equations which determine the \( q, f, \) and \( h \), which give the breakeven maximum economic benefit. Two of the three equations, (9) and (11), are the same as the corresponding equations determining the competitive equilibrium, but we now have equation (18) in place of equation (10) as the third.

**Monopoly**

In monopoly we assume that all the buses are operated by a single operator, who is free to maximise profit without constraints, and who is protected (by some mechanism) from the entry of competitors. This may not be realistic in general, but it is important to see what results it produces. Demand model (9) applies; there is no budget constraint; and the operator chooses \( f \) and \( h \) so as to maximise profit per minute, given by

\[
\frac{p}{h} = (f - m)q - \frac{F}{h}
\]
The first-order conditions for the maximum are
\[
\frac{q}{h} + \frac{(f-m)}{h} \frac{\partial q}{\partial f} = 0 \quad (19)
\]
and
\[
- \frac{1}{h^2} [(f-m)q - F] + \frac{1}{h} (f-m) \frac{\partial q}{\partial h} = 0 \quad (20)
\]
Substituting for \( dq/df \) from (15) into (19) gives
\[
f = v + m \quad (21)
\]
The monopoly price is thus a constant, depending only on the parameters \( v \) (average valuation of journeys) and \( m \) (marginal cost per passenger). Substituting for \( f \) from (21) and for \( dq/dh \) from (17) into (20) gives
\[
h v L e^{-f/v} e^{-c h/2 v} v q + F = 0 \quad (22)
\]
This condition and the demand equation (9) together determine \( q \) and \( h \). Equations (9) and (22) for monopoly are identical to the corresponding equations (9) and (18) for breakeven maximum net economic benefit. However, since the fare is determined differently in the two regimes, the solutions to the sets of equations are different.

**Maximum net economic benefit**

In this case we again assume a single (subsidised) operator; the planning authority chooses \( f \) and \( h \) so as to maximise net economic benefit per minute without a budget constraint. We spare ourselves the details of the maximising argument, which are similar to those for monopoly. The demand model (9) applies, and the other two equations for \( q, h \) and \( f \) are
\[
f = m \quad (23)
\]
and, again,
\[
h v L e^{-f/v} e^{-c h/2 v} v q + F = 0 \quad (24)
\]
In this case the fare is constant at the marginal cost per passenger, and all the bus-related costs of operation are covered by subsidy.

**Summary of this section**

We have considered four economic regimes for bus operation. Each leads to a set of three equations determining patronage, \( q \), fare, \( f \), and headway, \( h \). One of the three equations, the demand equation (9), is common to all sets, and other equations are common to more than one set. However, the sets are all different, and they lead to different outcomes. The algebra of all sets can be simplified by changing the variables, as is done in the Appendix, but it is still too complicated to enable us to give explicit expressions for the solutions to the equations. We now consider some numerical results.
4. NUMERICAL RESULTS

With five parameters \((m, \nu, c, F\) and \(L\)), and four regimes, a large amount of numerical calculation is possible with this model, and we have had to restrict our efforts to what is manageable. Among the parameters, we take greatest interest in the influence of the general demand level, represented by \(L\) (potential passengers per minute), because it can vary widely between different places and times, from very low to very high; it is largely determined by demographic factors outside the control of policy-makers and managers; and intuitively one might expect the relationships between the regimes on low-demand routes to be different from those on high-demand routes. The other parameters are less variable. We have therefore adopted fixed values of \(m, \nu, c,\) and \(F\) throughout, and considered the effects of varying \(L\) on the various measures of output under each of the economic regimes. The values chosen for the four other parameters are intended to be of the right order of magnitude, but they have been plucked out of the air for this purpose, and we do not claim any empirical validity for them. The kind of routes we have in mind are radial routes of 3 to 4 miles from the periphery to the centre of a sizeable town. Before we give the parameter values, it is useful to say that our qualitative results and some of the key quantitative results are invariant with respect to changes in the parameter values. The conclusions of the paper would therefore be similar for all reasonable parameter values, and we do not need to be over-concerned with them in this context.

The values adopted for the four fixed parameters are:

\(\nu:~60p\) per journey. This is the average sum at which the potential passengers value their journeys. Note that the exponential demand model implies a very skew distribution of valuations, so that the majority of potential passengers value their journeys at less than \(60p\), but a few value them very much more highly.

\(c:~3p\) per minute. This is the rescheduling cost.

\(F:~800p = £8\) per journey. This is the “fixed cost” (that is, not passenger-related) of bus operation. It is based on \(£1 – £1.50\) per mile on a six-mile round trip. We assume that the revenue from the return trip is small or negligible.

\(m:~10p\) per journey. This is the marginal cost per passenger-journey.

We let \(L\) vary over the range from zero to 9 potential passengers per minute. There is no upper limit to possible values for \(L\), but this range covers the interesting action. There is a minimum level of \(L\) below which it is not possible to provide a service in the three unsubsidised regimes (competition, breakeven maximum net economic benefit, and monopoly). This minimum value of \(L\) is the demand level at which monopoly profit becomes negative; if a monopolist cannot provide a service without subsidy, no other regime can. This minimum value of \(L\) with the data given above is 1.070 potential passengers per minute. With subsidy, it is possible to provide a service at any demand level; and there is an important range of \(L\), from 0.394 to 1.070 potential passengers per minute, in which maximum net economic benefit from the service is positive, but a subsidy is required to
provide it. This is the range of demand for which there is a consensus that "socially desirable, non-commercial" services should be provided. This range of $L$ is relatively large (a factor of 2.7 from bottom to top), but it is small in relation to the whole range of demand considered in this section. When the demand level is less than 0.394 potential passengers per minute, even maximum net economic benefit is negative. Services can still be provided (with subsidy) and the travel demands of the potential passengers still have to be attended to, but we do not consider these very low levels of demand in this paper. It is likely in any case that conventional bus services would not be appropriate.

It is worth making some general points about comparisons between the regimes before we look at the numerical results. The most interesting like-with-like comparison is between the first two regimes: that is, between competition and breakeven maximum net economic benefit. It is like-with-like in the sense that both regimes have zero subsidy and zero profit, and both as a consequence apply to the same range of demand, from 1.070 potential passengers per minute upwards. The other two regimes represent extremes in opposite directions. Monopoly has maximum profits, which increase indefinitely with demand, and unconstrained maximum net economic benefit has a large subsidy, which increases indefinitely in absolute terms with demand, though it decreases per passenger.

We are now ready to discuss the numerical results. To save space we abbreviate the words "maximum net economic benefit" to MNEB in the following description.

**Fares and frequencies**

As stated in section 4, we do not regard fares and frequencies as output measures; but they are interesting as descriptive measures, and we look at them first. Figure 2 shows fare levels in each of the regimes for all values of demand per minute, $L$, from the minimum, 1.070 or 0.394 potential passengers per minute according to regime, up to 9 potential passengers per minute. Figure 3 shows frequencies, measured in buses per hour.

Figure 2 shows that, at minimum demand of 1.070 passengers per minute, all three subsidised regimes have the same fare, which is the monopoly fare of 70p per journey — $v + m$ in terms of the parameters. The monopoly fare is constant at all demand levels, but the competitive and breakeven MNEB fares fall with increasing demand, sharply at first and then more gently. The competitive fare is higher than the breakeven MNEB fare at all demand levels above the minimum: for example, with demand at 5 potential passengers per minute, the competitive fare is 36p per journey and the breakeven MNEB fare is 25p. The unconstrained MNEB fare is constant at the marginal cost per passenger of 10p.

Figure 3 shows that, at all demand levels above the minimum, competition produces higher frequencies than breakeven MNEB. For example, with demand at 5 potential passengers per minute, the competitive frequency is 4.6 buses per hour and the breakeven MNEB frequency is 2.8 buses per hour. Competition thus gives a higher fare/higher frequency service than breakeven MNEB. If net economic benefit is taken as the measure of output of the bus service, breakeven
MNEB gives by definition the optimal zero-subsidy combination of fare and frequency. We consider below the effect of the non-optimal competitive combination of fare and frequency on net economic benefit. Figure 3 shows that the monopoly frequency is always low, so that monopoly gives high fares and low frequencies. The unconstrained MNEB frequency is always higher than the breakeven MNEB frequency, but only slightly so when demand is moderate or high, and at these demand levels the unconstrained MNEB frequency is lower than the competitive frequency. However, when demand is low unconstrained MNEB gives the highest frequency, and for levels of demand below 1.070 potential passengers per minute, it is the only regime to provide a service at all.

Net economic benefit
We take net economic benefit to be the main measure of output of the bus service; therefore this sub-section provides key comparisons between regimes. We should note that for the competitive and breakeven MNEB regimes producer surplus is zero, so that net economic benefit is equal to consumer surplus, which
is the measure of what the consumers get from the service. Net economic benefit, and the other economic output measures, rise naturally with the demand level, \( L \), so it is helpful for presentation and comparison to scale net economic benefit per minute by dividing by \( L \), to give net economic benefit per potential passenger. For any particular level of demand this scaling applies to all regimes in the same way, so it does not affect comparisons between them.

**Competition and breakeven MNEB**

Figure 4, then, gives net economic benefit per potential passenger for each regime for the usual ranges of levels of demand. For all regimes net economic benefit per potential passenger is zero at their respective minimum demand levels, and then rises with increasing demand, first very rapidly and then more slowly. Comparing the two breakeven regimes, we see that net economic benefit per potential passenger under competition is always less than under breakeven MNEB, as it must be. The gap between them widens as demand increases from zero to about 12 per cent, and then begins to narrow again, but it remains at around 10 to 12 per cent over a wide range of demand. Table 1 puts figures on this gap by showing in the second column the difference between the two net economic benefits as a per-
FIGURE 4 - RELATIONSHIPS BETWEEN NET ECONOMIC BENEFIT AND DEMAND LEVEL FOR FOUR ECONOMIC REGIMES (continuously variable headways)

FIGURE 4
Relationships Between Net Economic Benefit and Demand Level for Four Economic Regimes (continuously variable headways)
TABLE 1

Comparisons of Net Economic Benefit

<table>
<thead>
<tr>
<th>Demand Level (Potential Passengers/Minute, (L))</th>
<th>Difference in Net Economic Benefit per Potential Passenger from that under Breakeven MNEB</th>
<th>Competition</th>
<th>Monopoly</th>
<th>Unconstrained MNEB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>1.070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>−3.5</td>
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<td>+174</td>
<td></td>
</tr>
<tr>
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</tr>
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<td>−38.5</td>
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<td></td>
</tr>
<tr>
<td>3.0</td>
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<td>−36.3</td>
<td>+11.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
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<td></td>
</tr>
<tr>
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<td>−33.9</td>
<td>+3.7</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>−8.6</td>
<td>−33.2</td>
<td>+2.9</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>−8.1</td>
<td>−32.7</td>
<td>+2.3</td>
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</tr>
<tr>
<td>8.0</td>
<td>−7.6</td>
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<td></td>
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<tr>
<td>9.0</td>
<td>−7.2</td>
<td>−31.9</td>
<td>+1.7</td>
<td></td>
</tr>
</tbody>
</table>

centage of that under breakeven MNEB. The precise maximum difference is 12.22 per cent, which is attained at the demand level of 1.944 potential passengers per minute. It is remarkable that this maximum percentage difference is a constant of the model, and is invariant with respect to changes in any of the parameters. In other words, the maximum percentage loss in net economic benefit under competition, relative to that under breakeven MNEB, is the same, 12.22 per cent, for any combination of the four fixed parameter values. The demand level, \(L\), at which this maximum is attained does depend on the other parameters; but since, as is illustrated by Figure 4 and Table 1, the maximum is a flat one, a difference of the order of 10 to 12 per cent must apply over a wide range of parameter values. Having discovered the constancy of the maximum percentage difference through numerical examples, I eventually found a proof of this result, which is given in the Appendix. The result holds only for the exponential demand function, though other demand functions may have somewhat similar regularities.

The economic processes causing this difference in welfare are rather complicated; but an excellent understanding of them can be obtained from the paper by Salop (1979); this was developed in the context of spatial competition, but we shall here describe it as if it referred to bus competition. His model is similar to ours, and differs in only one respect. This is that his demand model is one in which all potential passengers place the same value on their journeys, and travel or
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not according to whether or not their generalised cost is below this common value. This is not realistic, but it does have the effect of illuminating the process of competition. In his model, as in ours, there is a minimum level of demand at which it is possible to provide an unsubsidised bus service: namely, the level at which a monopoly operator just breaks even. At this level net economic benefit is non-zero. Above this minimum unsubsidised level there are two effective ranges of demand, which have economically distinct properties. These ranges arise because the competitive demand facing individual operators is kinked, and the equilibrium has different properties according to whether it occurs at the kink or above the kink. The range of demand corresponding to the kink comes immediately above the minimum. It is relatively short, but it has very serious effects for welfare. This range has the remarkable property that net economic benefit falls with increasing demand, not merely relative to demand, but absolutely. The reason is that in this range the best strategy for operators is not to compete on fares with their neighbours, but to set fares so that the markets of neighbouring buses just touch without competing. The effect of increasing demand is to allow more buses to operate, each with a smaller market and higher fares. This leads to frequencies and fares that are higher than optimal. Welfare under competition reaches a relative minimum at the end of this adverse range. In the following range of demand, operators begin to compete on fares again, with overlapping potential markets. The following range therefore has conventional properties; increasing demand leads to higher frequencies, lower fares, and increasing welfare, eventually closing the gap with the optimum.

Our model does not have demand ranges with distinct properties, because, unlike Salop’s demand model, our exponential demand model is continuous. In consequence, the processes which are separated in Salop’s model are superimposed in ours. However, competition permits, and indeed promotes, higher frequencies and fares than are optimal in our model in the same way as in Salop’s. Another consequence of our demand curve is that our maximum welfare loss of 12.22 per cent relative to breakeven MNEB is modest compared with the maximum losses in Salop’s model.

Monopoly

Under the monopoly regime, Figure 4 shows that the high fares and low frequencies result in consistently low net economic benefit per potential passenger, and the third column of Table 1 shows that the gap is typically about 35 per cent of the breakeven MNEB. In other words, monopoly achieves only about 65 per cent of the theoretical unsubsidised MNEB. Moreover, part of that 65 per cent takes the form of profit (which we remind ourselves is actually superprofit) and goes to the operator, so even less is left for the consumer. Table 2 shows that the consumer surplus is typically only about 40 per cent of the theoretical maximum. Unregulated monopoly is therefore very bad for consumers. However, it is good for operators. It is the only one of the regimes considered which produces (super) profits, and Table 2 shows that they may be very substantial in relation to costs. They increase steadily with demand, and reach 100 per cent of costs with demand at about 4 potential passengers per minute. Nobody is proposing monopoly as a policy, but these figures serve to illustrate that in an unregulated regime the pay-
TABLE 2

Monopoly

<table>
<thead>
<tr>
<th>Potential Passengers/Minute</th>
<th>Consumer Surplus as % of that under Breakeven MNEB</th>
<th>Profit as % of Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>64.4</td>
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<td>1.2</td>
<td>56.3</td>
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<tr>
<td>2.0</td>
<td>45.3</td>
<td>44.3</td>
</tr>
<tr>
<td>3.0</td>
<td>42.5</td>
<td>75.2</td>
</tr>
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<td>4.0</td>
<td>41.2</td>
<td>98.5</td>
</tr>
<tr>
<td>5.0</td>
<td>40.5</td>
<td>117.3</td>
</tr>
<tr>
<td>6.0</td>
<td>40.1</td>
<td>133.1</td>
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<tr>
<td>7.0</td>
<td>39.7</td>
<td>146.7</td>
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<tr>
<td>8.0</td>
<td>39.5</td>
<td>158.6</td>
</tr>
<tr>
<td>9.0</td>
<td>39.3</td>
<td>169.3</td>
</tr>
</tbody>
</table>

offs to operators from establishing monopolies, even temporary or local ones, could be high.

Unconstrained MNEB

Finally, in this section, we consider net economic benefit from the unconstrained MNEB regime. Unconstrained maximisation of net economic benefit must always give a higher maximum than breakeven maximisation; this is illustrated in Figure 4 and the last column of Table 1. However, the gap is very much larger at low levels of demand, including of course the demand range in which only subsidised services can be provided, than when demand is high. Absolute levels of subsidy go the other way, being proportional to frequency. The consequence is that the net economic benefit per unit of subsidy is higher when demand is low than when it is high, and if subsidies are limited it is more efficient to subsidise routes with low demand than routes with high demand. In my judgement it is also generally more equitable. It may also be efficient to cross-subsidise from high-demand to low-demand routes, because the loss of net economic benefit resulting from profit-making on high-demand routes may be relatively slight compared with the gain from using the profit to subsidise low-demand routes. This argument was put by Gwilliam, Nash and Mackie (1985) in their comments on Department of Transport (1984). Cross-subsidy cannot occur under competition. The loss of net economic benefit resulting from this depends on the amount of subsidy available under competition and on the distribution of demand between routes. We have not attempted to estimate its magnitude.
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5. FINITE TIME CYCLES

All the previous results are based on the analysis of a representative time segment taken from a uniform day of infinite length. What difference does it make if we consider a time period of finite length? Periods of finite length present two different complications. The first stems from the fact that an integral number of buses must be fitted into a time period of finite length, which means that headways and frequencies may take only discrete values instead of being continuously variable. The second complication is that time periods of finite length may have beginnings and ends, which present their own problems. These have been discussed in the literature on spatial competition, starting with the original paper by Hotelling (1929). However, time periods (or spaces) of finite length do not have beginnings and ends if they are conceived of as a circle or cycle, with the end joined to the beginning. This is what we do here. I have not considered the problems stemming from beginnings and ends, as I have a hunch that they are not very important in bus operation.

However, finite time cycles are important, because much bus operation, particularly in urban areas, does indeed have an exogenously determined natural time cycle: namely, the time it takes for a bus to do a complete round trip on a route and be ready to start again. If buses are allocated to routes rather than to individual journeys, the number of buses on a route must be an integer, and if the headways are equal, they must be a value from the series \( t, t/2, t/3, \ldots \), where \( t \) is the cycle time. Novshek (1980) has shown that with the assumptions of our model any competitive equilibrium is symmetric; that is, it has equal fares and headways.

We have worked through much of our previous analysis and calculations with the assumption of a finite time cycle of 60 minutes, which restricts headways to a value from the series 60, 30, 20, 15, 12, \ldots minutes. The analysis and results for competition are qualitatively different, so much so that we have only begun to consider the topic. It would take another paper and perhaps a different, game-theoretic, approach to deal with it properly. On the other hand, the analysis and results for the other regimes — breakeven MNEB, monopoly, and unconstrained MNEB — are broadly similar to the previous ones. We briefly discuss these first.

**Effect of finite cycle time on non-competitive regimes**

For the three non-competitive regimes, the restriction on the headways makes quantitative differences, but not qualitative ones. Headways, and sometimes other quantities, change in jumps as demand increases, and this alters the details of the comparisons between the regimes. The previous maxima of net economic benefit or profit are achieved only at discrete demand levels; on the other hand, the new maxima do not generally fall very short of the old ones. As would be expected, the greatest differences arise when demand is low. In particular, the lowest level of demand at which it is possible to provide an unsubsidised bus service is 1.378 potential passengers per minute, in place of the previous 1.070. The new figure is the demand level at which one bus per hour with the monopoly fare of 70p per journey just breaks even.
Effect of finite cycle time on competition

We now turn to competition. The key qualitative difference is that previously competition was a zero-profit regime, but now it is not. Operators now generally make (super)profits. The reason is that competitive profits were previously kept to zero by the presumption of entry of new operators, with fares and headways continuously adjusting as the demand level changed. Now headways cannot adjust continuously, and there are ranges of demand in which $n$ buses make (super)profits but $(n+1)$ buses would be unprofitable, and therefore the $n$ buses have protection from entry. This is indeed the normal situation; the only exceptions to it, in which profits are zero, are the levels of demand at which the previous zero-profit equilibria happened to have an integral number of buses per hour. This is a discrete series: for example, with our data, it is $1.378, 2.156, 3.117, 4.256, 5.571, 7.060, 8.723, \ldots$ potential passengers per minute, at which $1, 2, 3, 4, 5, 6, 7, \ldots$ buses per hour just earn zero profit. For all other levels of demand above $1.378$ potential passengers per mile, the operators make (super) profits under competition.

There are further twists. The first is that, as indicated above, in the demand range $1.378$ to $2.156$ potential passengers per hour only one bus can operate profitably under competition, and therefore this demand range is a natural monopoly. The operator could charge the monopoly price and make the monopoly profit. In the next higher demand range, $2$ buses per hour can operate profitably in competitive equilibrium; if they are indeed in competition the fare will be kept down to the competitive level, which is $51p$ per journey. However, if the two buses are operated by the same operator, as they often will be, the two buses will not be in competition, and again the operator might charge the monopoly fare of $70p$ in the knowledge that a third bus would be unprofitable. Thus partial monopolies may exist over wide ranges of demand. It should be said that this is not nearly as serious for the consumer as a fully-protected monopoly would be, because the frequency of buses is much higher: for example, in a fully-protected monopoly the second bus is not introduced till the demand reaches $3.845$ potential passengers per minute.

The final issue raised by finite time cycles is the increased possibility of instability. As explained in Section 2, even in the infinitely long day, the attainment of competitive equilibrium requires potential entrants to make decisions not on the basis of the initial profit which would be made by a new bus, but on the profit after the existing operators have changed their timetables in response to the new bus. In finite time cycles, the attainment of equilibrium still requires potential entrants to base decisions on the long-run position. However, finite time cycles may be more demanding, because the addition of a new bus causes not just infinitesimal changes to long-run headways and fares, as in the infinitely long day, but step changes in both. The post-entry fares and headways may therefore be substantially different from the pre-entry fares and headways, and the attainment of equilibrium requires potential entrants to be correspondingly far-sighted.

Some numerical results for competition

Before we leave this section it seems useful to try to get a feel for the order of
magnitude of the (super)profit and other quantities in finite time cycles. To do this, we assume that monopoly applies in the lowest demand range, that thereafter the number of buses in operation is the maximum number that can be operated profitably under competition, and that the monopoly fare applies in the lowest range and competitive fares thereafter. We suppose therefore that potential entrants are indeed sufficiently far-sighted to make decisions on the basis of the post-entry position.

Figure 5 shows the competitive (super)profit and consumer surplus per potential passenger for the usual range of demand, where the route has a 60-minute cycle time. Figure 5 also shows for comparison the former competitive net economic benefit; this equals the old consumer surplus, since there were zero profits. The (super)profits are represented by the row of “teeth” above the horizontal axis. In each range of demand, profits rise from zero with increasing demand up to the point at which an extra bus can be operated; then they fall to zero, and the cycle starts again. Consumer surplus per potential passenger is represented by the rising step function. Consumer surplus per potential passenger is constant within each range of demand, because both headways and fares are constant; demand is therefore proportional to $L$; consumer surplus is proportional to demand; therefore consumer surplus per $L$ is constant. At the points where an extra bus is introduced, both headways and fares jump down, and therefore consumer surplus per potential passenger jumps up. It jumps up to meet exactly the old consumer surplus curve, because it was at the beginning of each range of demand (and only there) that the old model gave an exactly integral number of buses per hour.

Table 3 gives some figures relating to these graphs. The second column shows the profits at the peaks at the end of each demand range as a percentage of cost. These profits are substantial, ranging from 45 per cent at the first peak down to 15 per cent at the sixth. If we approximate the successive rising profit curves in each range by straight lines, we can estimate the average profit over each demand range as half that at the peak, which gives average profits of 22, 17, 13, 11, 9, and 7.5 per cent in the first six ranges. These are far less than monopoly profits, but they are not by any means negligible (remembering as always that they are superprofits), and they provide a theoretical response to those who argue that the generation of superprofits is impossible under competition.

The second column of Table 3 gives the loss of consumer surplus as compared with the infinite-cycle curve at the successive worst points. Again, we can halve these figures to get the rough average loss in each range. These average losses are 23, 11, 7, 5, 3.5 and 2.7 per cent in the first six ranges, and are again not negligible. They are more variable than, but of the same order of magnitude as, the 10 to 12 per cent difference in consumer surplus between competition and breakeven MNEB which we discussed in the previous section; and they have to be added to this. It should be said here that it is not quite fair to add these new losses in full, because breakeven MNEB itself suffers some loss in consumer surplus as a consequence of the 60-minute cycle time. However, the breakeven MNEB losses are much less. Apart from the demand range from 1.070 to 1.378 potential passengers per minute, in which the finite cycle time causes a complete loss of service in the unsubsidised regimes, the worst loss in the breakeven MNEB
FIGURE 5 CONSUMER SURPLUS AND SUPERPROFIT
UNDER COMPETITION WITH DISCRETE HEADWAYS
(bus cycle time 60 minutes)

FIGURE 5
Consumer Surplus and Superprofit Under Competition
with Discrete Headways
(bus cycle time 60 minutes)
TABLE 3

Competition with a 60-Minute Cycle Time
Profit, Consumer Surplus and Net Economic Benefit
at End of Demand Ranges

<table>
<thead>
<tr>
<th>Potential Passengers/Minute at End of Demand Range</th>
<th>Profit as % of Cost</th>
<th>Deviation from Quantity with Infinitely Long Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>2.156</td>
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<td>4.256</td>
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<td>5.571</td>
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</tr>
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</tr>
</tbody>
</table>

regime, at the end of the lowest demand range, is 8 per cent, implying an average loss over the relevant range of about 4 per cent, and the loss rapidly becomes negligible at higher demand levels. The final column of Table 3 gives the percentage difference in net economic benefit from that with the infinite cycle time. Apart from the first demand range, the loss in net economic benefit is negligible, and there are even some small gains. These gains arise because in some demand ranges the consumers’ losses resulting from the 60-minute cycle time are more than outweighed by the operators’ profits. These small increases in net economic benefit are surprising; they arise because the constraint of zero profits, which always applies with the infinite cycle time, only applies at discrete demand levels with the finite cycle time.

6. CONCLUSIONS

In this paper we have attempted a theoretical comparison of four economic regimes for bus operation: competition; maximisation of net economic benefit subject to the constraint that the bus service must break even; unregulated monopoly; and unconstrained maximisation of net economic benefit. We make use of the analogy between spatial competition, such as that between shops, and temporal competition in bus services, and we draw on the spatial competition literature, which is part of oligopoly theory. We couple the ideas from spatial
competition with the exponential demand model drawn from the public transport literature.

We make some crucial assumptions in the early stages of the paper, notably that potential passengers are indifferent between forward and backward re-scheduling of their journeys. We also assume that bus operating costs will be the same under all economic regimes, and we ignore limitations on the capacity of buses; but we guess that these assumptions could be relaxed without altering the whole structure of the analysis, though at the cost of additional complication.

In our main comparisons of regimes for bus operation we consider a representative time segment from an infinitely long day. The main point about an infinitely long day is that it permits bus headways to adjust continuously to external changes, so that there is always a zero-profit competitive equilibrium. In section 3, we derive a set of three equations which must be satisfied in the competitive equilibrium; these together determine unique values for the bus frequency, fare, and patronage. We also derive the corresponding equations for the other three economic regimes. The algebra of the exponential demand model is sufficiently tractable to allow us to deduce these equations but not to solve them explicitly, so we then resort to numerical solutions in Section 4.

For these, we pick some representative, but not empirically-based, values for four of the five parameters in the model. However, many of the conclusions are invariant with respect to changes in these parameters, so the choice of these is not important. We then vary the fifth parameter, the general level of demand, represented by potential passengers per minute, over a wide range, from zero to fairly high. We calculate the characteristics of the bus service for each regime over the range of demand levels. These characteristics are frequency, fare, and patronage, and then cost, profit, consumer surplus and net economic benefit. The most important comparison is between the two zero-profit/zero-subsidy regimes, competition and breakeven maximum net economic benefit. We highlight two points from this comparison here. First, competition leads to higher fares and higher frequencies than those consistent with maximum net economic benefit at all demand levels above the minimum. Secondly, the reduction in net economic benefit as a result of this is 10 to 12 per cent over a wide range of parameter values. This reduction looks modest if one believes, with the UK government (Department of Transport, 1984) that competition will save up to 30 per cent on costs. Supporters of competition might also accuse me of overstating the loss as 10 to 12 per cent, because it depends on comparing competition with a theoretical maximum that could only be achieved by omniscient planners and perfect implementation. My response to this is that the competitive net economic benefit is also a theoretical calculation: I would not expect either of the two welfare levels to be achieved in practice, but it is not obvious that competition would be more successful in this respect.

In section 5 we consider a bus service with a finite cycle time instead of a time segment from an infinitely long day. This restricts headways to discrete values. This restriction does not have a great effect on the three non-competitive regimes, but it changes competition in important ways. First, it means that competition is no longer a zero-profit regime. (Super)profits are now made; our numerical calculations suggest that these can average from 22 per cent of costs downwards,
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depending on the demand level, lower figures applying at higher demand levels. Secondly, the finite time cycle results in natural monopolies in certain ranges of demand, and generally makes monopoly control of route possible, though these would not be as adverse for consumers as a fully-protected monopoly would be. Thirdly, there may be a greater likelihood of instabilities with finite time cycles. Finally, the finite time cycle leads to a further loss in consumer surplus relative to the maximum, in addition to the 10 to 12 per cent mentioned above. This amount varies with demand level, but it is typically of the same order of magnitude.

APPENDIX

Proof that the maximum percentage loss in net economic benefit from competition relative to breakeven maximum net economic benefit as the demand level varies is invariant with respect to the other parameters

In section 4 we adopted fixed values of the parameters $c$, $v$, $m$ and $F$, and explored the consequences of varying $L$ for the segment of the infinitely long day. We found that with these fixed parameters the percentage loss in net economic benefit from competition relative to breakeven maximum net economic benefit (MNEB) increased with $L$ at low values of $L$, reached a maximum of 12.22 per cent, and then declined with further increases in $L$. The pattern is shown in Table 1 and Figure 4. We asserted that the maximum loss of 12.22 per cent would have been the same whatever values we had chosen for the four fixed parameters. The purpose of this appendix is to prove this assertion.

We begin with the three equations (9) to (11) of section 3, which together determine the equilibrium fare, frequency, and patronage under competition. We mentioned that these equations could be simplified by changing the variables. (I am indebted to David Ulph for this suggestion.) The new variables $x$ and $y$ replace $h$ and $f$ respectively, and are defined by

$$x = (1 - e^{-ch/2v})$$

and

$$y = \frac{f - m}{v}$$

Substituting for $h$ from (25) and $f$ from (26) gives new equations corresponding to (9) to (11) in terms of $x$, $y$, and $q$. They are the following:

$$q = \frac{2Lv}{c} e^{-m/v} e^{-y} x$$

(27)

$$q = \frac{Lv}{c} e^{-m/v} ye^{-y} (1 + x)$$

(28)
and

$$q = \frac{F}{vy}$$  \hspace{1cm} (29)$$

Eliminating $q$ from (27) and (28) gives

$$2x - y(1 + x) = 0$$  \hspace{1cm} (30)$$

Eliminating $q$ from (27) and (29) gives

$$xye^{-y}G = 1$$  \hspace{1cm} (31)$$

where

$$G = \frac{2Lv^2}{Fc} e^{-m/y}$$  \hspace{1cm} (32)$$

$G$ is a combination of all five parameters. Equations (30) and (31) together determine the values of $x$ and $y$ in competitive equilibrium (and thence the values of $q$, $h$, and $f$). Let the solutions to these equations be $x_c$ and $y_c$, where the subscript $c$ denotes "competitive". The values of $x_c$ and $y_c$ in any particular case depend on the five parameters, but since these appear in equations (30) and (31) only in the combination represented by $G$, all combinations of parameters giving the same value of $G$ give the same values of $x_c$ and $y_c$. We may therefore write $x_c$ and $y_c$ as functions of $G$ alone:

$$x_c = x_c(G)$$

$$y_c = y_c(G)$$  \hspace{1cm} (33)$$

We now return to section 3 for the three equations determining $q$, $h$, and $f$ for breakeven MNEB, namely (9), (11), and (18). We make the same change of variables as above, with $x$ and $y$ given by (25) and (26) replacing $h$ and $f$. Equations (9) and (11) are replaced by (27) and (29) above, and (18) is replaced by

$$q = \frac{2Lv}{c} e^{-m/y} e^{-y(1 - x) \log (1 - x)} + \frac{F}{y}$$

We can again eliminate $q$, and get the following equations determining the breakeven MNEB values of $x$ and $y$:

$$e^{-y} [1 + (1 - x) \log (1 - x)] G = 1$$

and

$$xye^{-y}G = 1$$

Let the solutions to these equations be $x_m$ and $y_m$ (m for "maximum"). The values of $x_m$ and $y_m$ again depend on all the parameters, but again these appear only in the combination represented by $G$. We may therefore write them as

$$x_m = x_m(G)$$

$$y_m = y_m(G)$$  \hspace{1cm} (34)$$
It is worth noting at this point that the equations determining $x$ and $y$ in the other two economic regimes also depend on the parameters only through $G$, but since we do not need these for the purpose of this appendix we do not pursue them here.

The next step is to note that, for both competition and breakeven MNEB, net economic benefit is equal to consumer surplus, given by $qv$, which by (29) is $F/y$. The quantity of concern to us is the proportionate reduction in net economic benefit from competition relative to breakeven MNEB, which we denote by $D$. This is given by

$$D = \frac{F}{y_m} - \frac{F}{y_c}$$

where the reduction is measured in the positive direction. $F$ cancels out in (35) and from (33) and (34) $y_c$ and $y_m$ are functions only of $G$. Therefore $D$ is a function only of $G$:

$$D = D(G)$$

The numerical example of section 4 shows that with fixed values of $c$, $v$, $m$, and $F$, which we denote here by $c_0$, $v_0$, $m_0$, and $F_0$, there is a unique value of $L$, say $L_0$, which maximises $D$ at 12.22 per cent. Let $G^*$ be the value of $G$ at this maximum, given by

$$G^* = \frac{2L_0 v_0^2 e^{-m_0/v_0}}{F_0 c_0}$$

$G^*$ has the properties that $D(G^*) = 12.22$ per cent, and that $D(G^*) \geq D(G)$ for all $G \neq G^*$. The first property follows directly from the numerical example, and the second holds because, if it did not, there would be some other value of $L \neq L_0$ giving a greater or equal value of $D$ than 12.22 per cent with the original fixed parameters, contrary to the numerical example.

Now suppose we are given any other values of the first four parameters, say $c_1$, $v_1$, $m_1$, and $F_1$. With these parameters there exists one and only one value of $L$ for which $G = G^*$, namely $L_1$, given by

$$L_1 = G^* \frac{F_1 c_1}{2v_1^2 e^{-m_1/v_1}}$$

With these parameters, when $L = L_1$, $D = D(G^*) = 12.22$ per cent. Any other value of $L \neq L_1$ gives a different value of $G$ for which $D(G) < D(G^*)$. Therefore $L_1$ is the demand level giving the maximum value of $D$. We have thus shown that, given any fixed values of the $c$, $v$, $m$, and $F$, the maximum value of $D$ is 12.22 per cent. That is what we set out to prove.
REFERENCES