Forecasting the Impact  
of Service Quality Changes  
on the Demand for Inter-urban Rail Travel

By Mark Wardman*

1. Introduction
The main purpose of the research reported here was to identify a preferred method of forecasting the effect of changes in principal service quality factors on the demand for inter-urban rail travel. This paper concentrates on two aspects of the research which have a significance wider than just rail demand analysis:

(i) the empirical findings have potentially important implications for the common practice of specifying demand expressions which contain variables, such as generalised cost, which conflate various travel attributes into a single representative measure;

(ii) the empirical findings challenge the wisdom of the conventional approach of specifying constant elasticity models.

Empirical results are presented for a large data set of non-London inter-urban flows. Given the commercially sensitive nature of the results, sufficient detail is presented to illustrate the differences between alternative models without revealing estimates of absolute elasticities or volume changes.

We commence with a discussion of British Rail's recommended procedure for forecasting the effect of changes to the principal service quality variables of journey time, frequency and interchange. The elasticities to time, frequency and interchange implied by this approach are identified before a discussion of the desirable properties of elasticities and how demand models can be specified to exhibit these properties. Having described the data set and modelling approach, we proceed to test British Rail's method, which can be regarded as a conventional approach, and then widen the discussion to include more general demand models. A preferred method of forecasting the effect of service quality changes is identified and the forecasts from alternative models are compared. We conclude with the findings of a validation test against subsequent observed demand changes.

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2. Background

2.1 British Rail's Generalised Time Approach

British Rail’s recommended procedure for forecasting service quality changes uses a variable termed Generalised Journey Time (GT), which represents the combined effect of station to station journey time (T), frequency (F) specified as service headway, and the number of interchanges (I). Fare is treated as a separate variable with its own range of elasticities. The GT expression is specified as:

\[ GT = T + \alpha_1 F + \alpha_2 I \]  

(1)

The parameters \( \alpha_1 \) and \( \alpha_2 \) are the service interval and interchange penalties which convert frequency and interchange into equivalent time units. The demand expression relating the volume of rail travel (\( V \)) to GT is:

\[ V = GT^\beta \]  

(2)

The GT elasticity (\( \beta \)) is constant and currently a value of \(-0.9\) is recommended. Evidence on the weights in the GT expression was largely obtained from Stated Preference studies (MVA Consultancy, 1985; MVA Consultancy, 1987; MVA Consultancy et al., 1987; Steer, Davies and Gleave, 1981). The GT elasticity was first estimated to the volume increases stemming from the introduction of high speed trains on London routes in the late 1970s (Operational Research Unit, 1985) and was confirmed in subsequent studies (Operational Research Unit, 1989, 1991a). However, more recent evidence has cast doubt on the recommended combination of GT weights and GT elasticity (Operational Research Unit, 1991b, 1992a, 1992b; Wardman, 1993a). The properties of this GT approach in terms of the implicit elasticities (\( \eta \)) to time, frequency and interchange are:

\[ \eta_T = \frac{\partial V}{\partial T} \frac{T}{V} = \beta \frac{T}{GT} \]  

(3a)

\[ \eta_F = \frac{\partial V}{\partial F} \frac{F}{V} = \beta \frac{\alpha_1 F}{GT} \]  

(3b)

\[ \eta_I = \frac{\partial V}{\partial I} \frac{1}{V} = \beta \frac{\alpha_2}{GT} \]  

(3c)

Given that interchange can be zero, it is more sensible to define its elasticity as the proportionate change in demand after a change in the number of interchanges, and equation (3c) is the point elasticity equivalent.

A feature of the elasticities implied by British Rail’s GT approach is their interaction with the level of GT. The time and frequency elasticities fall as these variables form a smaller proportion of GT. The interchange elasticity falls as GT increases, although this is moderated by the specification of interchange penalties which increase with distance. These impacts of GT on the separate service quality elasticities are plausible, but ideally we would wish to have empirical support for such elasticity variation, particularly since it can be large.

2.2 Desirable elasticity properties

Direct demand models of rail demand, which have been widely used in Great Britain to estimate the relationship between the volume of rail demand and relevant travel and socio-
economic characteristics, are typically of the constant elasticity form (Fowkes, Nash and Whiteing, 1985; Jones and Nichols, 1983; Owen and Phillips, 1987; Operational Research Unit, 1989, 1991a, 1991b; Preston, 1987; Shilton, 1982; Tyler and Hassard, 1973; Wardman, Bristow and Fowkes, 1992; White and Williams, 1976). If we are in a position to estimate directly separate elasticities to each service quality variable, what could be regarded as a conventional model form would be:

$$ V = T^{\beta_1} F^{\beta_2} D^{\beta_d} $$  \hspace{1cm} (4)

The time ($\beta_1$) and frequency ($\beta_2$) elasticities are constant and the interchange elasticity ($\beta_d$) denotes that the proportionate change in demand is the same for a given change in interchange. Constant elasticity model forms are often adopted by default but ideally they should form the starting point to a more detailed examination of functional form since they do not necessarily provide the best explanation of rail demand variation. It is desirable to make allowance for an elasticity to vary according to:

(i) the level its variable takes;

(ii) distance;

(iii) the level of other rail variables;

(iv) non-rail factors, such as inter-modal competition.

The first two forms of elasticity variation can be introduced by specifying the demand function as:

$$ V = \rho^1 \lambda^1 D \rho^2 \lambda^2 F^2 \rho^3 \lambda^3 D \rho^4 $$ \hspace{1cm} (5)

which implies time, frequency and interchange elasticities of:

$$ \eta_T = \beta_1 \lambda_1 T^\lambda_1 D^\lambda_1 $$ \hspace{1cm} (6a) \\
$$ \eta_F = \beta_2 \lambda_2 F^\lambda_2 D^\lambda_2 $$ \hspace{1cm} (6b) \\
$$ \eta_d = \beta_d \lambda_d D^\lambda_d $$ \hspace{1cm} (6c)

The most common alternative to the constant elasticity model is the model where the elasticity is proportional to the level of the variable (Owen and Phillips, 1987; Operational Research Unit, 1985; Preston, 1987). However, this is likely to form a too extreme departure from the constant elasticity position, for example, somewhat longer-distance flows are forced to have much higher time elasticities, and hence it is not surprising that it is rarely empirically superior to the constant elasticity model. The $\lambda$’s in equations (6a)-(6c) allow the elasticity variation with respect to the level of the variable to be dampened. 

Distance is included since individuals may evaluate the aspects of rail service quality relative to distance; for example, through the consideration of speed or by regarding lower frequencies to be less inconvenient for longer-distance journeys. Each elasticity can either increase or decrease with distance according to the sign of $\gamma$. The generality introduced by the power terms $\lambda$ and $\gamma$ in equation (5) comes at the cost of having to use non-linear least squares estimation.

An elasticity can be allowed to depend on the level of other variables by entering interaction terms. A problem with such an approach is that if, for example, we wished to allow the frequency elasticity to depend on the extent of through-train provision, and thus specify an independent variable which contains both $F$ and $J$, we automatically impose a relationship between the interchange elasticity and the level of frequency. The form of this
relationship may be inappropriate; an extreme, but not implausible, example is that the
certainty elasticity depends on the extent of through-trains but the interchange elasticity
is independent of frequency.

Given that the modelling approach examines changes in demand between two points
in time (see Section 3), an alternative approach is to relate an elasticity to the level some
other variable takes in the before situation. Equation (5) is then estimated as:

\[
\ln \frac{V_2}{V_1} = \beta_1 D_2^{T_2} (T_2^{\lambda_1} - T_1^{\lambda_1}) + \beta_2 D_2^{F_2} (F_2^{\lambda_2} - F_1^{\lambda_2}) + \beta_3 D_2^{I_2} (I_2^{\lambda_3} - I_1^{\lambda_3})
\]  

(7)

where subscripts 1 and 2 are the before and after years. If we wished to allow the time
elasticity to depend on frequency, the frequency elasticity to depend on interchange and
the interchange elasticity to depend on time, and specifying the interaction variable to
relate to the before year, equation (7) would be modified to:

\[
\ln \frac{V_2}{V_1} = \beta_1 D_1^{T_1} (T_1^{\lambda_1} - T_1^{\lambda_1}) + \beta_2 (I_{\max} - I_1)^{\alpha_2} D_2^{F_2} (F_2^{\lambda_2} - F_1^{\lambda_2}) + \beta_3 T_1^{\alpha_3} D_2^{I_2} (I_2^{\lambda_3} - I_1^{\lambda_3})
\]  

(8)

The frequency term interacting with time will allow the time elasticity to depend on
frequency but it will not allow time to enter the frequency elasticity. The interchange
interaction is specified differently since \(I_1\) can be zero and hence \(\tau_L\) could not be negative.
Thus \(I_{\max}\) is chosen so that \(I_{\max} - I_1\) is always positive.

In an analogous manner we can allow elasticities to interact with non-rail variables,
such as the degree of competition or socio-economic factors. Whilst such analysis was
beyond the scope of the study reported here, this direct demand approach has been
successfully employed to analyse the impact of competing modes on rail demand and rail
elasticities for inter-urban travel (Wardman, 1993b).

2.3 Generalising the GT approach
We can generalise the GT approach, in terms of both the demand expression relating
volume to GT and the GT expression itself, so that the implicit service quality elasticities
have more desirable properties. If we specify the demand expression as:

\[
V = e^{\beta GT + D^F}
\]  

(9)

the implicit service quality elasticities, given the standard GT expression of equation (1),
are:

\[
\eta_T = \beta \lambda D^T GT^{\lambda-1}
\]  

(10a)

\[
\eta_F = \beta \alpha_1 D^F GT^{\lambda-1}
\]  

(10b)

\[
\eta_I = \beta \alpha_2 D^I GT^{\lambda-1}
\]  

(10c)

The relationships between the service quality elasticities and GT are now more general
and the elasticities are allowed to vary with distance, although it may prove undesirable
to constrain both \(\lambda\) and \(\gamma\) to be the same for each elasticity. We can also obtain more
desirable elasticities by specifying a more general GT expression of:

\[
GT = D^T F^T F_{\theta_1} + \alpha_1 D^F \theta_2 + \alpha_2 D^I \theta_3
\]  

(11)

This expression allows the contribution of each of the variables to GT to vary according
to the level the variable takes and with distance. Given that distance enters the GT
expression, there is no need to enter it into the demand expression. Specifying the latter as:

\[ V = e^{\beta GT^\lambda} \]  

(12)

yields service quality elasticities of:

\[ \eta_T = \beta \lambda \phi_1 \psi_i^{1-\theta_1} GT^{\lambda-1} \]  

(13a)

\[ \eta_F = \beta \lambda \phi_1 \phi_2 \psi_i^{\theta_1} GT^{\lambda-1} \]  

(13b)

\[ \eta_I = \beta \lambda \phi_1 \phi_2 \psi_i^{\lambda-1} GT^{\lambda-1} \]  

(13c)

Again the magnitude of \( \lambda \) denotes the extent to which the service quality variables are, when taken together, dependent on the level of \( GT \). The implicit elasticities are more general than equations (10a)-(10c) because of the inclusion of the \( \theta \) terms, which are free to vary between \( T \), \( F \) and \( I \), and because the \( \psi \) parameters, which determine the effects of distance, are also allowed to vary between \( T \), \( F \) and \( I \). It is important to recognise that these elasticities are the same as those obtained from equation (5), which specifies separate time, frequency and interchange terms, except for the inclusion of the \( GT \) interaction.

3. Data and Modelling Approach

The data were initially collected for a study of the effect of interchange conducted for the Regional Railways sector of British Rail (Wardman, 1993c). Models are developed on changes in demand as measured by British Rail’s computerised ticket sales system (CAPRI), with annual data available for the years 1985/86 through to 1991/92. Pooled time-series and cross-sectional data are used for the following reasons:

(i) a pure cross-sectional model would suffer from simultaneity bias, that is, it would be unable to distinguish whether, for example, low frequency was the cause of low demand or the result of it. Such models tend to obtain large service quality and price elasticity estimates in relation to other models;

(ii) a pure time-series for a given route would not exhibit sufficient variation in the main variables of interest, and indeed distance would be constant, and it would not be sufficiently long.

We aim to explain the response in rail demand to changes in service quality and fare, exploiting the rich variation in these variables across routes. To illustrate the modelling approach, let us specify a simple demand function relating rail volume to \( GT \) and fare \( (P) \) between two points \( (i \) and \( j ) \) in time period \( t \) as:

\[ V_{ijt} = GT_{ijt}^{\phi_1} P_{ijt}^{\phi_2} \]  

(14)

Expressing this as a ratio of demand in two time periods \( (1 \) and \( 2 ) \) allows the estimation of the unknown parameters by multiple regression of:

\[ \ln \frac{V_{ij2}}{V_{ij1}} = \beta_1 \ln \frac{GT_{ij2}}{GT_{ij1}} + \beta_2 \ln \frac{P_{ij2}}{P_{ij1}} \]  

(15)

Time-trend dummy variables are added to capture the effect of factors not included in the model, such as the levels of economic activity, population and car ownership. We can
depart from this constant elasticity position, and more general functions, such as that represented by equation (5), can be estimated by non-linear least squares.

Analysis was based on 1,091 observations of changes in demand on non-London flows. Flows were selected in order to provide a range of variation in journey time, frequency, interchange and fare, with distance varying from 30 to 250 miles. The selected stations range from large regional centres to smaller market towns with a population of around 30,000. The volume data largely relate to leisure travel, since the distances involved are generally too long for large-scale commuting, and in any event season ticket sales have been excluded, whilst business travel is much less significant than on London services. The analysis centres on a year of significant change in service quality, which varies across routes, and this year is excluded from analysis in order to allow demand to settle down to the new conditions.

Since train services between two points can differ in terms of journey time and the number of interchanges, and departures are often not at fixed intervals, it is necessary to estimate representative measures of journey time, frequency and interchange. This was done using standard British Rail procedures given journey time values of frequency and interchange and a desired departure time profile. Fare is represented by revenue per trip, as measured in the CAPRI ticket sales recording system.

Having established representative measures of time, frequency and interchange, we can construct GT as we wish, and are therefore in a position to compare models which contain GT specified in a variety of ways and models which estimate separate elasticities to time, frequency and interchange.

4. Empirical Tests of the GT Approach

The author is not aware of models which contain composite variables, such as GT as defined here or the more usual Generalised Cost expression, which imply elasticities to the component variables which are independent of the level of the composite variable or indeed where the relationship is weaker than that apparent in equations (3a)-(3c). This section reports three tests of the extent to which the service quality elasticities for inter-urban rail travel depend on the level of GT:

(i) the estimation of a range of models which specify separate time, frequency and interchange elasticities which are allowed to vary with GT across models in the manner implicit in the conventional GT approach and specified in equations (3a)-(3c);

(ii) the inclusion of interaction terms in a model which directly estimates time, frequency and interchange elasticities;

(iii) the estimation of λ in equation (12).

4.1 Separate models

Since our data allow the direct estimation of each service quality elasticity, we have used equation (4) to estimate separate time elasticities for various categories of T/GT, separate
Table 1

*Time Elasticity According to T/GT*

<table>
<thead>
<tr>
<th>T/GT</th>
<th>&lt;0.50</th>
<th>0.50-0.55</th>
<th>0.55-0.60</th>
<th>0.60-0.65</th>
<th>0.65-0.70</th>
<th>&gt;0.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated $\eta_T$</td>
<td>$\hat{\eta}_T$</td>
<td>0.85$\hat{\eta}_T$</td>
<td>1.34$\hat{\eta}_T$</td>
<td>1.51$\hat{\eta}_T$</td>
<td>0.73$\hat{\eta}_T$</td>
<td>1.08$\hat{\eta}_T$</td>
</tr>
<tr>
<td>95% CI</td>
<td>±43%</td>
<td>±71%</td>
<td>±33%</td>
<td>±39%</td>
<td>±75%</td>
<td>±61%</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.43</td>
<td>0.50</td>
<td>0.63</td>
<td>0.48</td>
<td>0.55</td>
<td>0.78</td>
</tr>
<tr>
<td>Expected $\eta_T$</td>
<td>$\eta_T$</td>
<td>1.20$\eta_T$</td>
<td>1.31$\eta_T$</td>
<td>1.40$\eta_T$</td>
<td>1.53$\eta_T$</td>
<td>1.65$\eta_T$</td>
</tr>
</tbody>
</table>

Table 2

*Frequency Elasticity According to $\alpha_F/GT$*

<table>
<thead>
<tr>
<th>$\alpha_F/GT$</th>
<th>&lt;0.20</th>
<th>0.20-0.25</th>
<th>0.25-0.30</th>
<th>0.30-0.35</th>
<th>0.35-0.40</th>
<th>&gt;0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated $\eta_F$</td>
<td>$\hat{\eta}_F$</td>
<td>0.52$\hat{\eta}_F$</td>
<td>0.17$\hat{\eta}_F$</td>
<td>0.52$\hat{\eta}_F$</td>
<td>0.35$\hat{\eta}_F$</td>
<td>0.13$\hat{\eta}_F$</td>
</tr>
<tr>
<td>95% CI</td>
<td>±27%</td>
<td>±41%</td>
<td>±147%</td>
<td>±54%</td>
<td>±109%</td>
<td>±250%</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.77</td>
<td>0.62</td>
<td>0.59</td>
<td>0.57</td>
<td>0.52</td>
<td>0.55</td>
</tr>
<tr>
<td>Expected $\eta_F$</td>
<td>$\eta_F$</td>
<td>1.25$\eta_F$</td>
<td>1.56$\eta_F$</td>
<td>1.81$\eta_F$</td>
<td>2.12$\eta_F$</td>
<td>2.56$\eta_F$</td>
</tr>
</tbody>
</table>

Table 3

*Interchange Elasticity by $\alpha_I/GT$*

<table>
<thead>
<tr>
<th>$\alpha_I/GT$</th>
<th>&lt;0.08</th>
<th>0.08-0.10</th>
<th>0.10-0.12</th>
<th>0.12-0.15</th>
<th>&gt;0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated $\eta_I$</td>
<td>$\hat{\eta}_I$</td>
<td>1.33$\hat{\eta}_I$</td>
<td>1.44$\hat{\eta}_I$</td>
<td>1.22$\hat{\eta}_I$</td>
<td>0.67$\hat{\eta}_I$</td>
</tr>
<tr>
<td>95% CI</td>
<td>±39%</td>
<td>±22%</td>
<td>±19%</td>
<td>±36%</td>
<td>±77%</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.48</td>
<td>0.59</td>
<td>0.61</td>
<td>0.53</td>
<td>0.52</td>
</tr>
<tr>
<td>Expected $\eta_I$</td>
<td>$\eta_I$</td>
<td>1.33$\eta_I$</td>
<td>1.66$\eta_I$</td>
<td>2.00$\eta_I$</td>
<td>2.66$\eta_I$</td>
</tr>
</tbody>
</table>

frequency elasticities according to $\alpha_F/GT$ and separate interchange elasticities for various categories of $\alpha_I/GT$ in order to test whether the elasticity variation with GT implicit in the standard GT approach is empirically supported. Values of T, F and GT for the before year were used. The elasticity estimates are presented in Tables 1 to 3, along with their 95 per cent confidence intervals, the goodness of fit and the elasticity expected on the basis of equations (3a)-(3c). Each category contains at least 100 observations with half exceeding 200.
The time elasticities are generally estimated with a satisfactory level of precision but there does not appear to be any clear relationship between the estimated and expected elasticities. Although three of the frequency elasticity estimates have large confidence intervals, they do not appear to vary in a manner consistent with equation (3b). It could be claimed that the imprecision of some of the estimates is clouding the results. However, the actual elasticity variation did not resemble the variation implicit in the *GT* approach when the second and third categories and also the final three categories were combined for greater precision. This was also the case for other groupings. Nor does there appear to be a convincing relationship between the estimated and expected interchange elasticities. This evidence suggests that the impact of *GT* on the separate service quality elasticities is somewhat weaker than is apparent in the standard *GT* approach.

### 4.2 Interaction terms

We can specify models which include separate time, frequency and interchange terms in such a form that their elasticities are allowed to vary with the level of *GT*. The procedure first established a preferred form for the separate time, frequency and interchange terms which permitted their elasticities to vary with the level of the variable and with distance (equation (5)). A *GT* interaction term was then entered for each variable in turn, in the form specified in equation (8) given the values of λ and γ previously estimated, to determine whether this improved the fit and therefore indicated that *GT* was influencing the time, frequency or interchange elasticities. The interaction variable was *GT* for the before year raised to the power (τ) of 0.2 or −0.2. These values of τ were specified because stronger interactions would still improve the fit but larger τ would run the risk of failing to detect interaction effects which were not particularly strong.

Only the interaction of *GT* \(0.2\) with the interchange elasticity improved the goodness of fit and this relationship is the reverse of that implied by equation (3c). Thus there is again no empirical support for the time, frequency and interchange elasticity variation implied by the standard *GT* approach.

In addition to examining interactions with the level of *GT*, we also took the opportunity to examine interactions between each of the service quality elasticities and fare, fare per mile and the level of the two other service quality variables. The fit was improved only when the time elasticity was allowed to increase with the service interval and to decrease with interchange. However, the best fit implied only slight elasticity variation and it was concluded that there were no compelling reasons to include interaction terms.

### 4.3 Testing the joint dependency of the service quality elasticities on *GT*

The more general *GT* models presented in subsection 2.3 show that the separate service quality elasticities can be allowed to have a weaker relationship with *GT*. The estimation of λ in equation (12), with the *GT* expression specified as equation (11), would indicate the degree of joint dependency between the service quality elasticities and the level of *GT*. Unfortunately, the estimation of the parameters of equations (11) and (12) is a very complex task, particularly since distance enters three times, and convergence of the non-
linear least squares estimation procedure provided by the SAS package could not be achieved. However, if \( \lambda \) is close to one, as we suspect, then the elasticities of equations (13a)-(13c) and (6a)-(6c) are very similar. Given that we have obtained estimates of the parameters of equation (5), which is a less demanding but by no means straightforward task, these can be substituted into equations (11) and (12) leaving \( \lambda \) of equation (12) to be estimated by non-linear least squares. The estimate of \( \lambda \) obtained was 1.012, with a 95 per cent confidence interval of \( \pm 5\% \). Thus taken together, the service quality elasticities are again found to be independent of the level of \( GT \).

5. A Preferred Forecasting Approach

We have seen that there is no empirical support for the variation in the time, frequency and interchange elasticities implied by the standard \( GT \) approach. In this section, we identify a preferred approach to forecasting the effect of service quality changes. The models which we shall compare are:

(i) a model which estimates separate elasticities to time, frequency and interchange and takes what can be regarded as the conventional constant elasticity form — this is termed the standard separate elasticities (SSE) model;

(ii) a model which specifies separate terms for time, frequency and interchange and which allows their elasticities to vary with the level of the variable and with distance — this is termed the generalised separate elasticities (GSE) model. This model is equivalent to the generalised \( GT \) approach of equations (11) and (12) with \( \lambda \) set to one for which we have obtained empirical justification;

(iii) the standard \( GT \) model of linear-additive \( GT \) expression and constant \( GT \) elasticity. This is termed the SGT model. The results presented so far indicate that this will not be preferred, but it acts as a reference point, representing what can be regarded as the conventional form of this type of approach.

5.1 SSE model

This model takes the form of equation (4). The estimates of the elasticities to time, frequency and interchange were all plausible and precisely estimated; the 95 per cent confidence intervals of the respective coefficients being \( \pm 20\%\), \( \pm 18\% \) and \( \pm 12\% \). The fare elasticity was also consistent with other evidence and had a low confidence interval of \( \pm 12\% \). The \( R^2 \) was 0.576, which is quite acceptable in our experience of this type of model and data, with a highest correlation of \( -0.50 \) between the estimated time and interchange coefficients and no other correlations exceeding 0.3.

5.2 GSE model

This model takes the form of equation 5, with the time, frequency and interchange elasticities allowed to vary with the level of the variable and with distance. We concluded (section 4.2) that interaction effects were either absent or sufficiently small that their exclusion would make no material difference to the model’s demand forecasts.
Table 4

Non-linear Least Squares Estimates of GSE Model

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>λ</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME</td>
<td>β₁(±269%)</td>
<td>0.19(±182%)</td>
<td>-0.16(±125%)</td>
</tr>
<tr>
<td>FREQ</td>
<td>β₂(±152%)</td>
<td>0.38(±79%)</td>
<td>-0.56(±72%)</td>
</tr>
<tr>
<td>INT</td>
<td>β₃(±96%)</td>
<td>0.97(±31%)</td>
<td>0.41(±58%)</td>
</tr>
</tbody>
</table>

Table 5

Elasticities of GSE Model

<table>
<thead>
<tr>
<th>TIME</th>
<th>DIST</th>
<th>η₁</th>
<th>FREQ</th>
<th>DIST</th>
<th>η₂</th>
<th>INT</th>
<th>DIST</th>
<th>η₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>40</td>
<td>η₁</td>
<td>30</td>
<td>25</td>
<td>η₂</td>
<td>0.5</td>
<td>25</td>
<td>η₃</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>0.94η₁</td>
<td>30</td>
<td>50</td>
<td>0.68η₂</td>
<td>0.5</td>
<td>50</td>
<td>1.33η₃</td>
</tr>
<tr>
<td>120</td>
<td>80</td>
<td>1.02η₁</td>
<td>60</td>
<td>50</td>
<td>0.88η₂</td>
<td>1</td>
<td>100</td>
<td>1.73η₃</td>
</tr>
<tr>
<td>120</td>
<td>120</td>
<td>0.96η₁</td>
<td>60</td>
<td>100</td>
<td>0.60η₂</td>
<td>1</td>
<td>200</td>
<td>2.30η₃</td>
</tr>
<tr>
<td>180</td>
<td>150</td>
<td>1.00η₁</td>
<td>120</td>
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<td>1.69η₃</td>
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<td>250</td>
<td>0.47η₂</td>
<td>2</td>
<td>200</td>
<td>2.25η₃</td>
</tr>
</tbody>
</table>

The intention was to estimate all the parameters of equation (5) directly by non-linear least squares. However, we could not achieve convergence of the iterative estimation procedure, even though we had obtained sensible starting values as a result of some initial exploratory analysis which involved the estimation of the β's of equation (5) by linear least squares for a range of given values of the λ's and γ's. The estimation of the parameters of equation (5) is by no means a straightforward task, whilst inevitably high correlations between corresponding β's, λ's and γ's and distance entering the equation three times compound the problem. A pragmatic solution was to apply non-linear least squares to each of the variables in turn, thereby estimating each set of β, λ and γ separately, with the remaining two variables specified to have the constant elasticity form. The estimates of each of the λ and γ were then entered into equation (5) to leave just the estimation of the β's by linear least squares in order to obtain a model whose goodness of fit could be directly compared with the SSE model. Table 4 presents the non-linear least squares estimates.
The elasticities were found to lie in a plausible range, varying around central values which are consistent with other evidence. Table 5 illustrates the variation in the point elasticities across a range of situations and shows that it is not trivial. The time elasticity increases with journey time and falls with distance, but both effects are weak. The similar but opposite signs of the two effects, combined with the correlation between time and distance, operate to cancel each other out such that the journey time elasticity exhibits little variation. The frequency elasticity is higher, for a given distance, at higher service intervals whilst it is lower, at a given frequency, for longer distances. These are very plausible relationships; it is to be expected that frequency is less important for longer-distance journeys and that the benefits of frequency improvements will be less where there is already a frequent service. The interchange elasticity was found to be almost invariant with respect to the number of interchanges but it does vary quite strongly with distance. It indicates, quite plausibly, that interchange is more important for longer-distance journeys.

When the \( \lambda \)'s and \( \gamma \)'s in Table 4 were entered into equation (5), it was encouraging to find that the \( \beta \)'s were little different but, as would be expected, they had much narrower 95 per cent confidence intervals of \( \pm 18\% \), \( \pm 17\% \) and \( \pm 11\% \) respectively, whilst the \( R^2 \) statistic was 0.584. It was of some concern that the goodness of fit for the GSE model, which discerns appreciable elasticity variation, was only slightly better than the value of 0.576 obtained for the SSE model. This issue was examined further using a simulation exercise. This exercise created synthetic values for the dependent variable using the independent variables in our data set, pre-specified parameter values of the demand function of interest and a normally distributed error term satisfying classical assumptions. Equation (5) was used to generate the data, and the parameter values were those we have estimated for this function. The standard error of the disturbance term was chosen to give an \( R^2 \) broadly in line with what was achieved in practice. This was 0.608 for the GSE model which had actually created the data which was only slightly higher than the value of 0.600 which was obtained when the SSE model was estimated to the same data. Thus we conclude that the small variation in fit achieved in practice is not a cause for concern.

Given the difficulties involved in estimating the parameters of equation (5), it is important to make some assessment of how confident we can be that they accurately reflect actual elasticity variation. The coefficients are estimated somewhat less precisely than we would wish and high correlations are undoubtedly contributing to this. For the coefficients associated with time, which are the most imprecise, the correlations between the estimated coefficients are 0.77 between \( \beta_1 \) and \( \lambda_1 \), -0.56 between \( \beta_1 \) and \( \gamma_1 \) and -0.83 between \( \lambda_1 \) and \( \gamma_1 \). However, given that \( \lambda_1 \) and \( \gamma_1 \) are low, it is not as surprising that they are not estimated precisely. The frequency and interchange coefficients do not fare as badly; the only correlations exceeding 0.5 are -0.55 between \( \lambda_2 \) and \( \gamma_2 \) and 0.68 between \( \beta_3 \) and \( \gamma_3 \).

Whilst the imprecision of and the correlation between the estimated coefficients is undesirable, there is little we can do about it, and if the underlying demand function is of the form of equation (5) the results in Table 4 provide our best estimates of them. However, there are several reasons why we can be confident about the results we have obtained.
Firstly, the elasticities are reasonable and they vary in a plausible manner. Secondly, the estimates of the $\lambda$'s and $\gamma$'s reported in Table 4 are within the range of values which our initial exploratory analysis using linear least squares indicated that the best-fitting model would lie. Finally, we conducted a simulation exercise to compare the coefficient estimates and implied elasticities estimated by the same procedure as we have used here with known values used to create the synthetic data using equation (5). The results of the four tests which were conducted on a range of $\beta$'s, $\lambda$'s and $\gamma$'s, and which are reported in the Appendix, show that although we may have failed to discern the effect of time and distance on the journey time elasticity accurately, the estimates of the frequency and interchange elasticities are remarkably robust even in the presence of high correlations between coefficient estimates and large standard errors of estimates.

5.3 SGT approach
This is based on equations (1) and (2). Empirical evidence (Wardman, 1993a, 1993d) quite clearly indicated that the recommended frequency penalties were too high for the recommended $GT$ elasticity. The best-fitting model is when the frequency penalties are reduced to 40 per cent of their currently recommended values and these, combined with the recommended interchange penalties, yield a $GT$ elasticity of $-0.81$. The $R^2$ statistic for this model was 0.568, somewhat higher than the value of 0.519 obtained when the currently recommended frequency penalties were used. These revised frequency penalties and $GT$ elasticity are therefore used to represent this approach.

6. Comparative Forecasts
Having identified the GSE model to be the preferred explanation of the impact of service quality changes on the demand for inter-urban rail travel, we now examine the forecasts produced by the three models in order to:

(i) show that the differences between the forecasts are not trivial;
(ii) validate the forecasts.

6.1 Illustrative forecasts of demand changes
Table 6 presents forecasts of the effects of frequency and interchange variation for a range of circumstances. The percentage changes in demand for the SSE and SGT models are expressed relative to the figure for the GSE model. Forecasts of the effects of journey time variations are more similar and are not reported.

The relationships between the forecasts reflect the properties of the different approaches, for example, the GSE model provides relatively large forecasts for interchange variation at longer distances and relatively small forecasts at shorter distances. The main point, however, is that there can be very large differences between the forecasts of the different models, and thus the issue of identifying the best functional form with which to forecast demand is not a trivial one.
### Table 6
*Comparative Demand Forecasts*

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SSE</th>
<th>SGT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 Hours, Hourly Service, 0 Changes, 250 Miles</strong></td>
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<td></td>
</tr>
<tr>
<td>Hourly - 2 Hourly</td>
<td>2.22</td>
<td>0.83</td>
</tr>
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<td>Add Interchange</td>
<td>0.76</td>
<td>0.78</td>
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<td><strong>4.5 Hours, 2 Hourly Service, 1 Change, 250 Miles</strong></td>
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</tr>
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<td>2 Hourly - Hourly</td>
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<td>0.63</td>
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</tr>
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<td>0.44</td>
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<td>Remove 2 Interchanges</td>
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<td>0.49</td>
</tr>
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<td><strong>2.5 Hours, Hourly Service, 0 Changes, 150 Miles</strong></td>
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<td></td>
</tr>
<tr>
<td>Hourly - 2 Hourly</td>
<td>1.68</td>
<td>0.95</td>
</tr>
<tr>
<td>Add Interchange</td>
<td>0.91</td>
<td>1.12</td>
</tr>
<tr>
<td><strong>3 Hours, 2 Hourly Service, 1 Change, 150 Miles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Hourly - Hourly</td>
<td>1.74</td>
<td>0.60</td>
</tr>
<tr>
<td>Add Interchange</td>
<td>0.95</td>
<td>0.79</td>
</tr>
<tr>
<td>Remove Interchange</td>
<td>0.89</td>
<td>0.94</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>3 Hourly - 2 Hourly</td>
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<td>1.28</td>
</tr>
<tr>
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<td>1.48</td>
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<tr>
<td><strong>40 minutes, Half Hourly Service, 0 Changes, 50 Miles</strong></td>
<td></td>
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</tr>
<tr>
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<td>1.16</td>
</tr>
<tr>
<td>Add Interchange</td>
<td>1.37</td>
<td>2.81</td>
</tr>
<tr>
<td><strong>1.5 Hours, Hourly Service, 1 Change, 50 Miles</strong></td>
<td></td>
<td></td>
</tr>
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<td>Hourly - Half Hourly</td>
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<td>0.41</td>
</tr>
<tr>
<td>Add Interchange</td>
<td>1.43</td>
<td>1.38</td>
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<tr>
<td>Remove Interchange</td>
<td>1.46</td>
<td>1.99</td>
</tr>
<tr>
<td><strong>40 Minutes, Half Hourly Service, 0 Changes, 30 Miles</strong></td>
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</tr>
<tr>
<td>Half Hourly - Quarter Hourly</td>
<td>1.18</td>
<td>1.06</td>
</tr>
<tr>
<td>Half Hourly - Hourly</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>Add Interchange</td>
<td>1.67</td>
<td>2.84</td>
</tr>
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</table>
6.2 Validation of forecasting models
The subsequent availability of data for 1992/93 provided the opportunity to test the forecasts of the various models against observed demand changes not included in the calibrated models. Flows which had experienced appreciable service quality changes in 1991/92 were identified, along with similar flows where there had been little change in service quality which act as 'controls'. Given that the after year is 1992/93, we use a before year of 1990/91 and omit the year in which the changes occurred in order to allow demand to settle down to the new conditions.

Clearly, any attempt to compare actual and forecast changes in demand stemming from service quality changes must isolate the effects of changes in factors other than service quality. This was done by estimating equation (16):

$$\ln \frac{V_2}{V_1} = \alpha_0 + \alpha_1 \ln \frac{GT_2}{GT_1} + \alpha_2 \ln \frac{P_2}{P_1}$$

(16)

where \( P \) is fare. The \( \alpha_0 \) term detects underlying changes in demand not associated with \( GT \) and fare, largely related to GDP changes and changes in car ownership levels. The fare elasticity and \( \alpha_0 \) were both estimated precisely, with 95 per cent confidence intervals of ±49% and ±50%, and are used to amend the observed demand changes to account for fare changes and underlying demand changes.

The comparison of observed and forecast changes in demand was conducted only for the 43 flows which experienced appreciable service quality changes, since the differences between the forecasting models could only be slight for the control flows. The percentage actual demand changes net of fare and underlying effects were compared with the percentage changes in demand forecast by each of the three methods. The mean deviation between actual and forecast demand changes was −4.48, −5.81 and −7.67 percentage points for the GSE, SSE and SGT models respectively. We also regressed the percentage actual demand change net of fare and underlying effects on the forecast percentage demand change. Ideally, we require that the intercept is close to zero and that the slope is close to one. Table 7 presents the parameter estimates, \( t \)-ratios and \( R^2 \)'s for each model. The GSE model has a slope which is very close to one and the intercept which is closest to zero, whilst the SSE model is the second best in terms of both the intercept and slope.

The evidence of the validation exercise substantiates the findings of our empirical analysis in favouring a model which allows variation in the time, frequency and interchange elasticities. However, our conclusions cannot be as firm as we would wish because of the relatively small sample size and the small differences in predictive performance. The latter stems from relatively small differences between the forecasts of each model since the flows are fairly similar in nature and experienced similar changes. Almost all the flows experienced interchange variation with relatively minor variations in frequency and then not independent of the interchange variation. It was therefore not possible to test the models in terms of their predictions of the effects of frequency which is the variable for which the forecasts of each approach differ the greatest. The distance band of 42-167 miles is also somewhat narrower than the range of 30-250 miles in the data set on which the models were calibrated. By no means does the range of situations covered in the validation exercise reflect the variation in circumstances depicted in Table 6.
Table 7

Regression of Actual and Predicted Demand Changes

<table>
<thead>
<tr>
<th></th>
<th>GSE</th>
<th>SSE</th>
<th>SGT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-4.69 (1.07)</td>
<td>-5.04 (1.14)</td>
<td>-6.16 (1.37)</td>
</tr>
<tr>
<td>SLOPE</td>
<td>1.01 (6.00)</td>
<td>0.96 (6.03)</td>
<td>0.94 (5.98)</td>
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<tr>
<td>$R^2$</td>
<td>0.468</td>
<td>0.461</td>
<td>0.460</td>
</tr>
</tbody>
</table>

7. Conclusions

The study reported here constitutes a most detailed examination of the effect of service quality changes on the demand for inter-urban rail travel in Great Britain. It has examined the standard GT approach, focusing on its implied elasticities to time, frequency and interchange, and discussed how the model can be generalised to have more desirable elasticity properties. Models which estimate separate elasticities to time, frequency and interchange and which allow more desirable elasticity variation than has hitherto been the case have also been advanced. These theoretical considerations have formed the basis of subsequent empirical analysis and testing, with the identification of a preferred method of forecasting service quality and validation against subsequent observed demand changes. In conclusion, we make the following points.

- The exponential model which implies point elasticities to some variable $X$ of $\beta X^p$ is a particularly attractive function. It departs from the restrictiveness of the conventional constant elasticity model but, unlike the most common alternative functional form of direct demand model, it does not impose the potentially undesirable proportional relationship between an elasticity and the level of its variable. We recommend that this function receives more widespread application than has previously been the case.

- A drawback of this more general exponential model is that it must be estimated by non-linear least squares and convergence problems may require, as here, pragmatic solutions to parameter estimation. Even then, concerns may arise about the reliability of the parameter estimates because of large correlations and imprecision. Simulation exercises based on synthetic data have a role to play in determining how confident we can be about the results we have obtained, and we recommend their use.

- The difference between the GT approach and the approach which specifies separate terms for time, frequency and interchange, is not as great as might appear at first sight, because they can be made to yield similar elasticity functions for the separate service quality variables. Indeed, our preferred model which estimated separate terms for time, frequency and interchange is a special case of a generalised form of GT approach.
• What we have termed the standard GT approach, and indeed the typical form of models which are based on variables such as generalised cost which conflate various travel variables into a single term, implies strong relationships between the elasticities of the component variables and the level of the composite variable. Various tests have found quite conclusively that the elasticities to time, frequency and interchange for inter-urban rail travel in Great Britain are independent of the level of GT. This casts serious doubt on the wisdom of adopting by default the standard linear-additive form of GT expression combined with constant GT elasticity.

• We recommend that more attention is paid to the functional form of direct demand models; in particular, examination of the properties of the elasticities implied to each variable in a composite term such as GT as defined here, or the more common generalised cost, has been a neglected issue. Identifying the appropriate functional form to explain the relationship between demand and travel variables is not merely an academic exercise in model building since we have seen that the differences between the forecasts produced by various models can often be substantial.

• We are encouraged by the plausibility of both the magnitude and the direction of elasticity variation that has been estimated, and that we have been able to obtain empirical support for a departure from the conventional constant elasticity position. The frequency elasticity was found to be higher at higher service intervals and to be lower at longer distances. The interchange elasticity was found to increase with distance.

• This paper has examined some forms of elasticity variation, but other sources of variation should be examined, such as the impact of the strength of competition from other modes on rail elasticities. We would expect rail elasticities to be higher, other things being equal, where rail has a lower market share.

• The evidence of a validation exercise comparing predicted and actual changes in demand substantiates the empirical findings, although there are reasons why the conclusions cannot be as firm as we might wish. Whilst validation remains desirable, we are limited by the number and nature of changes which actually occur.

• For pragmatic reasons, primarily because the results reported here relate to Regional Railways’ services and there are good reasons why they may not transfer to other sectors of British Rail, such as InterCity and Network SouthEast services to and from London, and because the modification of the computerised forecasting model would be a costly exercise, British Rail’s demand forecasting procedure remains based on the standard GT approach. However, the frequency and interchange penalties recommended for Regional Railways’ services have been amended so that when used with the currently recommended GT elasticity the forecasts provide a closer approximation to the forecasts of our preferred model than is currently the case.
• The analysis has been made possible because of the large number of service quality changes which have occurred on Regional Railways’ services over recent years. This has yielded a particularly rich data set, in stark contrast to the situation elsewhere on British Rail where the variation in service quality is insufficient to allow the calibration of robust models examining elasticity variation. A possible way forward for these other services is to develop direct demand models based on ‘pseudo’ ticket sales data obtained from Stated Preference exercises which would ensure adequate variation in the main variables of interest.

Appendix

Given the concerns about the parameters estimated for equation (5) because convergence could not be achieved in the iterative non-linear least squares estimation procedure and because the estimates obtained to each variable separately exhibited collinearity and imprecision problems, we conducted a simulation exercise using synthetic data to assist in assessing the reliability of the results obtained. The synthetic dependent variable data were constructed using the independent variables in the data set upon which we have calibrated the models reported in this paper, along with a range of values for the β's, λ's and γ's and an error term satisfying classical assumptions. The standard deviation of the errors was chosen to give a goodness of fit similar to that obtained to the real data.

The aim is to determine how closely the elasticity variation implied by the estimated parameters approximates the elasticity variation implied by the β's, λ's and γ's used to generate the synthetic data. The same two-stage process was used as in practice. Table A.1 reports the given values of the β's, λ's and γ's, the estimates of them obtained by applying non-linear least squares (NLLS) to each variable in turn and the estimates of the β's obtained by linear least squares (LLS) on the basis of the estimated values of the λ's and γ's. We then report the actual and estimated point elasticities for time, frequency and interchange for the six sets of (round trip) circumstances listed in Table A.2 which were selected to cover a representative range of situations. The estimated elasticities use the β's estimated by linear least squares.

The imprecision of and large correlations between some of the parameter estimates apparent in the real data are also apparent here. The correlations arise because there is freedoem for the β's, λ's and γ's associated with a variable to vary in such a way as to maintain a similar relationship between demand and the level of the variable. We are therefore not so much interested in the accuracy of the parameter estimates but rather the extent to which the elasticities obtained from the estimated parameters approximate to the elasticities implied by the given parameter values.

It can be seen that although there can be problems discerning the effects of journey time and distance on the journey time elasticity, the estimates of the frequency and interchange elasticities are remarkably accurate even in the presence of collinearity.
### Table A.1

**Actual and Estimated Parameters and Elasticities**

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<th>( \beta_1 )</th>
<th>( \lambda_1 )</th>
<th>( \gamma_1 )</th>
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<th>( \lambda_3 )</th>
<th>( \gamma_3 )</th>
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<td>-0.40</td>
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</tr>
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<th>( \beta_2 )</th>
<th>( \lambda_2 )</th>
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<th>( \lambda_3 )</th>
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<th>( \gamma_1 )</th>
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<th>( \lambda_2 )</th>
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<td>-0.09</td>
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Table A.2

Round Trip Values Used in Elasticity Calculations

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<th>INT</th>
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</table>

Note: since frequency is specified as a quarter of the service interval, the value of 15 corresponds to a half hourly interval on each leg of a round trip.

References


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