A Model of Travelling to Shop with Congestion Costs

By Robert W. Bacon*

1. Introduction

An earlier paper in this journal (Bacon, 1992b) analysed a model of shopping behaviour in which the frequency of shopping was an endogenous variable. From such a model aggregate travel characteristics (for example, the total number of trips per period arriving at a shopping centre) were derived. The sensitivity of these variables to changes in economic parameters, such as the cost of travel per mile, were explored with numerical simulations.

An important limitation of that paper, in common with almost all formal models of the frequency of travel, is the failure to allow for congestion. Not only is this an extra cost determined by the actions of all travellers together, it is also likely to vary in importance according to the location of the household in relation to the object of travel. Models of the journey to work, such as those of Solow (1972) and Dixit (1973), have allowed for the increased costs caused by congestion but, since the journey to work has a fixed frequency per period of time, these make no allowance for the feedback on to the frequency of travel itself.

Models of shopping behaviour, often the largest generator of trips after the journey to work (Vickerman and Barnby, 1984), usually assume that the frequency of shopping is variable — the higher the travel cost the fewer the number of trips made per period. So improvements in the road or parking system, which are designed to reduce congestion and thus travel costs, will have an important repercussion as shoppers start to travel more, thus raising congestion back towards its former level.

Congestion of the road system is likely to be felt most acutely at the time of the journey to work, both because of the high frequency of such trips relative to other purposes of travel, and because of the concentration of such trips in a narrow time interval. However, shopping trips will also suffer from some effects of congestion on total travel time.

* Lincoln College, Oxford. The author would like to thank Giuseppe Mazzarino for programming the calculations, and an anonymous referee for some helpful suggestions.
Parking also suffers from congestion in terms of the increased time and difficulty of finding a space at a time when many others are using the same centre. Models which include a fixed cost of parking implicitly assume that there is such a large supply of parking that the chance of finding a space is not affected by changes in the overall frequency of shopping. Experience with city centre shopping suggests that this is rarely a realistic assumption. Centres out of town or on the edge of the town can often offer larger amounts of parking, because of lower land values, thus increasing their attractiveness to shoppers relative to city centre sites. It also appears that the journey to work will often be associated with the lowest parking cost while taking account of difficulty in finding a space, and shoppers who travel later in the day will find greater parking congestion. Local authorities often attempt to compensate for this by reserving parking for short stay or by using a scale of parking charges that increases more than proportionately with time spent.

Improvements in road and parking capacities can thus be expected to generate more traffic as the costs of shopping trips are lowered. They can also have an impact on the relative attractiveness of different shopping centres and lead to a diversion of traffic as well as to the generation of new traffic. Here improvements in parking are much more likely to be asymmetric in their effects — an improvement in the road system will generally improve access to several centres, while parking changes are implemented individually.

The purpose of this paper is, first, to explore the effects of varying travel costs, caused by traffic congestion, on the frequency of shopping. The second aim is to evaluate the impact of changing the capacity of the road and parking system on the frequency of shopping and on congestion, as extra traffic is generated by the improved facilities.

The paper is structured as follows. Section 2 describes the basic model of household shopping behaviour, when there are no congestion costs, for a single-centre town of given size. Section 3 discusses the modelling of congestion, and Section 4 provides some numerical illustrations of the effects of altering the capacity of the road and parking facilities on the total amount of travel. Section 5 extends this methodology to the case of a two-centre town, where asymmetric changes in parking facilities lead to trip diversion as well as trip generation. Section 6 discusses methods of evaluating the benefits created by an improvement in road or parking facilities. The final section reviews the results obtained and discusses possible extensions of the models used.

2. A Model of the Frequency of Shopping in the Absence of Congestion

Models of shopping behaviour, in which frequency is an endogenous variable, were first proposed by Reinhardt (1973). Travel costs are assumed to be proportional to the distance to the shopping centre plus a fixed cost element per trip. The cost of travel is assumed to be unaffected by the quantity purchased per trip, unlike the "delivered price" model. With such a model the household would minimise the total costs of shopping per period by
A Model of Travelling to Shop with Congestion Costs

R. W. Bacon

purchasing very large amounts on very infrequent trips, so an inventory cost proportional to the bundle size is introduced. It is assumed that the consumer uses up the inventory at a constant rate and goes shopping immediately the inventory is empty. This gives a balance between shopping very frequently to minimise the inventory cost element, but incurring high travel costs, and shopping very infrequently, thus lowering travel costs but increasing inventory costs per period. The travel cost assumption, which has a constant cost per mile, reflects the absence of congestion effects. The cost per mile can be shown to include both the money cost and the implicit value of the time taken, as in the models of Bacon (1984, 1991) and Ingene (1984), while the fixed cost represents the money and time cost of parking.

The consumer’s shopping problem can be stated: choose the bundle size (amount per trip) and the frequency of shopping that maximise the total purchased per period subject to a constraint, in which the disposable income must equal the purchase cost plus travel cost plus inventory cost per period. Formally, choose \( f(t) \) and \( q(t) \) so as to maximise

\[
Q(t) = f(t) \cdot q(t)
\]

given that

\[
Y = p \cdot f(t) \cdot q(t) + G(t) \cdot f(t) + c \cdot p \cdot q(t)
\]

where

\[
\begin{align*}
 f(t) &= \text{frequency of shopping for a consumer located } r \text{ units from the shopping centre;} \\
 q(t) &= \text{bundle size per trip for a consumer located at } t; \\
 Q(t) &= \text{total purchased per period by a consumer } r \text{ units from centre;} \\
 Y &= \text{disposable income per period;} \\
 p &= \text{price of one unit of the good;} \\
 G(t) &= \text{total costs of making one trip to the centre for a consumer located at } t; \\
 2c &= \text{cost of storing an inventory of unit value for one period;} \\
 t &= \text{the distance of the household from the shopping centre.}
\end{align*}
\]

In the simple model without congestion the travel cost is assumed to be of the linear form described:

\[
G(t) = g \cdot t + x
\]

where

\[
\begin{align*}
 g &= \text{cost of travelling unit distance;} \\
 x &= \text{fixed cost per trip.}
\end{align*}
\]

The equations for optimal frequency and bundle size are:

\[
f^*(t) = -c + c \cdot \sqrt{1 + Y[c \cdot G(t)]}
\]

\[
q^*(t) = \frac{(-G(t) + G(t) \cdot \sqrt{1 + Y[c \cdot G(t)]})}{p}
\]

Equation (4) shows that the frequency of shopping is an increasing function of the level of income and of inventory costs, while the travel-related cost has an inverse relation with frequency. Given the assumption about the nature of travel costs, it follows that frequency is lower at greater distances from the centre, so that different consumers have different travel patterns according to their location even when all other factors are identical. The impact of changes in travel cost per unit distance (g) and of parking charges (x) can be seen

279
to be different at various locations and to depend on the values of the other parameters. A numerical example given by Bacon (1992b) illustrates a case where, for a linear town with uniform population density, a 1 per cent increase in the travel cost per unit distance reduces the total volume of shopping trips made per period by about 0.3 per cent, and a 1 per cent increase in the parking charge has a similar order of effect on the total number of shopping trips.

3. The Frequency of Shopping in the Presence of Congestion Costs

In order to allow for congestion it is necessary to relate the behaviour of each household to that of all other households using the same road or parking facilities, since the total number of trips determines the level of congestion. Such models have been developed in urban economics by a number of authors (see Mohan (1979) and Straszheim (1987) for reviews). Models involving the journey to work, such as those developed by Solow (1972) and Dixit (1973), have the feature that, in the absence of migration, the total number of trips arriving at the centre per period is independent of the road system or parking provision because each household is assumed to make exactly one journey to work each day. Changes in road capacity do affect residential location decisions, house rents and hence the road congestion at various intermediate points on the road, but parking use is the same whatever the levels of the economic variables determining endogenous behaviour.

One model of congestion, which relates the travel time to the amount of traffic on the road, has been described in detail by Mohan. Here, the cost of travelling a given small distance is a constant plus a power function of the ratio of the number travelling at that point relative to the "capacity" of the road. Formally this can be written:

$$ h(t) = \{a + a^* [N(t)/K]^b\} \, dt $$

(6)

where

- $a$ is the fixed cost of travelling unit distance;
- $a^*$, $b$ are parameters;
- $N(t)$ is the total number travelling to the shops at a location $t$ units from the centre;
- $K$ is a measure of road capacity.

In this formulation, the total travel cost of one trip (excluding parking) from location $t$ to the centre, meeting different degrees of congestion along the road, is

$$ g(t) = \int_0^t \{a + a^* [N(\tau)/K]^b\} \, d\tau $$

(7)

where $\tau$ is a dummy variable to integrate travel costs over all distances from the centre between zero and $t$, and function $N(t)$ is the total number of all trips per period passing point $t$ on their way to the centre. This number depends on the nature of the town — the simplest case analyses a linear town of length $T$, with a uniform population density $D$ per unit distance. In this case, for shopping trips with variable frequencies, the total number of trips per unit of time passing point $t$ is the sum of all frequencies originating further away than $t$: 280
A Model of Travelling to Shop with Congestion Costs

R. W. Bacon

\[ N(t) = D \int_{T}^{\infty} f(\sigma) d\sigma \]  

(8)

where \( \sigma \) is a dummy variable to integrate over the set of distances between the city limit \( T \) and \( t \). This formulation reveals how the optimal behaviour of each individual is related to that of all other households.

The cost function indicates that, when no one else uses the road, the cost of travel from location \( t \) is \( at \), as in models of shopping behaviour which have ignored congestion. The formulation of the costs of travel when there is heavy traffic is less satisfactory — for a given measure of road capacity \( (K) \) there is no finite traffic flow which would produce an infinite cost (that is, the equivalent to a complete standstill). This in turn means that for heavy traffic the feedback on to household behaviour as travel costs rise may be quite weak, whereas when a road actually approaches saturation households will use it much less, thus ensuring that the actual traffic flow never passes the capacity level. For the rest of this paper an alternative formulation, which does produce infinite costs when traffic flow equals road capacity, is adopted:

\[ h_1(t) = \{a K/[K - N(t)]\} dt \]  

(9)

In this formulation the cost of travelling a small distance is proportional to the cost of travel on an empty road \( (a) \) multiplied by a factor depending on the degree of road use at that point. The factor increases from unity, when there is no other traffic, to infinity, as the traffic approaches the capacity of the road. The cost increases ever more rapidly as full capacity is approached.

A parking congestion cost is introduced in the same way. The cost per trip depends on a basic cost \( m \), multiplied by a factor which depends on the degree of use of the parking capacity, which is proportional to the number of trips arriving at the centre per period. The parking cost per trip \( (x) \) is given by:

\[ x = m Z/[Z - N(0)] \]  

(10)

where

- \( m \) = cost of parking in the absence of congestion;
- \( Z \) = car park capacity.

The elements of travel costs based on models (9) and (10) can be introduced into the optimal frequency of shopping equation by using total travel cost and traffic flow equations analogous to (7) and (8).

It can be seen that the frequency of travel for an individual depends on two integrals: the first expresses the individual’s total travel costs as the sum of travel costs along a road experiencing different congestion at every point, while the second expresses the total traffic flow at a point as the sum of all frequencies of travel from more distant locations. Thus the behaviour of each household depends on that of all other households.

If these integrals could be evaluated at a particular location then the shopping frequency of the individual located at that point could be determined. The set of frequencies at all locations can be solved by approximation techniques for numerically specified cases, even though it would be difficult to derive an explicit analytical solution. An iterative scheme of solution is employed: for a given set of parameters an initial
solution is found on the assumption that there are no congestion effects (that is, that each household is the only one shopping). This allows estimates of frequency at each location to be calculated. From these values, the integrals for total traffic at each point and the associated travel time for an individual, based on this traffic flow and its congestion effect, can be calculated. This calculation then yields a revised set of shopping frequencies, based on the estimates of traffic flow, and the process is iterated until it converges (the total traffic reaching the centre is taken as the variable whose values are checked for convergence to a pre-assigned tolerance). In order to evaluate the integrals at each round of the iteration a numerical approximation is required since exact expressions are not available. The length of the town is divided into ten equal steps and the integrals are evaluated by taking the average values of the functions between adjacent points and multiplying by the step width — these values are then summed over all the steps. Experiments with increasing the number of steps showed that, for the parameter values chosen below, an approximation based on ten steps gave the same answer, to four significant figures, as one based on twenty steps.

4. A Numerical Illustration for a Single Shopping Centre

For this Section it is assumed that the length of the town \((T)\) is one unit and that the uniformly spread population has a density \((D)\) of one per unit length. Each household has an income \((Y)\) of five units per period, the cost of storing unit value of shopping for one period \((2c)\) is 0.05, and the price of a unit of the good is 1. The cost of travel of unit distance in the absence of congestion cost \((a)\) is 0.2 per trip, while the road capacity \((K)\) is 1. The cost of parking in the absence of congestion \((m)\) is 0.5 units per trip and the capacity of the parking area \((Z)\) is 1. The single shopping centre is located at one end of the town.

The model is solved to yield the total traffic arriving at the centre per period \((N(0))\) which is also equal to the total parking use; the total travel cost per trip \((g(1))\) and the frequency of travelling \((f(1))\) from the most distant household (at the city “edge”); and the total parking cost per trip \((x)\) which is the same for all households. In addition it is possible to calculate, from the first iteration, the total traffic flow and travel times, in the absence of congestion effects on travel and parking costs. Table 1 gives the results for the basic set of parameter values.

As expected, ignoring congestion costs generates a substantially lower travel and parking cost per trip and a higher frequency of shopping and total traffic flow. The impact of the congestion costs is quite strong even though the capacity of the road and parking systems are less than 50 per cent utilised before allowing for the impacts of the congestion (the road has a capacity of unity and in the absence of any congestion costs would experience a use of 0.434 cars per period). Proportionately, the parking costs are forced up by more than the travel costs at the city edge. This is because the maximum congestion effects of the extra traffic flow are felt most strongly at the car park. For those travelling to the centre some of the journey is at lower levels of congestion, so that proportionately the cost rise for travel is less than for parking.

282
A Model of Travelling to Shop with Congestion Costs

R. W. Bacon

Table 1

Characteristics of Travel for Base Case Parameters

<table>
<thead>
<tr>
<th></th>
<th>With congestion</th>
<th>Without congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total traffic at centre per period</td>
<td>0.350</td>
<td>0.434</td>
</tr>
<tr>
<td>Frequency at city edge</td>
<td>0.327</td>
<td>0.398</td>
</tr>
<tr>
<td>Total travel cost from edge</td>
<td>0.248</td>
<td>0.200</td>
</tr>
<tr>
<td>Total parking cost per trip</td>
<td>0.769</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 2

Elasticities with Respect to Travel Parameters

<table>
<thead>
<tr>
<th></th>
<th>Frequency of shopping at city edge</th>
<th>Total traffic arriving at centre</th>
<th>Total travel cost per trip at city edge</th>
<th>Total parking cost per trip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Road capacity</td>
<td>+0.031</td>
<td>+0.023</td>
<td>-0.213</td>
<td>+0.010</td>
</tr>
<tr>
<td>Parking capacity</td>
<td>+0.179</td>
<td>+0.208</td>
<td>+0.049</td>
<td>-0.457</td>
</tr>
<tr>
<td>Travel costs (i)</td>
<td>-0.150</td>
<td>-0.083</td>
<td>+1.000</td>
<td>+0.000</td>
</tr>
<tr>
<td>Travel costs (ii)</td>
<td>-0.116</td>
<td>-0.057</td>
<td>+0.980</td>
<td>-0.031</td>
</tr>
<tr>
<td>Parking costs (i)</td>
<td>-0.380</td>
<td>-0.450</td>
<td>+0.000</td>
<td>+1.000</td>
</tr>
<tr>
<td>Parking costs (ii)</td>
<td>-0.319</td>
<td>-0.362</td>
<td>-0.092</td>
<td>+0.811</td>
</tr>
</tbody>
</table>

(i) in the absence of congestion effects
(ii) allowing for congestion effects

The effects of variations in the travel cost parameters, with all other parameters held constant, can be calculated for each parameter in turn. These calculations are repeated, ignoring the impacts of congestion for the non-capacity parameters.

In order to summarise these results and to put them on an equal footing by removing scale effects, the percentage change in response for a one per cent change in each of the four travel parameters is calculated at base case values with the arc elasticity formula:

\[ e_{vi} = \frac{(v_2 - v_1)(v_2 + v_1)}{([i_2 - i_1][i_2 + i_1])] \]  

(11)
where
\[ v_2 \] is the value of the variable of interest corresponding to the travel parameter at one step above the mean value;
\[ v_1 \] is the value at one step below the mean value;
\[ t_2 \] is the value of the travel parameter one step above the mean value;
\[ t_1 \] is the value of the travel parameter one step below the mean value.

The step sizes are 0.25 for the road (K) and parking (Z) capacities, 0.05 for the travel cost (a) and 0.1 for parking cost (m). The results are shown in Table 2.

The elasticities show how, in proportional terms, the impacts of changes in travel and parking costs have a substantially smaller effect once the feedback from congestion is taken into account. The impact of variations in the two capacities is seen to be sensitive to the relative importance of these elements in total travel costs. The relatively high parking charge gives greater weight to a change in parking capacity than does the lower cost of travel per mile.

5. Traffic Flows When There Are Rival Centres

So far the analysis has concentrated on the case where there is a single shopping centre, so that all shopping trips must be made to that centre. Improvements in travel conditions can generate more trips from each household but they do not divert trips from one centre to another. As a way of exploring trip diversion as well as trip generation, a very simple case is considered. A linear town with uniform population density is assumed to have a shopping centre at each end of the town. The shops charge identical prices for the good so that the complications raised by price competition are ruled out. The road system is assumed to have uniform capacity at every point so that any improvement is equally beneficial to both centres in the absence of congestion. Consumers are assumed to shop at that centre which allows them to maximise their total purchases per period. If parking provisions were equal at the two centres, then the market boundary would be halfway between the two centres. However, if the centres offer different levels of parking provision, then one would become more attractive and would draw some trade from consumers living nearer to the other centre. The aim of this section is to illustrate this traffic diversion effect. There is also an associated congestion feedback since the increased attraction for existing customers plus the diverted customers will raise congestion cost on the road and modify the effects of the reduction in parking costs at the car park.

The length of the town is assumed to be two units, so that under symmetrical costs the boundary for each centre would give identical behaviour to the single centre case discussed above. All the other parameter levels are set to the same levels as in the single-centre case, except for the parking capacities \( Z_1 \) and \( Z_2 \). The capacity at centre 2 is fixed at one unit, while that at centre 1 is varied. The market boundary is where the optimal quantity per period purchased at either centre is equal.\(^1\) This value is identified by a further

\(^1\) Since the constrained optimum from equations (1) and (2) can be expressed as an unconstrained optimum problem in the single variable \( f \), it follows that when the quantities per period are equal the frequencies will be also.
iteration. An initial market boundary (halfway between the two centres) is chosen and the equilibrium traffic flows are calculated for each centre as detailed above. The quantities purchased per period at the boundary for the two centres are evaluated using equations (4) and (5). If these values are different, by more than the tolerance level of the iteration, then a new market boundary is calculated by extrapolating expenditure per period as a linear function from the city centre to the old market boundary. The two extrapolations meet at a point and the new market boundary is taken to be halfway between this point and the old market boundary. The calculations are then iterated for the new market areas and the process is repeated until it converges with the optimal expenditures for the two centres equal at the boundary. Table 3 gives the results for the case where the parking capacity of centre 1 is varied.

The increase of parking provision at centre 1 has the effect of moving the boundary towards centre 2. In the absence of price competition this effect is quite strong and would clearly have a powerful effect on the amount of trade going to each of the centres as well as on the amount of traffic. Even though the boundary moves away from centre 1, so that consumers at the market boundary have further to travel, the reduction in parking costs still results in an increase in the frequency of travel for all shoppers. At the same time, the traffic visiting centre 2 is reduced, but not by an equal amount. The extension of the market area for centre 1 means that the travel cost from the boundary increases, both because the distance is longer than before and because of the extra road congestion created by trip generation and trip diversion. Travel costs to centre 2 fall for similar reasons. Parking costs fall at both centres — the extra capacity at centre 1 more than compensates for the extra

### Table 3

*Traffic Flows to Competing Centres when the Parking Provision for Centre 1 (Z1) is Altered*

<table>
<thead>
<tr>
<th></th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of boundary from centre 1</td>
<td>0.769</td>
<td>0.913</td>
<td>1.00</td>
<td>1.056</td>
<td>1.143</td>
</tr>
<tr>
<td>Frequency to centre 1 from boundary</td>
<td>0.311</td>
<td>0.318</td>
<td>0.327</td>
<td>0.332</td>
<td>0.342</td>
</tr>
<tr>
<td>Frequency to centre 2 from boundary</td>
<td>0.311</td>
<td>0.318</td>
<td>0.327</td>
<td>0.332</td>
<td>0.342</td>
</tr>
<tr>
<td>Total traffic to centre 1</td>
<td>0.244</td>
<td>0.305</td>
<td>0.350</td>
<td>0.378</td>
<td>0.427</td>
</tr>
<tr>
<td>Total traffic to centre 2</td>
<td>0.405</td>
<td>0.371</td>
<td>0.350</td>
<td>0.335</td>
<td>0.311</td>
</tr>
<tr>
<td>Travel cost to centre 1 at boundary</td>
<td>0.176</td>
<td>0.217</td>
<td>0.248</td>
<td>0.263</td>
<td>0.295</td>
</tr>
<tr>
<td>Travel cost to centre 2 at boundary</td>
<td>0.313</td>
<td>0.270</td>
<td>0.248</td>
<td>0.229</td>
<td>0.205</td>
</tr>
<tr>
<td>Parking cost at centre 1</td>
<td>0.917</td>
<td>0.843</td>
<td>0.769</td>
<td>0.717</td>
<td>0.636</td>
</tr>
<tr>
<td>Parking cost at centre 2</td>
<td>0.818</td>
<td>0.794</td>
<td>0.769</td>
<td>0.752</td>
<td>0.726</td>
</tr>
</tbody>
</table>
traffic that is attracted, while there is a fall in traffic to centre 2 with unchanged provision. A comparison with Table 2, in which parking provision for a single centre is altered by the same amount, indicates the importance of trip diversion. The elasticity for the total traffic flow with respect to a change in parking provisions (around a mean value of 1) is 0.208 in the absence of trip diversion, but with the extra traffic from trip diversion this rises to 0.428.

6. Evaluating Improvements in Travel Costs

The modelling of travel costs and parking costs incorporates one element for the cost in the absence of other users (which will be independent of capacity which is irrelevant) and a second element to measure the degree of utilisation of capacity. It is this second element which can be affected by the action of the public authority or the shopping centre owner as they alter capacity itself. The model thus relates the frequency of travel of individuals to changes in capacity and by adding these together allows for the feedback as total traffic alters and impinges on individuals through the congestion costs. Thus, for a change in capacity, the final impact on individual frequencies and travel costs can be calculated as in Sections 4 and 5. Furthermore, the fact that frequency is derived from a specific objective function (quantity purchased per period), makes it possible to evaluate proposed capacity changes in terms of the changes this would make to the objectives of the consumers.

For a population spread uniformly along a linear town, this evaluation would require the integration along the line of the optimal quantity purchased (in the before and after situations). Although that is, in principle, possible with the model developed in this paper, a simpler approximation to the change in benefits is available. The unconstrained objective function can be written:

\[ Q(t) = \{ f(t)/(p f(t) + p c)\} \{ y - G(t) f(t)\} \]  

(12)

where \( f(t) \) is the single choice variable. Hence, by the "envelope" theorem, the impact on the optimal quantity per period — \( Q^*(t) \) — of a change in the parameter \( G \), is given by:

\[ \frac{\partial Q^*}{\partial G} = -f^* [f^*/(p f^* + p c)] \]

(13)
evaluated at the optimal frequency \( f^* \). If this frequency is large relative to the inventory cost parameter, then:

\[ \frac{\partial Q^*}{\partial G} = -f^*/p \]

(14)

This result can be approximated for finite changes as:

\[ \Delta Q^* = -f^* \Delta G/p \]

(15)
The interpretation of (15) is very simple — the change in consumer benefits resulting from a change in the transport cost parameter is approximately equal to the change in travel costs per trip (allowing for the feedback through congestion effects) multiplied by the number of trips at the original optimum divided by the price of a unit of the good (that is, the extra number of goods that could be purchased if the consumer continued with the same travel pattern under the new costs).
To illustrate these ideas the impact of an improvement in capacity is evaluated in three different ways:

(i) using the objective function to calculate the exact change in benefits allowing for the new equilibrium set of frequencies;
(ii) using the approximation (15) for this change in benefits;
(iii) using the approximation (15) based on the assumption that the change in capacity will leave, for the individual, total traffic unchanged.

Cases (i) and (ii) both require the new equilibrium set of frequencies to be worked out and, as shown above, the solution for any household depends on that for all other households. The third case ignores this interaction and evaluates the capacity improvement for an individual on the basis that total traffic flow is unchanged so that the induced extra traffic is ignored.

In order to evaluate these alternatives a particularly simple case is examined. It is assumed that the parking capacity \( Z \) in the single-centre case of Section 4 is increased to two units. The household evaluated is one which has zero distance to travel and which just pays parking costs. At the original optimum \( Z = 1 \) the total traffic arriving at the centre — \( N(0) \) — was 0.350, so that the parking cost for this household per trip is given by equation (10) as \( 0.5 \times 0.65 = 0.769 \). Using the expressions for the optimum (4) and (5) it follows that the maximum number of goods purchased per period would be 4.417 units (\( p = 1 \)). With a parking capacity of 2, the model was solved to yield a total traffic flow at the centre of 0.384 per period and hence a parking cost of 0.619. Using this parking cost the new optimal quantity — \( Q^* \) — rises to 4.474, giving a benefit of \( \Delta Q^* = +0.057 \). The approximating equation (15), allowing for the full change in travel costs produced at the new equilibrium, identifies the change in travel costs per trip as \(-0.150\), which when multiplied by the original optimal frequency (0.379) yields exactly the same estimate of the change in benefits (to three decimal places). The approximation is very good, despite the large percentage change taken for the travel cost. The ratio of the optimal frequency to the inventory cost parameter \( (c = 0.025) \) is sufficiently great to show why the method works.

The “short cut” method, which takes travel flow as unchanged when evaluating the change in travel costs for the household, would have a traffic flow of 0.350, which when combined with the new parking capacity yields a parking cost of 0.606. Using this in the “envelope” theorem, approximation gives an estimate for the increase in the quantity purchased per period of 0.0618, which is about 8 per cent greater than the true value. The error in ignoring the induced traffic caused by the travel improvement will clearly be greatest when the system is nearest saturation. In the present case the system starts with a car park that is used only to 35 per cent of its capacity, so that the induced traffic created by doubling its capacity is likely to be relatively small.

The method illustrated above for a single household can be extended to the whole town, but the calculation will be considerably more complicated for households not living at the centre because congestion is changed at every point along the road so that the change in the total travel cost per trip will also require a complex set of calculations.
7. Conclusions

The modelling of the interaction between shopping behaviour and congestion is of considerable interest because of the endogeneity of the frequency of shopping. In the presence of congestion costs, households can partially offset their impact on the journey to shop by making less frequent trips and buying more on each trip. This in turn implies that any change to the urban system will produce a smaller end result on the congestion observed in the system than would be the case where the frequency of shopping was fixed (as is the case of the journey to work). This might be thought to imply that policies to affect congestion should concentrate solely on the journey to work. However, it is argued in this paper that congestion in parking may be more acute for shoppers than for those travelling to work, so that parking provision can be an important variable for both planning authorities and for retail establishments seeking to compete by non-price methods. The numerical examples given in the paper, to illustrate the potential effects of alterations in travel-related factors, reveal that, even in the single-centre case where there is no trip diversion between centres, changes in road and parking capacity can have large effects on the total amount of traffic coming to a centre. The relative importance of such changes will depend on the relative importance of travel costs per mile to parking charges in the absence of congestion. Once a second shopping centre is taken into consideration the impact of improved parking facilities at one of the centres has powerful effects on traffic diversion, as consumers not only shop more often but also switch to the centre with the less congested parking. Even with the feedback of higher traffic volumes on to the degree of congestion and thus on travel costs, the total sensitivity of shopping traffic to changes in the capacity of the congestible infrastructure is great.

The models developed in this paper allow, in principle, an exact evaluation of the benefits of improvements in road and parking capacities. Because the frequency of travel is explicitly linked to the household’s objective function, changes in travel costs can be linked to consumer welfare. Hence a measure designed to improve consumers’ welfare by reducing congestion can be evaluated both in terms of its impact on travel time and in terms of the benefits associated with this. If the total travel cost can be calculated for the before and after situations, then a particularly useful approximation to the benefits is available from using the “envelope” theorem.

The models used are simple, being designed to capture just the endogeneity of the frequency of shopping and congestion effects due to the total use of roads and parking. Several other potential feedbacks on to household behaviour have been ignored. The presence of varying amounts of congestion along the road will alter the attractiveness of different locations and house rents could be expected to respond to this, thus altering shopping patterns as in the models of Fujita and Thisse (1986), Ohta, Asami and Kohlhase (1990), and Bacon (1992a). Household preferences for leisure as well as goods affect shopping patterns, and the time effects of congestion on the roads and at the parking area could be separated from money costs, to allow for the relative strength of preference for leisure as opposed to goods, in a model similar to that developed above. On the supply side, shops might compete by price as well as by the provision of parking. The decision on the
A Model of Travelling to Shop with Congestion Costs

amount of parking to provide, which clearly can be an important strategic variable, will depend not only on its impact on traffic and trade but on the price of land at different locations, which in turn would require an integrated model of the land market. These models would be more complicated than that presented in this paper but the key insights would carry over.

References


Date of receipt of final manuscript: January 1993