THE SPATIAL DISTRIBUTION OF RETAIL EXPENDITURES

Joint Estimation of a Polychotomous Discrete-Continuous Choice System

By Peter O. Barnard* and David A. Hensher†

INTRODUCTION

A growing area of interest in transport economics is the relationship between discrete and continuous choices. Discrete-continuous choice models have been used to study choices in a number of areas such as residential appliance holdings and consumption (Dubin and McFadden (1984), and Brownstone (1980)) and automobile holdings and use (Manering and Winston (1985), Train (1986), Hensher and Milthorpe (1987), and Hensher et al. (1992)).

In the present study the economic links between discrete and continuous choices are used to analyse shopping behaviour. The empirical study forges a link between two hitherto disparate approaches to examining shopping behaviour. One approach, characterised by discrete choice shopping models, analyses the decision of where to shop in isolation of how much to spend; for example the contributions of Domencich and McFadden (1975), Recker and Kostyniuk (1978), Koppelman and Hauser (1978), McCarthy (1979), Gautschi (1981), Weisbrod et al. (1984), Parcells and Kern (1984) and Eagle (1984). Another set of models has examined shopping expenditure or retail sales patterns, largely ignoring how this is related to individual decisions of where to shop; for example, Curhan (1972), Guy (1984) and Morey (1980). To the extent that these two choices are interrelated, these models will be less than complete and the results may be biased. From an information perspective it is beneficial for developers and planners to know both the number of persons using a shopping centre and the expenditure at that centre.

* Australian Meat and Livestock Corporation.
† Institute of Transport Studies, Graduate School of Business, University of Sydney.
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We extend recent work on discrete-continuous choice modelling by jointly estimating a model system where the discrete choice is characterised by a polychotomous choice. In contrast to two-stage methods, joint estimation allows for the imposition of a number of cross-equation restrictions implied by economic theory. As a result of this, the link between economic theory and the empirical model is stronger than in past studies that have used two-stage estimation techniques.

The remainder of the paper is divided into four sections. The next section sets out a general theoretical framework for analysing shopping destination and expenditure choices. This framework is refined in the following section in order to derive an empirical and jointly estimable destination-expenditure choice system. The penultimate section reports results from an empirical application. The final section contains our conclusions.

THE THEORY OF SHOPPING DESTINATION AND EXPENDITURE CHOICE

The conventional economic paradigm of self-gratification assumes that a representative consumer selects a shopping destination and levels of shopping expenditure, leisure and consumption of other goods as if to maximise utility. The representative consumer may be identified as the main household shopper for an analysis conducted at the household level. A general form of the consumer’s utility function from the analyst’s perspective is written as:

\[ U = U(G, B_1, B_2, \ldots, B_N, Z, L) \]  

(1)

where \( G \) is a vector \((g_1, g_2, \ldots, g_N)\) representing consumption of shopping items by the representative consumer from destinations \( 1, 2, \ldots, N \), respectively; \( B_i \) is a vector \((b_{i1}, b_{i2}, \ldots, b_{ik})\) of \( K \) quality variables associated with the consumption of shopping items from the \( i \)th destination; \( Z \) is the Hicksian composite commodity encapsulating consumption of other goods; \( L \) is leisure time and the subscript referencing the consumer has been suppressed.

Maximisation of utility is subject to income and time constraints:

\[ Y = \sum_{i}^{N} p_i g_i + Z \sum_{i}^{N} \xi_i c_i \]  

(2)

\[ L = T - \sum_{i}^{N} \xi_i t_i \]  

(3)

where \( p_i \) is an index of shopping prices at the \( i \)th destination, \( \xi_i = \xi_i (g_i) \) is an indicator function with \( \xi_i = 1 \) if \( g_i > 0 \) and \( \xi_i = 0 \) if \( g_i = 0 \); \( c_i \) is the cost of travel to the \( i \)th destination; \( Y \) is income; and \( T \) is total time available. In equation (2) income, shopping prices and travel costs have been normalised by the price, \( p_{i0} \), of the Hicksian composite commodity. Alternative \( i \) is strictly a destination/mode combination, since travel times and costs vary by alternative modes as well as destinations. It is conceptually easier, however, to think of \( i \) solely in terms of destination choice.
An element of discreteness can be introduced into the model by assuming that in any time period the consumer selects one destination for shopping purchases. A possible behavioural source for this restriction is that the consumer views alternative shopping destinations as perfect substitutes, but one destination must be chosen since shopping represents an essential activity. This effectively concentrates attention on the destination choice of shopping travel behaviour. It implies that \( Z, L \), and one of the \( g_j \)'s is positive with all \( g_i \) (\( i \neq j \)) equal to zero. The discrete element of the solution relates to which of the \( g_i \)'s are to take zero values. A continuous dimension is also evident because the non-zero \( g_i \), \( Z \) and \( L \) can be consumed in any quantities.

In obtaining optimal values of the \( g_i \)'s, \( Z \) and \( L \) the consumer can be thought of as applying a two-stage maximisation process. Assuming that shopping destination 1 is chosen, and if \( g_1 = 0 \), then

\[
\frac{\partial U}{\partial b_{11}} = \frac{\partial U}{\partial b_{12}} = \ldots = \frac{\partial U}{\partial b_{1k}} = 0
\]

(Hanemann, 1984), the maximisation problem can be redefined as:

\[
\text{max } U_1 = U_1(g_1, B_1, Z, L)
\]  

subject to:

\[
Y = p_1 g_1 + Z + c_1
\]  

\[
L = T - t_1
\]  

The solution to (4a), (4b), (4c) is a set of demand equations, describing shopping purchase \( (g^*)_1 \), other goods purchases \( (Z^*)_1 \) and time spent on leisure activities \( (L^*)_1 \) conditional upon the choice of shopping destination 1. The process can be repeated for \( g_2 \) > 0, \( g_1 = g_3 = \ldots = g_N = 0 \), and so on.

Associated with each set of conditional demand functions is a conditional indirect utility function (CIUF). The CIUFs define the maximum attainable levels of utility conditional upon the choice of particular shopping destinations. The CIUF associated with shopping destination \( i, V_i \), is defined by:

\[
V_i = U_i(g^*_i, Z^*_i, L^*_i, B_i) = V_i(p_1, B_1, T - t_1, Y - c_1)
\]

The global utility maximising problem can now be expressed as:

\[
\bar{U}(\bar{p}, \bar{B}, T - \bar{t}, Y - \bar{c}) = \max \{V_1(p_1, B_1, T - t_1, Y - c_1),
\]

\[
V_2(p_2, B_2, T - t_2, Y - c_2), \ldots ,
\]

\[
V_N(p_N, B_N, T - t_N, Y - c_N)
\]

Shopping destination \( j \) will be chosen if

\[
V_j(p_j, B_j, T - t_j, Y - c_j) > V_i(p_i, B_i, T - t_i, Y - c_i) \text{ for all } i \neq j
\]

The \( V_j \)'s are the functions encountered in conventional derivations of discrete choice models (for example, McFadden (1981), Small (1982), Domencich and McFadden (1975), Hensher and Johnson (1981), Greene (1990)).

Note that the \( V_j \)'s contain variables describing prices at destination \( i \), other attractiveness variables associated with destination \( i \), and travel times and costs to destination \( i \)
all the variables normally included in a behaviourally based shopping destination choice model (Recker and Kostyniuk (1978), Koppelman and Hauser (1978), McCarthy (1979), Gautschi (1981), Weisbrod et al. (1984), Parcels and Kern (1984), Eagle (1984)).

The convenience of working with the indirect utility function derives from the knowledge that demand equations which are consistent with utility-maximising behaviour can be obtained by applying Roy’s identity (Roy, 1942) to $V_{ij}$, rather than explicitly solving the maximisation problem in equation (4). In particular, the conditional demand equation corresponding to the CIUF shown in equation (5) can be derived as:

$$g_i^* = -\frac{\partial V/\partial p_i}{\partial V/\partial U} = g_i^*(p_{ij}, B_{ij}, T - t_i, Y - c_i)$$ (8)

Equation (8) can be expressed in expenditure form as:

$$p_i g_i^* = E_i = p_i g_i^*(p_{ij}, B_{ij}, T - t_i, Y - c_i)$$ (9)

Equation (9) and the corresponding indirect utility function (equation (5)) establish the link between shopping centre choice and shopping expenditure.

ISSUES IN THE EMPIRICAL ESTIMATION OF A JOINT SHOPPING DESTINATION AND EXPENDITURE CHOICE MODEL

In deriving an empirically estimable model, it is necessary to allow for variation in the CIUFs across individuals. Different individuals will face different travel times and costs to each destination and may perceive price and quality aspects of the destinations differently. Furthermore, it is necessary to introduce error terms into the CIUFs and expenditure functions.

With these additions, the CIUF and expenditure function associated with destination $i$ and individual $q$ may be specified as:

$$V_{iq} = \overline{V}_{iq}(p_{iq}, B_{iq}, T - t_{iq}, Y - c_{iq}) + \epsilon_{iq}$$ (10)

and

$$E_{iq} = p_{iq} \overline{g}_{iq}^*(p_{iq}, B_{iq}, T - t_{iq}, Y - c_{iq}) + u_{iq}$$ (11)

where $\overline{V}_{iq}$ is the observable or representative component of the CIUF; $\overline{g}_{iq}^*$ is the representative component of the conditional demand functions; and $\epsilon_{iq}$ and $u_{iq}$ are error terms in the CIUFs and expenditure functions, respectively.

To simplify computational aspects of the model system, we have chosen to specify a form for the $\overline{V}_{iq}$ that will yield, after application of Roy’s identity, a linear-in-the-parameters shopping expenditure model. A family of CIUFs that meet this requirement is defined by:

$$\overline{V}_{iq} = f_2(f_2(p_{iq} B_{iq} Y - c_{iq}, T - t_{iq}) p^{\gamma_{iq}})$$ (12)

where $f_2$ is linear in its arguments. A specific form of equation (12) is:
\[ \bar{V}_{iq} = \left[ \alpha_1 - \alpha_2 \log(p_{iq}) + \sum_{k=3}^{K+2} \alpha_k b_{iqk} + \alpha_{K+3} (Y_q - c_{iq}) + \alpha_{K+4} (T - t_{iq}) \right] p_{iq}^{\alpha_{K+3}} \]  

(13)

which is the form for the “representative” component of the CIUFs utilised in the current study. Equation (13) is a variant of the form of CIUF used by Dubin and McFadden (1984) in a binary choice context. Applying Roy’s identity to equation (13) provides the expected shopping expenditure level for consumer \( q \):

\[ \bar{E}_{iq} = \alpha_1 - \alpha_2 \log(p_{iq}) + \sum_{k=3}^{K+2} \alpha_k b_{iqk} + \alpha_{K+3} (Y_q - c_{iq}) + \alpha_{K+4} (T - t_{iq}) + \frac{\alpha_2}{\alpha_{K+3}} \]  

(14)

For equation (13) to represent a valid CIUF, with \( \bar{V}_{iq} \) defined by equation (16), it must conform to a number of conditions (Diewert, 1974):

(i) \( V(.) \) is continuous for all prices and income > 0,
(ii) \( V(.) \) is homogeneous of degree zero in prices and income,
(iii) \( V(.) \) is non-increasing in prices and non-decreasing in income, and
(iv) \( V(.) \) is quasi-convex in prices.

Condition (ii) is automatically met by the formulation of the model. The other conditions are tested upon estimation of the model.

The specification adopted in equations (10) and (11) implies treatment of the unobservable components in the CIUFs and conditional expenditure functions outside the strict theoretical framework. To treat the error terms within the theoretical framework introduces almost impossible complexities into estimation. The shopping choice model developed in this paper is inherently more complex than previously estimated model systems dealing with interrelated discrete and continuous choices (for example, Dubin and McFadden (1984), Mannerling and Winston (1985)). The discrete choice modelled here is polychotomous, whereas most previous discrete/continuous modelling efforts have concentrated on dichotomous choices (for example, in the case of Dubin and McFadden (1984), choice of electricity versus gas space and water heating). Further, the great number of shopping destinations available forces the analyst to select a model specification in which the alternatives are unranked.

Rather than fully integrating the error terms in the estimable models with the theoretical framework, a statistical procedure is used instead that is in harmony with the theory developed above.

In presenting the statistical procedure we economise on notation by defining a row vector \( Z_{iq} \) containing \( \log(p_{iq}), B_{iq} \ T - t_{iq} \) and \( Y_q - c_{iq} \). Equations (10), (13), and (11), (14) can then be rewritten as:

\[ V_{iq} = \bar{V}_{iq}(Z_{iq}, \alpha) + \varepsilon_{iq} \]  

(15)

and,

\[ E_{iq} = Z_{iq} \beta + u_{iq} \]  

(16)

where \( \alpha \) and \( \beta \) are the unknown parameter vectors in the discrete and continuous choice models respectively, with \( \beta = f(\alpha) \).
There are two important features of equation system (15) and (16). Firstly, shopping expenditure for each individual is only observed at the chosen destination. From the theoretical model, in choosing a shopping destination the individual calculates optimal expenditure levels at each destination; but the optimal expenditure levels at non-chosen centres are hidden from the analyst. The result is systematic missing data on the $E_{iq}$. Secondly, it is likely that the error terms $e_{iq}$ and $u_{iq}$ will be correlated because the disturbances in both the discrete destination choice model and continuous shopping expenditure model arise from the same source, namely, uncertainty concerning the CIUFs.

In accordance with most past studies of shopping destination choice, it is assumed that the $e_{iq}$ are independently and identically distributed extreme value type 1, leading to a multinomial logit (MNL) destination choice model, and the $u_{iq}$ are normally distributed. Recognising the conditionality of observed data points in the expenditure model, equation (16) may be respecified in estimation form as:

$$E_{iq} = Z_{iq} \beta + E(u_{iq} | V_{ij} > V_i, \forall i \neq j)$$

where $E(\cdot)$ denotes "the expected value of". The last term on the right-hand side of equation (17) will, in general, be non-zero resulting in biased estimates of $\beta$ when using OLS.

Using a technique developed by Lee (1983), building upon the work of Heckman (1976), the term $E(u_{iq} | V_{ij} > V_i, \forall i \neq j)$ can be evaluated by the following method. Let

$$\eta_{iq} = \max \{ V_{ij} - e_{iq} (i = 1, 2, \ldots, N_q; i \neq j) \}$$

Shopping destination $j$ will be chosen by consumer $q$ if $V_{ij} > V_i$ for all $i \neq j$, or

$$\tau_q = j \text{ iff } \eta_{iq} < \frac{V_i}{Z_{iq} \alpha}$$

With the $e_{iq}$ iid extreme value type 1, the distribution of $\eta_{iq}$ is:

$$D(\eta_{iq}) \propto \exp(\eta_{iq}/\mu) \left[ \sum_{i=1, i \neq j} \exp \left( \frac{V_i}{Z_{iq} \alpha}/\mu \right) \right]$$

where $\mu$ is the scale parameter of the logit distribution

$$\mu = \sqrt{\frac{3}{\pi}} \sigma_{e_{ij}}$$

with $\sigma_{e_{ij}}^2$ the variance of $e_{ij}$. In turn, $\eta_{jq}$ can be transformed into a standard normal variate, $\eta^*_{jq}$, by applying:

$$\eta^*_{jq} = J(\eta_{jq}) = \phi^{-1}[D(\eta_{jq})]$$

where $\phi^{-1}$ is the inverse of the standard normal distribution. Computationally accurate methods are available for approximating the inverse of the standard normal distribution (NAG, 1984). With this transformation $j$ will be chosen if $\eta^*_{jq} < J(\frac{V_i}{Z_{iq} \alpha})$.

Given that the $u_{iq}$ are also normally distributed, the bivariate distribution between $\eta^*_{jq}$ and $u_{jq}$ can be specified as

$$\mathcal{N}(0, 0, 1, \sigma_{u_{jq}}, \rho_{u_{jq}} \eta^*_{jq})$$

The equation system (15) and (16) should only be estimated independently when the correlation coefficient, $\rho_{u_{jq}} \eta^*_{jq}$, is equal to zero. In the more general case, the conditional...
expectation $E(u_{iq} \mid I_q = j)$ needs to be included as a regressor in equation (16). Through integration this can be shown to equal:

$$\sigma_{u_j} \eta_j \Phi \left( J \left[ V(Z_{ijp}, \alpha) \right] \right) / D \left[ V(Z_{ijp}, \alpha) \right]$$

(22)

where $\sigma_u \eta_j$ is the covariance between $u_j$ and $\eta_j$ and $\Phi$ is the density function of the standard normal, so the shopping expenditure model becomes:

$$E_{ij} = Z_{ij} \beta - \sigma_{u_i \eta_j} \Phi \left( J \left[ V(Z_{ijp}, \alpha) \right] \right) / D \left[ V(Z_{ijp}, \alpha) \right] + \theta_{ij}$$

(23)

where $\theta_{ij}$ is a new error term with $E(\theta_{ij} \mid I_q = j) = 0$.

The log-likelihood function for the model system for a sample of $Q$ individuals can now be specified as:

$$\log L = \sum_{q} \sum_{i} \left[ k_{iq} \left[ -\log \left( \sqrt{2\pi} \right) \sigma_{uu} - 0.5u_{iq}^2 / (\sigma_{uu})^2 \right] + k_{iq} \log \phi \left( \frac{1}{\sqrt{2\pi\sigma_{uu}}} \left( \frac{u_{iq} - \sigma_{u_i \eta_j} \Phi \left( J \left[ V(Z_{ijp}, \alpha) \right] \right) / D \left[ V(Z_{ijp}, \alpha) \right]}{\sigma_{uu}} \right) \right) \right]$$

(24)

where $k_{iq} = 1$ iff $I_q = i$. The log-likelihood function is a member of the set of log-likelihood functions considered by Lee (1983). The first partial derivatives of equation (23) with respect to the structural parameters, $\alpha$, are:

$$\frac{\partial \log L}{\partial \alpha_{e}} = \sum_{q} \sum_{i} \left[ -k_{iq} \frac{u_{iq}}{(\sigma_{uu})^2} \frac{\partial u_{iq}}{\partial \alpha_{e}} + k_{iq} \frac{\phi(\kappa_{iq})}{\phi(\kappa_{iq})} \times \left\{ \phi[D(V_{iq})] - 1 \right\} \right]$$

(25)

$$\left\{ D(V_{iq}) \frac{\partial (V_{iq})}{\partial \alpha_{e}} - \sum_{j} D(V_{ij}) \frac{\partial (V_{ij})}{\partial \alpha_{e}} \right\} \frac{\rho_{u_i \eta_j} \frac{\partial u_{iq}}{\partial \alpha_{e}}}{[1 - (\rho_{u_i \eta_j})^2]^{0.5}}$$

where

$$\kappa_{iq} = \left\{ \phi^{-1} \left[ D(V_{iq}) \right] - \rho_{u_i \eta_j} (u_{iq} / \sigma_{\eta_j}) \right\} / [1 - (\rho_{u_i \eta_j})^2]^{0.5}$$

Equation (24) can be maximised using a number of algorithms, including the Davidson-Fletcher-Powell algorithm.

**EMPIRICAL RESULTS**

The model of equation (24) was used to study the distribution of grocery shopping expenditure across stores. The data obtained from a survey of shoppers in Adelaide (Australian Road Research Board, 1981) consisted of two parts. First, participating households were required to fill out diaries documenting one week of activities. At the end of that period, diaries were collected and the main household shopper interviewed regarding the household’s food shopping arrangements.
In the shopping interview, information was sought on outlets patronised by the household for grocery shopping, and possible alternatives to those outlets. This information defined the household's grocery shopping choice set. For each reported shopping outlet, respondents were asked to rate the outlet in terms of price, selection and store convenience. Selection and store convenience ratings were measured on a 5 point scale with a value range from "far above average" (5) to "far below average" (1). The price rating is based on a basket of commonly purchased grocery shopping goods (Choice, 1981) and expressed in monetary units. In addition, information on possible methods of travel to each outlet by the respondent was obtained, and for each mode specified, travel time and cost data collected.

It was important that the same store codes were used in the activity diaries as in the shopping questionnaire. Blank diary pages were divided into two parts. The lower half was designed to facilitate personal documentation of the nature, time, location and level of expenditure associated with each activity episode. The upper half was designed to allow the respondent to provide further information on each trip undertaken (that is, travel activity).

Data for estimating the model system were obtained from merging the shopping questionnaire information with shopping episodes recorded in the activity diaries. Diaries for main household shoppers who filled in the shopping questionnaire were interrogated for records of activity episodes involving grocery shopping with associated travel to and from the household's residence. Records were rejected if no expenditure information was provided, or if the store visited was not one of the set of stores provided by the shopper in the shopping questionnaire. Further records were excluded when no income information was provided.

The estimation data set consisted of observations on 236 store choices. In each case the choice set for an individual comprised the list of mode/store alternatives provided in the shopping questionnaire with the chosen alternative being the mode/store combination observed in the activity diary. Definitions of variables used in this study, along with summary statistical information, are shown in Table 1.

The data used to estimate the model rely on reported information by shoppers on centre and transport characteristics. Some contend that such information is unreliable or, worse still, that shoppers will deliberately bias information to justify use of their currently selected centre. An extensive analysis of this issue is to be found in Barnard (1987). In general there was little evidence of respondents supplying deliberately biased information.

Results from maximising the log-likelihood function of equation (24), with \( B'_{iq} = (SEL_{iq}, CONV_{iq}) \) using the Davidson-Fletcher-Powell algorithm are included in Table 2. A number of initial starting values for the unknown parameters were used to ensure that the global maximum of the log-likelihood function had been attained. This procedure is superior to generating initial parameter estimates from two-stage estimation when \( V_{iq} \) is non-linear in the parameters since the MNL log-likelihood function may be characterised by multiple local maxima. This study and Krishnamurthi and Raj (1988) represent the only known applications of FIML to the joint estimation of a polychotomous discrete/

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TABLE 1

Variable Definitions and Summary Statistics

<table>
<thead>
<tr>
<th>Mnemonic</th>
<th>Definition</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PRICE_{dq} )</td>
<td>An individual rating of prices at store ( d ) based on a basket of commonly purchased grocery shopping goods ($).</td>
<td>25.91</td>
<td>4.88</td>
</tr>
<tr>
<td>( SEL_{dq} )</td>
<td>An individual rating on a 5 point scale of the selection of grocery items available at store ( d ).</td>
<td>2.76</td>
<td>0.81</td>
</tr>
<tr>
<td>( CONV_{dq} )</td>
<td>An individual rating on a 5 point scale of the convenience of using store ( d ).</td>
<td>2.22</td>
<td>1.01</td>
</tr>
<tr>
<td>( TCOST_{mdq} )</td>
<td>The cost of individual ( q ) travelling to and from store ( d ) by mode ( m ):&lt;br&gt;– if mode is bus, ( TCOST = ) reported bus fare;&lt;br&gt;– if mode is car, ( TCOST = ) network highway distance ( \times ) 0.23;&lt;br&gt;– if mode is walk or bicycle, ( TCOST = 0 ) ($).</td>
<td>1.06</td>
<td>0.66</td>
</tr>
<tr>
<td>( TTIME_{mdq} )</td>
<td>The reported time for individual ( q ) to travel to store ( d ) by mode ( m ) (minutes).</td>
<td>7.47</td>
<td>4.25</td>
</tr>
<tr>
<td>( INCOME_{q} )</td>
<td>Weekly household income ($).</td>
<td>288.96</td>
<td>114.61</td>
</tr>
<tr>
<td>( EXPEND_{mdq} )</td>
<td>Grocery shopping expenditure by household ( q ) at store ( d ) when using mode ( m ).</td>
<td>15.70</td>
<td>17.35</td>
</tr>
</tbody>
</table>

continuous choice model system. McFadden et al. (1986) is an example of joint estimation of a dichotomous discrete/continuous model system.

When interpreting the parameter estimates of Table 2, from equations (13) and (14), the model specification includes the price-related parameter estimates as negatively signed. A positive estimate for \( \alpha_2 \) therefore indicates that as log \( (p_{iq}) \) increases, \( V_{iq} \) decreases. Similarly, a positive estimate for \( \alpha_2 \) indicates as \( p_{iq} \) increases, \( V_{iq} \) decreases. A number of factors serve to engender confidence in the model. These can be grouped under two headings; compatibility of the results with economic theory, and reasonableness of the parameter estimates. With respect to the former, the estimated CIUFs conformed to all necessary properties of indirect utility functions. The non-increasing price condition implies,
TABLE 2

Joint Store Choice/Shopping Expenditure Model Parameter Estimates and Statistics

<table>
<thead>
<tr>
<th>Variable/Parameter</th>
<th>Description</th>
<th>Parameter Estimate</th>
<th>T-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>$\alpha_1$</td>
<td>-13.0779</td>
<td>-10.76</td>
</tr>
<tr>
<td>log (PRICE)</td>
<td>$\alpha_2$</td>
<td>0.7012</td>
<td>3.02</td>
</tr>
<tr>
<td>SEL</td>
<td>$\alpha_3$</td>
<td>0.3758</td>
<td>2.94</td>
</tr>
<tr>
<td>CONV</td>
<td>$\alpha_4$</td>
<td>0.2070</td>
<td>1.79</td>
</tr>
<tr>
<td>(INCOME – TCOST)</td>
<td>$\alpha_5$</td>
<td>0.0469</td>
<td>17.51</td>
</tr>
<tr>
<td>(60 – TTIME)</td>
<td>$\alpha_6$</td>
<td>0.1212</td>
<td>4.19</td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td>0.2421</td>
<td>3.63</td>
</tr>
<tr>
<td>$\rho$</td>
<td></td>
<td>0.3357</td>
<td>8.26</td>
</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td>16.8174</td>
<td>11.47</td>
</tr>
<tr>
<td>Sample Size</td>
<td></td>
<td>236</td>
<td></td>
</tr>
<tr>
<td>log L at convergence</td>
<td></td>
<td>-986.6</td>
<td></td>
</tr>
<tr>
<td>log L (0)*</td>
<td></td>
<td>-1825.6</td>
<td></td>
</tr>
<tr>
<td>Likelihood ratio</td>
<td></td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

* log L(0) is defined as the value of the log-likelihood function with $\alpha_1 = \alpha_2 = \ldots = \alpha_6 = \rho = 0$ and $\mu = 1$.

\[
\frac{\partial V_{iq}}{\partial p_{iq}} = -p^{-(\alpha_5 + 1)}(\alpha_5 [\alpha_1 - \alpha_2 \log p_{iq} + \alpha_3 SEL_{iq} + \alpha_4 CONV_{iq}]
+ \alpha_5 (Y_q - c_{iq}) + \alpha_6 (T - t_{iq}) - \alpha_2) \leq 0
\] (25)

In the sample this condition was met for CIUFs associated with all alternatives in all choice sets. The non-decreasing income condition implies,

\[
\frac{\partial V_{iq}}{\partial Y_q} = \alpha_5 p^{-\alpha_5} \geq 0
\] (26)

and since $\hat{\alpha}_5 \geq 0$ this condition was also met for all estimated CIUFs. Finally, a test of the quasi-concavity condition is that the diagonal elements of the Slutsky matrix be non-positive (for example, Hausman (1981)). That is,

\[
S_{ii} = \frac{\partial Y^*_{iq}}{\partial p_{iq} \partial p_{iq}} = \left( \alpha_5 - 1 \right) [\alpha_1 - \alpha_2 \log p_{iq} + \alpha_3 SEL_{iq} + \alpha_4 CONV_{iq}]
+ \alpha_5 (Y_q - c_{iq}) + \alpha_6 (T - t_{iq}) - \alpha_2 / \alpha_5 \right) / p_{iq}^2 \leq 0
\] (27)
where \( P^*_q \) is the conditional cost function. In contrast to some other studies which experienced difficulties in meeting this condition (for example, Wales and Woodland (1977), Brownstone (1980)), all estimated CIUFs satisfied equation (27). Evaluated at mean levels for the explanatory variables

\[
\delta V_q/\delta P_{iq} = -0.0326, \quad \delta V_q/\delta Y_q = 0.0403 \quad \text{and} \quad s_q = -0.0308.
\]

The influence of each variable on store choice and grocery shopping expenditure can be gauged from an examination of the relevant elasticities, given in Table 3. Two types of expenditure (= \( x \)) elasticities are shown. One is indicative of the impact of a change in a variable on expenditure at a particular store; the other is indicative of a change in a variable on shopping expenditure in general.

Examination of the store choice elasticities shows, as anticipated, that an increase in travel time or travel cost to a store results in a decreased probability of that store being selected, as does an increase in store prices. Conversely, an increase in the perceived quality of a destination, as encapsulated in the variables \( SEL \) and \( CONV \), is predicted to increase patronage of that destination.

A comparison of the store-specific expenditure elasticities with the store choice elasticities, suggests that a change in the value of a variable is predicted to affect store expenditure in the same direction as store patronage, but with greater force. This result has a basis in theory. An increase in prices at a particular store, for example, will not only cause the utility associated with that store to decrease, and thus the probability of choosing the store, but will also cause those individuals who continue to use the store to spend less there.

### Table 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Store Choice Elasticities</th>
<th>Store-specific Expenditure Elasticities</th>
<th>Unconditional Grocery Shopping Expenditure Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>-0.274</td>
<td>-0.312</td>
<td>-0.038</td>
</tr>
<tr>
<td>SEL</td>
<td>0.597</td>
<td>0.630</td>
<td>0.039</td>
</tr>
<tr>
<td>CONV</td>
<td>0.277</td>
<td>0.340</td>
<td>0.037</td>
</tr>
<tr>
<td>TCOST</td>
<td>-0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TTIME</td>
<td>-0.358</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INCOME</td>
<td></td>
<td></td>
<td>0.717</td>
</tr>
<tr>
<td>TIME</td>
<td></td>
<td></td>
<td>0.892</td>
</tr>
</tbody>
</table>

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The unconditional grocery shopping expenditure elasticities possess the same signs as the store-specific expenditure elasticities, but are of significantly less magnitude. Expenditure on groceries is predicted to be virtually unaffected by a change in grocery prices, in the perceived selection of grocery items available, or in the perceived convenience of using grocery stores. An across-the-board 10 per cent increase in household incomes is predicted to result in a 7.2 per cent increase in grocery shopping expenditure. The time elasticity may be interpreted as the change in expenditure expected if more time for shopping became available.

All the elasticity estimates are of the expected magnitude. The store choice accessibility-related elasticity estimates are within the range suggested by other studies which have examined shopping destination choice behaviour (for example, Domenich and McFadden (1975), Richards and Ben-Akiva (1975)). The income inelasticity of grocery expenditures as found in the current study conforms with similar income-inelastic estimates for food expenditures obtained in classical studies of consumer demand using substantially different data and statistical methods (Houthakker (1957), Barten (1964), Theil et al. (1981), Podder (1971)). Store-specific expenditures are predicted to be more sensitive to the perceived range of merchandise available than to prices, an intuitively appealing result. Generally, however, there is a dearth of published estimates of store attribute elasticities with regard to grocery shopping expenditure with which to compare the values in column 2 of Table 3.

CONCLUSION

In this paper a model which fuses the shopping destination choices made by individuals with shopping expenditure decisions was developed. Economic theory was used to demonstrate a close relationship between these two facets of shopping behaviour. The form of this relationship was then used to develop an empirical model of shopping behaviour.

The use of FIML jointly to estimate the models associated with the store choice and shopping expenditure decisions meant that a set of cross-equation parameter restrictions implied by theory could be imposed on estimation. By basing the empirical model of shopping behaviour firmly on economic theory, a number of tests could be applied that are unavailable when ad hoc approaches are used.

Although work reported in this paper has specifically involved the analysis of shopping behaviour, the methods used are applicable to other choice processes, such as those examined by Dubin and McFadden (1984) and Brownstone (1980) or Train (1986), Mannering and Winston (1985) and Hensher et al. (1992), where a discrete and a continuous component is evident.

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REFERENCES


