NOTES AND COMMENTS

COMPETITION ON AN URBAN BUS ROUTE

A Comment

By Tristan E. Galvez*

A model of bus/minibus competition has been reported in this Journal by Glaister (1985a), commented upon by Nash (1985) and Glaister (1985c), and updated by Glaister (1985b). This model is an important tool for predicting the outcome of unregulated competition on urban bus routes. Dr. Glaister kindly made the details of the original and updated models available for review (Galvez, 1985). This note reports on two technical issues related to the modelling of boarding and alighting by passengers, and on the results obtained with a revised version of the computer model.

PASSENGERS BOARDING

In a world with big and small buses offering different fares and journey speeds, passengers waiting at bus stops face a decision on mode choice. For example, as a minibus approaches, they must decide whether to board or to wait for a big bus, trading off the fare differential against the expected additional waiting and journey time. In Glaister's model the queue of passengers at bus stops is divided into three, corresponding to users with high, medium and low time values. The model selects a queue at random, and offers the first user in it the chance to board. For each queue, there is a probability of a user refusing to board, obtained from the modal split model. If the user refuses to board, a random queue is selected and a new user is interrogated. The process continues until somebody boards or until all the users from all the queues have refused to board.

In the original model, whether or not a user boarded, the bus stayed at the stop, and in the next step all the users were again interrogated. The process continued until the bus was full or the queue was empty. If the bus was the only

* Departamento de Ingeniería Civil, Universidad de Chile. I thank P. J. Mackie, of the Institute for Transport Studies, University of Leeds, for helpful comments.
one at the stop, it was then certain that all the users in the queue would board, if there were enough seats in the bus, whatever the value of the probability of refusal. The updated programme (Glaister, 1985b) has been improved so that, if at some step all the users refuse to board, the bus is forced to leave the stop. However, this only mitigates the problem, because if at any step somebody boards, at the next step the users who have previously refused to board are interrogated again. This has the effect of raising the probabilities of boarding above the level obtained from the model of modal split.

In the revised programme (Galvez, 1985), each user in the queue is asked once and only once. A register of the accumulated number of refusals to board is stored, and the bus is forced to leave the stop when the number of refusals equals or exceeds the number of users remaining in the queues. This ensures that, if the probability of refusal is 90 per cent, on average only 10 per cent of the users in the queue will board. This change seems to be a subtle one, but it is really the core of the modal split process, and its influence over the final results is dramatic.

To illustrate this, we consider the High Flows case at the stops at the far end of the route. In Glaister’s original work, in base conditions each of these stops has an average demand of 176 passengers/hour and a frequency of service of 9.7 buses/hour; this implies a mean headway of 371 seconds. This gives an average number of boarders of $176/9.7 = 18$ passengers/bus, with an average load time around $2.5 \text{ seconds} \times 18 = 45$ seconds. So on average the stop will be empty (with no buses) about 88 per cent of the time, and in general one would rarely find more than one bus at the stop. This means that almost any bus will be alone at the stop and, irrespective of the probabilities involved, all the passengers in the queue will try to board. So the average load of a big bus in this section of the route will be about 18 passengers (20 per cent) and the average load of the first minibus added will be 15 passengers (its total capacity). As the fares are set to break even at loads of 40 and 7 passengers, the big buses will be making losses in this critical section and the small buses will be making profits. It is not surprising, then, that the minibuses displace the big buses.

**PASSENGERS ALIGHTING**

Glaister’s original and updated models both use the same approach for the process of alighting. When a bus reaches a stop, the number of passengers that will alight is calculated by means of a binomial distribution. It is assumed that the probability of any user remaining on the bus is given by an exponential function

$$q = \exp(-s/t)$$

where $t$ is the average trip length and $s$ is the distance travelled from the last stop. The shape of this function is shown in Figure 1. The number $k$ of passengers alighting is assumed to follow a binomial distribution:

$$k = B(n, p, a)$$

where $n$ is the total number of passengers on the bus and $p = 1 - q$. The calcula-
Probability distribution of $q$

- exponential
- stepwise
- stepwise without stopping at Stop 1

**FIGURE 1**

*Illustration of the Bias in the Alighting Process*

$q$ represents the probability that a user will remain on the bus at distance $s$ from stop 0
<table>
<thead>
<tr>
<th></th>
<th>CASE 1 (base)</th>
<th>CASE 4 (test)</th>
<th>CASE 9 (base)</th>
<th>CASE 12 (test)</th>
<th>NEW CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HIGH FLOWS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of big buses</td>
<td>88</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No. of small buses</td>
<td>0</td>
<td>0</td>
<td>222</td>
<td>0</td>
<td>89</td>
</tr>
<tr>
<td>Big bus fare</td>
<td>3.7</td>
<td>4.3</td>
<td>4.1</td>
<td>4.5</td>
<td>4.9</td>
</tr>
<tr>
<td>Small bus fare</td>
<td>n.d.</td>
<td>n.d.</td>
<td>12.1</td>
<td>n.d.</td>
<td>17.2</td>
</tr>
<tr>
<td>Total pass-miles/hour</td>
<td>23,122</td>
<td>20,459</td>
<td>22,479</td>
<td>21,129</td>
<td>16.7</td>
</tr>
<tr>
<td>Total user benefits</td>
<td>n.d.</td>
<td>n.d.</td>
<td>4,851</td>
<td>4,032</td>
<td>3,354</td>
</tr>
<tr>
<td>Total subsidy paid (£/hour)</td>
<td>n.d.</td>
<td>n.d.</td>
<td>434.16</td>
<td>102.68</td>
<td>289.80</td>
</tr>
</tbody>
</table>

**Notes:**
- Base cases: Big buses only, non-competitive costs, 1/3 subsidy.
- Test case: Big and small buses, non-competitive costs, no subsidy.
- New case: Big and small buses, non-competitive costs, 1/2 subsidy.
- n.d.: No data available.
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T. E. Galvez

tions are made separately for each type of users. This method of modelling introduces an important upward bias in the resulting average trip lengths. The bias comes from the fact that the alighting process is discrete, concentrated at the stops, and so the distribution of $q$ is actually a stepwise function. It can be shown that in Figure 1 the shaded area $A$ is equal to the target average trip length, and the shaded area $B$ is equal to the added trip length. The bias is $(A + B)/A$. The effect of this added trip length is an artificial increase in the numbers and frequencies of both sizes of buses. But the bias is still greater if a bus does not stop at all the stops, as shown in Figure 1. As minibuses are more likely to miss some stops, the bias tends to increase their relative profitability.

To overcome this problem, the alighting process has been modelled in a completely different way in the revised programme. The main difference from the old method is that the binomial distribution is used to obtain the stop where a passenger will alight instead of the number of passengers that will alight at the next stop.

THE RESULTS

The main results for four of the twelve cases reported in Glaister (1985a) are presented in Table 1. For the High Flows base situation (Case 1, big bus only, non-competitive costs, one-third subsidy), the total passenger-miles/hour generated, compared with those reported formerly, shows that the likely effect of the bias in the original alighting model was an increase of about 13.6 per cent in the total passenger-miles. Case 4 reported in Glaister (1985a) was selected for the test run. In this case there is no subsidy; costs are set to competitive levels, and there is free competition between big and small buses. Several runs were performed, starting from different initial numbers of big and small buses in order to check the existence of local optima. In all cases the result was the same: the small buses were unable to survive, and the final equilibrium contained only big buses. This is completely different from Glaister's result. Detailed analysis shows that the main factor explaining this difference is the correction made to the boarding process. The revised (B) results in Table 1 for Case 4, compared with those for Case 1, show that break-even has been obtained with very similar fares and frequencies. This is because the cost reduction due to competition is supposed to be about 30 per cent, which approximately compensates for the elimination of subsidy. User benefits shown in Table 1 are negative, but smaller (in absolute value) than the reduction of subsidy. In as much as the reduction in costs represents an efficiency gain, rather than a transfer, a net social benefit is generated.

In the Low Flows (Case 9), the total patronage obtained shows that the bias of the alighting process is about 20.3 per cent. Case 12 (analogous to Case 4) was selected for the test run. Again, serveral runs were performed in order to check the existence of local optima. In all cases the result was the same: the big buses were unable to survive, and the final equilibrium contained only small buses. This is exactly the reverse of the High Flows case. Glaister's original paper also forecast the elimination of big buses, but the results obtained were dramatically different. The new equilibrium obtained with the revised programme contains
only 66 small buses instead of 165. Patronage is reduced by 17 per cent instead of being increased by 31 per cent. From more detailed output (not presented here, but available from the author), it can be shown that the only “winners” are the high-value-of-time users from the far end markets, and that the “gains to those who gain” no longer outweigh “the losses of those who lose”. User benefits shown in Table 1 are negative, in relation to base, and are almost equal to the reduction in subsidy.

From all this it is clear that Glaister’s model tests the impact of a policy which simultaneously causes a cost reduction for all buses and permits competition between buses of different sizes. Because the effect of the postulated cost reductions is so great, it is unclear from Glaister’s results whether it is justifiable to allow competition between buses of different sizes. In order to test this question, a completely new case was run, and the results of this also are included in Table 1. In this case, costs remain at their base level and a one-third subsidy is maintained, but competition by small buses is allowed. The final equilibrium contains only small buses, but the users’ welfare is reduced and the subsidy is greater, both in relation to base. So, for the case investigated, the effect of bus size competition in isolation is clearly disadvantageous.

Finally, several runs were performed for the Medium Flows case. In this case the model failed to achieve a unique solution, as the final equilibrium depended on the starting point. If the level of service offered by the big buses was initially low, the final solution contained only small buses. Similarly, a low initial level of service by small buses led to a solution with only big buses.

CONCLUSIONS

1. The procedure used by Glaister to model passengers’ boarding behaviour led to a significant over-estimation of the number of passengers choosing small buses.

2. The Passenger Alighting model over-estimates mean trip length and also creates a bias towards small buses.

3. Amending the model to allow for these effects changes the nature of the results:
   (a) The mix of big and small buses forecast in the original work does not appear, whatever the case investigated. This suggests either that a mix cannot survive, or more probably that the conditions necessary to enable it to do so may be different from those assumed in the present exercise. For example, a big bus service might cover a long route, with small buses covering only the section with high demand, or vice versa.

   (b) Small buses can displace big buses in a market such as the one modelled for the low flow, but this implies a reduction in welfare for nearly all the users instead of the gains predicted by Glaister (1985a). The reduction in welfare is mainly due to the higher fares.
A Rejoinder

By Stephen Glaister †

Tristan Galvez has reviewed my model with care, and I am pleased that he has been able to improve it. I believe the conclusions to be drawn from his results are similar to my own.

1. At high flows he has big buses only in the competitive equilibrium, and at low flows small buses only (as I had). Therefore, he confirms my conclusion that high flows tend to favour big buses and low flows to favour small buses. I acknowledged in my response to Dr. C. A. Nash's (1985) comment in this Journal that the flows I used were untypically high. Galvez's results are consistent with the proposition that big buses will find themselves uncompetitive at passenger flow levels that are found in a great many situations. It is unfortunate that he was unable to achieve a unique equilibrium in the medium flow case — yet this does raise the interesting possibility that in some circumstances the final outcome would depend upon the nature of the operating concern at the time of deregulation.

2. He estimates net benefits of deregulation of the order of \((434 - 39) = £395/\text{hour}\) in the high flow case, and disbenefits of the order of \((103 - 80) = £23/\text{hour}\) in the low flow case. It is clear that over most of the range of flows considered his estimate of the benefits of deregulation will substantially exceed the costs. The heaviest flows are those that suffer most from regulation. Galvez's new case confirms that to achieve cost reductions through competition is important for the success of the policy of deregulation. This is common ground.

3. As is clear from my exchange with Dr. Nash (1985), both my original work and that of Galvez are limited, as we consider only one of many possible pricing strategies for competitive bus operators. Galvez's low flow and new cases turn out badly because the high-price small bus is able to oust the low-price big bus without providing sufficient compensation to passengers in the form of improved services. But this will not necessarily be the end: there may well be scope for reductions in fares on small buses through price competition among themselves, or for re-entry of big buses at fares higher than those they charged in the base. These are issues that I have considered more carefully in subsequent work with the later version of the model. Even with the inflexible pricing strategies considered by Galvez, it seems plausible to

† London School of Economics.
expect to find flow rates between his two extremes at which the two types of vehicle would survive in competition.

Galvez's two poor outcomes at low flows reflect his substantial fall in patronage relative to the base, where my version of the model showed a substantial increase. The reason for the difference is not clear. It cannot be due to the different passenger boarding processes, because in these cases there is no choice between buses. Nor does the difference in alighting formulations appear to be sufficient to account for a difference in patronage of a factor of about two.

In his discussion of the differences between our boarding processes, Galvez assumes that buses arrive at stops at random, with the implication that there will seldom be two or more vehicles at a stop. In fact, bunching together of vehicles was a marked feature of my model, and substantially increased the probability that several vehicles would be on offer at one time. Also, because of the stochastic nature of the arrivals of passengers at stops, the queues at the ends of the routes were often shorter than their average elsewhere. It was therefore most unlikely that the average load of departing minibuses could be equal to their capacity, as Galvez suggests. Both these factors serve to narrow — but not to eliminate — the effects of the differences between our boarding processes. My impression is that these differences are more important than those between our alighting processes.

There is reason to expect that the users of the fast, but more expensive, small bus will in practice make trips of more than average length. Though I did not intend it, the "bias" in trip lengths in favour of the occupants of small vehicles, introduced by my formulation of the alighting process and pointed out by Galvez, may be the kind of thing that one would expect (though for other reasons) to happen in practice. In fact, other critics have commented that I have neglected an important competitive weapon of the small bus: the ability to offer express, or limited stop, relatively long-distance trips. The taxi is an illustration of this.

REFERENCES


