SOME CURIOUS OLD PRACTICES AND THEIR RELEVANCE TO EQUILIBRIUM IN BUS COMPETITION

By Christopher Foster and Jeanne Golay*

1. INTRODUCTION

Gilbert Ponsonby, in his book on London’s Passenger Transport Problem (Ponsonby, 1932), revealed himself as escaping from the intellectual attitudes he had learned as an assistant to Sydney and Beatrice Webb. His rigour led him to demolish almost every argument for combining the multitude of London’s buses, railways and trams into the monolith which is now London Regional Transport. However, when he came to consider some ancient practices which bus drivers then still used to compete with each other, he concluded that the instability they caused might be an argument for intervention in the market, and even for the London Passenger Transport Board created in 1933.

Forty years later, in his mild manner, he told one of the authors of this paper that one of the follies of his youth that he regretted was that he had ever been persuaded to endorse the LPTB. Our aim now is to set the older Ponsonby against the younger Ponsonby by unravelling the economic meaning and the effects on stability of those practices. We are helped by the recent development of a substantial literature which illuminates the problem, though it is, as yet, unrelated to the technology of bus transport. Our object is to see how far the recent literature rebutting, or developing from, Hotelling’s famous article (Hotelling, 1929) may be used to help to analyse these issues.

They have come to have a sudden importance in England and Wales because the Transport Act, 1985, has abolished the bus licensing system which has existed outside London since 1930.1 It would be replaced by something like the free competition that existed before, but with stricter controls over vehicles, drivers and operators, mainly in the interests of safety, and some new controls as well over timetables and dangerous driving.2

Many people are afraid that deregulation may create such rapid changes in bus schedules, and therefore uncertainty, that users will incur information costs that

* Coopers and Lybrand Associates and the London School of Economics. We are grateful for comment from Richard Allsop, Andrew Evans, Stephen Glaister, Peter Mackie, Corinne Mulley, Christopher Nash and Gabriel Roth.
1 Deregulation has not been taken as far in London, to which therefore the argument of this article does not apply.
2 Most of the policies and procedures set out in the White Paper Buses (Department of Transport, 1984) survive into the Act. There will still be regulation in the interest of safety (Sections 7–9 and Sections 26–31).
will outweigh the benefits from more frequent, better quality, and possibly cheaper services. This has already been said to have happened as a consequence of experimental deregulation since the Transport Act, 1980, particularly in Hereford; and it is also said to have been common before 1930.3

An economist recognises these doubts as concerning the existence, stability and optimality of equilibrium in a competitive bus industry. Disequilibrium may be defined here as occurring when there are frequent and unpredictable changes in timetables and fares caused by alterations in the strategy of those operating a route, as well as by exits and entries of buses or operators to and from the route. In stable equilibrium the buses operating a route, the timetables and the fares change only if there are corresponding changes in the external environment: in demand, relative prices, bus technology or bus costs. After any such change a new equilibrium will be attained.

Safeguards under deregulation

Changes in timetables, fares and the numbers of buses plying a route were allegedly frequent before the 1930s. So were other old bus practices, the names of which have come down to us as words which, to the best of our knowledge, have never been carefully defined (see Hibbs, 1972). The questions that interest us are:

(i) Which of these practices may recur, and in what circumstances?
(ii) Which are likely to be evidence of the non-existence of equilibrium?
(iii) What could be done — not only in relation to these old practices — to make equilibrium more likely?

If any of these practices were to be adopted it would imply that the driver and, in some cases, the conductor shared the bus operators’ objectives — their remuneration wholly or in part depending on their actual takings. This will be assumed throughout this article, as it seems a not unreasonable requisite for restoring bus competition. Some practices are unlikely to be revived. For example, “lifting” and “chucking” seem to have been horse-bus practices (the conductor heaved unsuspecting passengers on to a bus going in a direction they did not want, and then let them repent at leisure). If buses are clearly marked, passengers who suffered such conduct ought to be able to gain redress. Nor, in view of the requirements of the Act, should “diverting” be a problem. Diverting occurs when buses wander off the published route to put down and possibly pick up passengers, no doubt missing out some of their usual pick-up points on the way. The law will allow “flexible routing”, but only if an operator announces this policy in advance.

Practices not inconsistent with equilibrium

However, we will argue that some other old practices may revive but need not necessarily be inconsistent with the existence of equilibrium.

---

3 See on the Hereford experiment Fairhead and Balcombe (1984); Evans and Hoyes (1985); and Evans (1985).
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

(a) The most important may well be “hanging back”, “hanging the road” or “crawling”, which means that buses go slowly so as to pick up as much traffic as possible that would otherwise go to the bus behind. A variant of this is “waiting” at a bus stop till the bus has a fuller load or till the next bus comes over the horizon.

(b) “Missing” out a bus stop and “racing” a bus (which generally implies missing out some bus stops) are examples of practices which may well be in an operator’s self-interest. The reasons for their being objectionable need to be carefully thought out. If a driver decides there are too few passengers to stop for and that it would be more profitable to race and start picking up passengers further ahead, the objection is, in a sense, a distributional one. We commonly feel that a bus should stop for even one passenger, whatever its cost in terms of bus profitability. Indeed the law requires operators to keep the service registered, including stops. Therefore such behaviour can lead to a prosecution. Where buses are “bunched”, however, missing stops may be a rational way for a bus to break away from the bunch: so may “hanging back”, but without detriment to the passenger.

(c) “Turning” a bus when it is empty or nearly empty is turning it round before the end of the route and sending it back in the opposite direction. If the service is infrequent, a driver who finds his bus is empty, if he foresees little traffic offering ahead, may decide to turn it in his own and his company’s self-interest. The issue is partly distributional again – what weight to give to those disappointed by the non-appearance of a bus on which they had counted – but the driver will himself give this some weight also, since he knows that disappointed would-be passengers may be discouraged from turning up for that bus in future. If the route is congested and buses are bunched, turning a bus may be in everyone’s interest. Many operators appreciate this, and turning was a frequent practice in congestion in a regulated environment.

We will argue that the practices described above are unlikely to arise, or that if they do – where demand is stochastic or there is congestion, or both – their adoption may contribute to the attainment of equilibrium.

Practices adequately controlled under the Act

(d) The meaning of “passing” or “overtaking” another bus is obvious. “Tailing” is keeping just behind another bus to overtake it or “cut in”, whenever seems likely to be the more profitable. “Chasing” is catching a bus up to overtake or “cut in”; “leapfrogging” or “jockeying” would seem to describe situations where buses continually pass and re-pass each other. “Head running” occurs where one operator schedules a bus just before a rival’s bus. As it will attract retaliation, the effect will be an attempt at leap-frogging through the timetable. “Schedule-matching” is putting on a bus at the same published time as a rival’s. “Nursing” is setting a bus to accompany a rival’s bus and get as much traffic away from it as possible. In its more violent form it was trying to run your rival into the ditch. “Blanketing” may be nursing with two buses – one before and one after the rival bus. “Cutting in” is interposing your bus between a rival bus and a bus stop so as to cream off the traffic waiting there.

193
We will argue that where demand is predictable these practices are likely to be irrational, but that where there is stochastic demand, congestion or both, they may contribute to equilibrium. Besides, before being overawed by such a catalogue of practices, we must remember that, by current standards, the behaviour of all vehicles, not only buses, was appalling during the 1920s. That is well documented in government reports at the time. Control by traffic lights and signs and parking controls barely existed. Bus traffic was far heavier on almost all routes, so that many more buses were plying, even on country routes. So there were many more opportunities for "gaming" behaviour between competing buses than is possible now. We will not discuss the practices listed under (d) further. Like "lifting", "chucking" and "diverting", their prevention, when necessary, should be enforceable within provisions likely to be laid down by regulations under the Section 6 of the Transport Act 1985 requiring operators to adhere to a published timetable. "Cutting in" would also seem to be a prime example of dangerous driving liable to prosecution and the withdrawal of an operator's or driver's licence.

Predatory pricing

(e) But there is one case which is always inconsistent with a competitive equilibrium. This is when one bus or operator deliberately prices below marginal cost in order to drive his competitors off the road through predatory action. The literature on competitive equilibrium tends to rule predatory behaviour out by assumption (Eaton, 1976). That behaviour can only be sustained if the predator can meet his losses by cross-subsidy from elsewhere, or from financial reserves which have been accumulated in the past. (The National Bus Company may have such reserves, though they will be capitalised on flotation.) If predatory behaviour were possible, the object of deregulation would be thwarted. In the past, bus transport has been exempted from competition law. These exceptions have been removed by the Act (Sections 114 and 115). Therefore we assume that the Traffic Commissioners and competition law will prevent this from happening in future, as they have not been able to do previously.

Future practices and competitive equilibrium

In the rest of this paper we will test the hypotheses set out in (a), (b) and (c). (c) raises different issues from (a) and (b). Whether these practices return must in part depend on the particular circumstances of a bus route. The six sets of circumstances we examine are:

(i) where demand is reasonably predictable over the day and where headways are sufficiently great for buses on a route to be well separated in time. Experience suggests that on such routes passengers will know the timetable and arrive at bus stops in time for a bus;

---

4 See Royal Commission on London Government (1923) and especially the copious references to this conduct in the evidence of the borough representatives in the Minutes of Evidence, and Royal Commission on Transport (1929).
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

(ii) where conditions are otherwise as in (i) but headways are much closer, so that buses are more likely to bunch and interact if they choose to do so. Where headways are as close as this, passengers may be expected to arrive at random and take the next bus approaching;

(iii) and (iv) are the same as (i) and (ii) but there is a stochastic element in the demand;

(v) and (vi) are as (iii) and (iv) with congestion added. Congestion itself causes the bunching of buses, as is well established in the literature (see Beckman et al., 1956, chapter 1).

How far the deregulation envisaged by the 1985 Act will achieve competitive equilibrium will also partly depend on how effectively competition law is enforced.

(i) Before regulation collusion between operators was often highly formalised. For example, the old London Horse Bus Association operated a system in which all the receipts on a route were pooled and allocated between proprietors. "Times" were also allocated. Generally, when net receipts per "time" rose above an allocated minimum, another bus was run, since "starving" a route was "asking for trouble" (see Barker and Robbins, 1974, Appendix 1, p. 417). Similar arrangements still exist overseas, for example in Bogota. As long as there are no restrictions on entry to allow the cooperative to enjoy a monopoly profit, the self-regulation it entails is likely to make equilibrium more stable without necessarily reducing efficiency.

(ii) The temptation to act as a predator will also be great in the short run.

If competition is to be realised, the Traffic Commissioner, the MMC and the courts must discourage any aspect of collusion against the consumers' interest, and also predatory behaviour.

Section 2 of the paper considers the relevance of the literature on competitive equilibrium for testing our hypothesis, and in particular for determining on what assumptions equilibrium might be reached. Section 3 discusses how a route timetable might be built up under competition, on the assumption that demand is predictable. Section 4 deals with the consequences of assuming stochastic demand and congestion.

2. THE LITERATURE ON COMPETITIVE EQUILIBRIUM

Our problem is to adapt to bus competition the large volume of literature that has developed from comments on the Hotelling paradox, and also to simplify the issues so that their analysis is tractable without too much sacrifice of realism or of theoretical sophistication.

That literature is almost all concerned with spatial competition. Entrepreneurs are considering where to locate their plans in space and what prices to charge.⁵

---

⁵ There is a partial survey of the literature in Grafton (1982).
Most models describe circles, lines of finite length, lines of infinite length or circumferences. The first (that is, a disc) has no relevance to our problem. Any of the last three could be used as an analogy, since, instead of a number of plants located on a line, straight or circular, one may postulate the timetable of a route on which competitors "locate" their buses at points in time. Their problem is to optimise in both time and price (instead of space and price). The principal advantage of assuming a finite straight line is a rather limited realism: the finite line seems not unreasonable as a graphic representation of a daily timetable, since one may suppose that there is a period of "night" when demand does not justify any buses, and another period, "day", when buses run, though at headways that vary with demand. (Where demand arises only at peaks, the timetable may be two or more discontinuous straight lines.) Whether continuous or not, a straight line raises the theoretical difficulty of corner problems at either end because one must fit a finite number of buses into a timetable of finite length while trying to require that buses make only normal profits. An infinite line avoids that problem but creates another: there is no end to the number of buses in a daily timetable, and buses adjacent in a timetable need never compete. Therefore, even at the sacrifice of superficial realism, it seems to us most useful to regard a daily timetable as a continuous line or circumference. (In practice corner solutions should not arise, because the beginning and end of any daily timetable will be flexible.)

We will also argue against the assumption of normal profits. The circle is not the 24 hours of the day, but the hours when effective demand is high enough to run buses profitably.

We have already defined equilibrium as a timetable that does not change until external conditions change. Now we need to decide: (a) whether equilibrium exists; (b) how it will be reached.

(a) The existence of equilibrium

Much of the work on the existence of equilibrium assumes zero conjectural variation (ZCV): that is, "that each firm views the strategies of all other firms as fixed, regardless of its own strategy" (Novshek, 1980). Adapting Novshek's objection to ZCV to the circumstances of a route timetable, we find that ZCV is an inappropriate assumption for one or other of two opposite reasons. Either each bus on the timetable is a monopolist in the sense that there is no substitutability between its market and that of any earlier or later bus — if so, ZCV holds but is irrelevant — or a change in the price or time of any one bus will affect the demand and therefore the profits of adjacent buses. Thus ZCV cannot hold. Therefore Novshek proposes a modified ZCV assumption (MZCV), which will be ours also6: "Each firm views the strategy of other firms as fixed so long as its own strategy does not cause its delivered price to match or undercut any other firm's price at that other firm's own location . . . and that other firms will reduce price if they are undercut at their own locations".

Thus in effect a bus timetable is in equilibrium when each bus on it believes that, if it changes its price and time, other buses will retaliate, so that its own

---

6 In this he follows Eaton (1972).
profits will be less than they were initially.

Novshek then demonstrates that a set of price equations will exist with free entry and exit in a spatial market where demand is continuous and uniform. He assumes identical cost functions consisting of a fixed cost plus a constant marginal cost. His results apply to the bus industry in which over a substantial range costs per seat fall as a bus fills up. He shows that equilibrium will be attained, but that there is no reason to suppose that the configuration of demand and of decreasing costs will mean that profits will be normal. Even if they are above normal, the best a potential entrant could do would be to earn a negative profit (and cause some at least of the other buses on the route to lose profits too). The result depends on the fixed cost being small, because only then is it likely that all the competitors may be sure of covering their fixed cost in equilibrium as well as their marginal costs.

This again applies well to the bus industry, where there is a well-developed second-hand market in buses and the possibility of running minibuses. Further, there is no need for substantial administrative or other overhead costs, which competition should keep down anyway. Therefore entry can be considered virtually costless.

For the purpose most relevant to this article — establishing schedule equilibrium — labour costs are mainly fixed. (At present labour costs tend to be fixed if bus frequencies are increased within an existing crew shift, and variable where overtime must be paid or an additional crew engaged. In future, labour practices are much more likely to be adaptable, and both split-shift and part-time workers will be common, as in the taxi trade.)

(b) Building competitive equilibrium

If one starts by assuming there are no buses on the route and several competitors enter simultaneously, each picking his own "time" and fare strategy in ignorance of his rivals' intentions, then it is very unlikely that equilibrium will be achieved (Richardson, 1970; Novshek, 1980). Moreover, that assumption is unrealistic. Deregulation will seldom lead to many buses all competing on new routes where there have been no buses before, unless costs fall substantially. Rather, there will usually already be an operator with a timetable; and the choice facing a new entrant is to pick the combination of "times", fares and quality which will maximise his profits. Even then more than one operator could enter simultaneously and this increases the probability of disequilibrium (Grace, 1970). It is too restrictive to assume that only one operator enters at a time, but the Department of Transport has introduced a process to prevent this source of instability.\(^7\) There is to be an interactive process, so that operators will be made aware of each other's intentions before a timetable is finally settled.

Another influence on the likelihood of equilibrium, and on how close it will approach to competitive equilibrium, is the size of costs of entry into the market. The smaller they are assumed to be, the easier will be entry and exit: but each equilibrium will last a shorter time, as the market will be more easily influenced

---

\(^7\) See Department of Transport (1984b), para 14.
by changes in demand and costs. Because economies of scale and scope in bus operation are small\(^8\) (except those caused by the existence of regulation itself), a model in which entry is costless is not unrealistic\(^9\) for the bus industry, which can therefore have an approximately competitive equilibrium, as Novshek demonstrates.

Moreover, there will be a greater chance of reaching and sustaining equilibrium if there is a product differentiation between buses on the same route — for example, if there is a difference between express and ordinary buses or a variation in the comfort of different operators’ buses, since this will reduce substitutability in demand. Moreover, buses will often be in competition with taxis as well as with minibuses: as elsewhere in the world, one may well find that there is a continuum between a bus and a taxi, not a sharp difference. For this reason one can expect differentiation to be stimulated by competition as a protection against instability (see for example Shaked and Sutton, 1983, pp. 1469–83). If there are costs of entry, it pays existing operators to adopt various strategies to deter competition (see Hay, 1976, pp. 240–57, and Prescott and Visscher, 1977, pp. 378–93). Those strategies make an equilibrium more likely, but at the expense of competition.

Our approach rests on these postulates:

(i) that a daily timetable may be represented by the circumference of a circle, so as to avoid corner solutions;

(ii) that bus operators’ behaviour reflects a modified ZCV assumption, so as not to require normal profits in equilibrium;

(iii) that the costs of entry are small and the government will not allow them to rise; and

(iv) that the timetable will be built up by sequential entry from the current monopoly timetable.

Theoretically, equilibrium will exist if these conditions are fulfilled. The next section will demonstrate how equilibrium might be achieved on a route under free competition, and what type it would be. There are obstacles to achieving and maintaining that equilibrium, and these may be manifested as the revival of some of the old practices described earlier. Where they are disequilibrating, the question arises how far they can be prevented under the provisions of existing legislation and of the 1985 Act.

3. BASIC CASE

Let us consider two applications of a simplified model of a single bus route or service. Many of the initial assumptions are unrealistic. Their purpose is to establish the circumstances in which it can be shown that there will be stable equilibrium in these two applications. Application A refers to a bus operator who is able to decide on fare and frequency without needing to consider the poss-

---

\(^8\) For a discussion of this see Gwilliam et al. (1985) and Beesley and Glaister (1985). Our views coincide with those of Beesley and Glaister.

\(^9\) For a model with these characteristics see Salop (1979), pp. 141–56, especially page 145.
ability of interaction with other operators. His presumed objective is to maximise his total profits from the operation. Application B relates to two or more operators interacting; competition determines headways, but operators choose the fares at which profits are maximised. Our applications both show that there is a maximum (profit-maximising) and a minimum (MZCV) headway consistent with the different degrees of competition assumed. Later it will be seen how far stable equilibrium survives after some of these assumptions are relaxed.

The basic assumptions are:

(i) The year is divided into days, assumed identical in every relevant respect; and demand is not substitutable between days. The day may be divided into periods of "night" when no demand offers and one or more of the "day" during which demand arises uniformly;

(ii) The route is a simplified one in which the market is the demand offering at location A over the day. The journey is from A to B;

(iii) Individuals' demands for trips are independent. Everyone may be assumed to have a preferred time of day for making the journey. If a passenger has to leave later or earlier than this preferred time or to wait at the bus stop, he experiences a disutility in the form of a personal rescheduling cost, expressed as a function of the difference between the preferred and actual departure time;

(iv) Bus operators have a free entry into the market, and to move from one slot to another in the timetable is almost costless;

(v) Operators have pricing freedom and may be assumed to set fares which maximise profits. For legal reasons, however, they cannot practise price discrimination. This rule is enforceable and enforced;

(vi) Because bus costs per passenger fall as occupancy rises, fares are influenced by the degree of monopoly a bus enjoys and will also be inversely related to bus occupancy;

(vii) Modified ZCV obtains as defined in section 2;

(viii) The level and time distribution of demand in a day is predictable;

(ix) All individual demand curves are identical. They do not vary during day-time. If the total disutility of travel to an individual rises above the utility of trip, he may be assumed to go by car or stay at home;

(x) The disutility of rescheduled time is symmetrical: that is, users are indifferent between advancing or delaying their departure by the same number of minutes. The disutility of travel is the rescheduling disutility plus that of the fare;

(xi) All buses on a route provide the same quality of service in terms of comfort and any other factor; they have the same cost functions;

(xii) On most routes, unrealistically, buses are assumed to have capacity to carry all traffic offering over a day;
(xiii) Traffic and road conditions are such that journey times are the same throughout the day (this may roughly be taken as equivalent to an absence of congestion).

On these assumptions, whether there are one or more buses, and the length of the headways between buses, will depend entirely upon the (identical) values users set on the disutility to them from rescheduling their travel times. Let us consider the profit-maximising policy to be pursued by the first operator who plans to operate the first bus (of infinite capacity) on a route. As indicated earlier, its market may be seen as a circumference. Of the demand offering in the day it would secure a segment; the length of the segment in time will depend on the elasticity of passengers' disutility of rescheduling with respect to time.

The model which seems the best to adopt for our purpose is that of Hay, which was primarily developed to analyse sequential entry into a market (Hay, 1976; see also Prescott and Visscher, 1977). Application A supposes a single operator setting up a route; the analogy with application B is that we may suppose that, first, one operator picks a time or times for one bus and then he or other operators select their times. We start by using one operator who has already entered before deregulation. Other operators are able to enter after deregulation. As was explained in section 2, an interactive procedure is used, which means operators and potential operators know each other's intentions.

Because entry costs are high in Hay's model, and plants are assumed to have effectively fixed locations thereafter, it pays firms to locate their plants so as to discourage other firms from entering. That assumption does not seem appropriate in the bus industry if entry costs are low and it is easy to alter timetables. In parentheses one might add that entry-deterring strategies will only become profitable if timetables are hard to change. Hay's model is nevertheless appropriate for our purpose, since under the 1985 Act timetables are fixed unless a bus operator gives formal advance notice of his intention to change.

The demand for bus services at a point in time, \( t \), is given by:

\[
B_t = y(t)(a - bF) \quad \text{for} \quad a > bF
\]

where \( y(t) \) is the population and \( (a - bF) \) is the proportion of the population wanting to make that journey at time \( t \), their preferred time of travel; \( F \) is the fare and \( a \) and \( b \) are constants. The linear form of the proportion equation is assumed here purely for the sake of simplicity.

The demand function for travel by a bus departing at time \( t^* \) includes that of individuals who would have preferred to travel at another time. The demand function of those wishing to travel at time \( t \) will be of the form:

\[
D(t^*) = y(t) [a - b \{ F + U(t^* - t) \}]
\]

where \( U(t^* - t) \) is the disutility of rescheduling the trip from the preferred to the actual time of departure.

Then any bus operator choosing to operate at a particular time, \( t^* \), which is not too near the beginning or end of the day, will have the total demand for his bus given by:

200
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

\[ B(t^*) = \int_{t^* - z}^{t^* + w} y(t^* - t) \left[ a - b \left\{ F + U(t - t^*) \right\} \right] d(t^* - t) \text{ for } z \geq 0, w \geq 0. \]

where \( z \) is the earliest time band and \( w \) the latest band where effective demand arises, and \( U \) is the rescheduling disutility. This equation can be rewritten, without loss of generality, as:

\[ B(t^*) = \int_{z}^{w} y(t) \left[ a - b \left\{ F + U(t) \right\} \right] dt \tag{1} \]

by changing the variable in the integrand by \( t^* \) and by changing the sign of \( z \).

To keep this expression simple, as so to show its role in building up a timetable as already mentioned, we are assuming that this disutility is related to the extent of rescheduling only; is symmetrical backwards and forwards in time; and influences demand to the same extent as fares do. These assumptions could be relaxed. Then the total demand for a bus will be (if \( U(t^* - t) = u|t| \) and \( |z| = w \)):

\[ B(t^*) = yz (2a - 2bF - buz) \]

We now need to make an assumption on the size (in time) of the market over which the bus operator attracts custom:

(a) the market is unlimited and \( z \) is a variable for the bus operator;

(b) the market is constrained by external conditions and \( z \) is fixed.

In both models we will assume that the cost of a bus trip is of the form:

\[ \text{Cost} = K + k \cdot B(t^*). \]

Application A: \( z \) is unconstrained

In this model, there is no limit to the extent to which passengers will schedule their trip, except the constraint embodied in the \( D(t^*) \) demand function (p. 15):

\[ D(t^*) \text{ falls to zero when } \]

\[ a - b (F + uz) = 0 \quad \text{or} \quad z = \frac{a}{bu} - \frac{F}{u} \]

Then, \( B(t^*) \) becomes:

\[ B(t^*) = \frac{y}{bu} (a - Fb)^2 \]

The form of this demand function reflects the double effect of a fall in fare. First, for any given rescheduling \( t^* - t \), the proportion increases of people willing to travel at time \( t \) who find it now worth while to use bus at time \( t^* \). Also, the maximum rescheduling \( z \) has now increased.

The profit function for a bus operating in such unconstrained circumstances is:

\[ P = (F - k) \frac{y}{bu} (a - Fb)^2 - K \]

This is a monopolist profit function, since the bus operator determines demand by setting an optimising fare. Differentiating \( P \) with respect to \( F \), we obtain:
\[ F = \frac{1}{3} (2k + a) \text{ and } e = \frac{2}{3u} \left( \frac{a}{b} - k \right) \]

where \( e \) is the effective maximum rescheduling that will take place at this fare. Second order conditions show this fare to be a profit-maximising solution.

The model just described applies to routes where bus services are infrequent and do not directly compete with each other. The length of a bus market in time will stretch to its economic maximum: passengers whose preferred time of departure is before or after the time of departure of the bus will join it so long as the marginal cost (fare plus re-scheduling cost) to them is less than the value they put on the trip. The length of its market will increase as the disutility attached to rescheduling decreases (this will also reduce the fare through higher occupancy). If the disutility were zero, then, on these assumptions, all the traffic offering during the day would be carried in one bus. Alternatively, if the fare were to vary for some other reason, the headway would be affected.

**Application B: \( z \) is fixed**

The bus operator now assumes that the extent of rescheduling which his passengers are willing to undergo in order to use his bus is predetermined. His aim is to maximise profit, given \( z \), and the optimising fare will only determine what proportion of the population willing to travel at time \( t \) will choose to travel at time \( t^* \). The profit function is:

\[ P = (F - k) y z (2a - 2bF - buz) - K \]

where \( K \) is the fixed cost of running a bus. Setting \( \partial P / \partial F \) to 0, we get the profit-maximising fare:

\[ F = \frac{a}{2b} + \frac{k}{2} - \frac{uz}{4} \tag{2} \]

This price equation expresses the relationship between fare and bus catchment period, for a profit-maximising monopolist. This model applies to a variety of circumstances, particularly to cases when a bus operator is constrained by other buses before and after \( t^* \). An example is given in the Appendix.

If we assume that \( z \) is not arbitrarily fixed but is determined by the maximum rescheduling acceptable to passengers, that is,

\[ z = \frac{a}{bu} - \frac{F}{u} \]

then the results obtained in model A obtain here also. We also find that, given any

\[ z \leq \frac{a}{bu} - \frac{F}{u} \tag{3} \]

the bus operator will always set a fare which enables him to attract passengers willing to incur rescheduling amounting to \( z \). Indeed, substituting the limiting equation for \( F \) derived from (3) into the price equation (2), we obtain a limit for \( z \) such that:

202
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

\[ z \leq \frac{2}{3u} \left( \frac{a}{b} - k \right) \]  \hspace{1cm} (4)

Moreover, substituting the same equation for \( F \) in the profit function we have:

\[ P = \frac{1}{2} byz \left( \frac{a}{b} - k - \frac{uz}{2} \right)^2 - K \]  \hspace{1cm} (5)

and

\[ \frac{\partial P}{\partial z} = \frac{1}{2} by \left( \frac{a}{b} - k - \frac{uz}{2} \right) \left( \frac{a}{b} - k - \frac{3uz}{2} \right) \]

As the expression in the second bracket is always positive for all non-negative demand functions (that is, when (4) holds), then

\[ \frac{\partial P}{\partial z} > 0 \]

over the relevant range.

The shortest length of time, \( x \), defining the minimum market for a bus is where profits fall to zero, or are as low as is consistent with non-negative profits.

**Timetable constructed by free entry**

Once one bus has chosen its departure time, the construction of a timetable by free entry follows straightforwardly. The second bus may time its departure at any other time in the day, provided that it does not overlap with the market of the first bus, as that would reduce the profits of both. We may then suppose a number of buses filling up the intervals between the earlier scheduled buses till we have a timetable of buses in sequence, each securing a maximum market in time. On our assumptions the service frequency will be the same (equal to the maximum) throughout the day.

What happens if as the timetable fills up there is an interval between two buses which is substantially more than the maximum? Then if the cost to an operator of altering its slot in the timetable is zero, it will pay for another operator to enter midway between the two existing buses and for them to shift further apart. This may induce other buses to move also, so that each bus ends by achieving a maximum interval. Such a process may be the means by which a timetable is achieved, but it is not a competitive equilibrium, since we have shown that all buses will be earning monopoly profits. Therefore it will still pay operators to introduce new buses into the schedule at a lower fare. Then, through shunting, a new timetable with less than maximum intervals will be created, till a minimum frequency timetable is established in which no new bus entering could earn a positive profit. In this way, stable competitive, or more strictly MZCV, equilibrium is reached.

Some interesting results are indicated even by this simple analysis (expressed in a diagrammatic form in the Appendix):

(i) A profit-maximising monopolist would, if rational, choose a timetable which has maximum headways, since that maximises profits. Free entry
and competition would increase the frequency to the minimum headway level.

(ii) Fares may be higher under free competition because bus occupancy is lower, but this will be more than offset by greater consumers' surplus through a reduction in rescheduling disutility. This is in principle the main economic benefit of bus deregulation. (It is illustrated in Figure 3 in the Appendix.)

(iii) It will never pay any bus after the first to enter at the same time as another bus. Thus the Hotelling solution to his Ice Cream Sellers' problem (they partitioned the market by standing alongside each other) will never arise. The reasons for this are plain. A necessary and sufficient reason is that a bus departing at a given time cannot realistically be the same product for all users (including those incurring rescheduling cost) unless the value they put on their rescheduling time is zero. Otherwise it will be a different product depending upon differences between the preferred and actual times of departure. This is a sufficient reason for a solution in the Hotelling style to be invalid, and it is analytically similar to the refutation of Hotelling made in the literature on spatial competition.\textsuperscript{10} Even if that were not a sufficient objection, a moment's thought shows that the difference between space and time is equally damaging. Two sellers can divide a spatial market by being adjacent, but one bus following another cannot so divide a temporal market. The first bus scoops the market.

(iv) It will always pay a new bus entering the market to enter midway between two existing buses, in order to take most advantage of the preferred departure times of those passengers who are most disadvantaged by the departure times of the buses already in the schedule.

If assumptions are relaxed

The effect of relaxing some base case assumptions is obvious. For example, if lower cost bus operators or buses appear, they will (other things being equal) undercut and drive out existing buses. So will buses showing superior quality in achieving faster journey times through higher speed or shorter boarding and alighting times. The days can be varied between weekdays and weekends, or high season and low season, without essentially affecting the results. Moreover, the bus day can be short, long or interrupted. There may be only enough demand for one bus a day to operate, or for peak operation alone. In all these cases, there should still be equilibrium.

Substantial bus entry costs could easily be handled by adopting Hay's analysis of this problem (Hay, 1976, pp. 245—9). If the Traffic Commissioners' proceedings continued to involve operators in substantial costs, if timetables were hard to

\textsuperscript{10} This fallacy was indicated by earlier critics of Hotelling, and was demonstrated by d'Aspremont et al. (1979).
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

shift, and if quality licensing were used to deter entry, then entry-deterring
strategies for operators already in the market would be profitable. To banish these
practices is a formidable argument for deregulation.

By contrast, to produce a dynamic model which would allow for the fact that
buses operate through time and over space would require complications which we
do not attempt. Nor do we consider the problems caused by the substitutability
of demand between days or the interdependence of trip demand during the day.
None of this, we believe, would substantially affect the results.

Our results can be shown to survive the relaxation of other assumptions:

(a) If demand remains predictable, but its density varies over the day,
service frequency and profitability will be greater in the peak than in the
off-peak.

Let us assume that competition has achieved a minimum competitive (MZCV)
headway spacing so that, approximately, from equation (5):

\[ P = \frac{1}{2} b y x \left(\frac{a}{b} - k - \frac{ux}{2}\right)^2 - K = 0 \]

where \( x \) is the minimum or competitive headway.

Mathematical developments in Hay's article show that, provided \( x \) stays within
its relevant range, so that

\[ 0 < x < \frac{2}{3u} \left(\frac{a}{b} - k\right) \]

then \( \partial P/\partial y \) is positive. Where demand is highest profits will be greatest, even
though service frequency is highest.

The same analysis can be used for maximum or monopoly headways, with the
difference that, while a monopolist will charge his highest fares in the peak, in a
competitive situation peak fares should be lower than off-peak.\(^{11}\)

In both cases, there will be stable equilibrium and the results of the last section
remain valid.

(b) If bus capacity were limited so that there was less capacity than the
demand offering in an otherwise optimal period, the maximum and
minimum periods would be reduced; the fares and the frequencies would
be increased, provided that enough demand was still offering at the higher
fares; but buses would still be optimally spaced and there would still be
stable equilibrium. If the demand were greater at a point of time than
would fill one bus, one would expect enough buses to enter to take
advantage of that demand. Provided they all left at the same time and
passengers had equal access to them, the standard results would follow.

\(^{11}\) An analogous point is made by Hay (1976), pp. 251–2.
(c) Again, the results would not in the main be affected if there were product differentiation between buses in other ways than through differences in rescheduling time — for example, if some were quicker or more comfortable than others. There would still be a tendency for buses to spread their service over time, unless there were zero inelasticity of substitution between these other aspects of bus quality and rescheduling disutility.

(d) If the disutility of rescheduling increases more than proportionately with rescheduling time, then, other things being equal, frequencies will be greater and fares higher, but no other result will be affected. If the disutilities of rescheduling backwards and forwards in time are asymmetrical, the departure time of a bus will not be in the midpoint of its market, and the expressions given earlier will have to be modified; but again equilibrium will be stable and the results will hold. Even if rescheduling disutility and utility of travel vary over time, the optimal service intervals will vary, but there should still be stable equilibrium and the four results will hold.

Their robustness can be demonstrated by using the general form of the demand function equation (1). We need only assume that the individual demand function can be integrated and that \( y(t) \) is well-behaved and can be differentiated. Together, these assumptions mean \( B(t^*) \) is defined as a function of \( F \), of the number wishing to travel at each point in time, and of the two limits to the forward and backward rescheduling of the trip. These two points need not be, and indeed are not likely to be, equidistant in time from the departure time. For example, commuters in the morning peak may attach a much higher disutility to being late than to being early. Given an individual demand function, the two feasible bounds to the market for a bus at time \( t^* \) can be defined. Within that double constraint, the total demand function can be defined, and so can the optimal headings and timetable over the day. It remains true that equilibrium should be stable and that it will be optimal for buses to be separated in time, unless the market at a point in time is large enough to fill more than one bus, or unless there is zero elasticity of substitution between the products represented by different buses.

Two possible old practices

Many of the curious old practices listed in section 1 simply cannot arise on our assumptions so far, but two are worth considering:

(1) **Missing Out Bus Stops**

Missing out bus stops may be profitable, since its profitability in the circumstances so far described depends not on the main model but on a side-calculation. At any bus stop a profit-maximising bus driver would consider whether the extra fares to be gained from stopping exceed the costs of stopping and starting again, and the delay time. Without congestion and with predictable demand these costs are likely to be less, and therefore buses are more likely to pick up a passenger.
(2) *Hanging Back*

It will not pay buses to hang back and pick up later demand that would otherwise be picked up by the bus behind. There are two reasons for this. First, it means a delay for all passengers who believed the timetable. Secondly, it disappoints the expectation of the passenger whose preferred travelling time was earliest and who was at the margin of indifference between taking that bus and the bus before. If the bus made a practice of doing this, he and perhaps some intra-marginal passengers would switch to an earlier bus. Thus in the long run hanging back would not pay. But this assumes that passengers wait at the kerb side from their preferred to their actual times of departure: that is it's irrational and against experience. On a bus route with predictable departure times passengers arrive in just enough time to catch a scheduled bus. Thus for hanging back to be worthwhile, the bus would have to dawdle till it arrived at the same time as the next bus; this would have no obvious advantage, and would encourage retaliation from many of its indignant passengers.

All these arguments seem robust enough to suggest the kinds of circumstances in which one would expect stable equilibrium — where demand is moderately predictable and there is no substantial congestion: that is, on most country and small town and on many suburban routes.

4. STOCHASTIC DEMAND AND CONGESTION

Let demand for buses on a route be predictable over the day. Assume that:

(i) headways between buses are sufficiently long for users to present themselves at the timetabled departure time for a bus, rather than at random;

(ii) those headways are optimised as described in the last section. This means that, averaging out their daily experience, operators choose the departure times and frequencies that earn them profits as near normal as is consistent with modified ZCV; and, if the pattern of demand shifts, they learn from experience and alter the frequencies;

(iii) the demand offering for each bus is unpredictable, and the variations in demand for buses at different times are independent of each other;

(iv) initially all passengers are carried from origin to destination without intermediate stops, as in section 3.

Let us now suppose a bus on the point of departure which finds much less demand than expected. From assumption (iv) above it cannot gain by chasing and overtaking an earlier bus. Even if assumption (iv) is relaxed, the cost of catching up and overtaking the bus in front, on an uncongested road where speeding may be presumed to be difficult and where overtaking may not take place till after several bus stops, must be greater than that of waiting at the bus stop. Thus it should never pay a bus in such circumstances to race and overtake if the distribution of demand over time is unpredictable.
Even so, the bus operator has three alternatives:

(a) He may decide not to run a bus at all. This may be costly in loss of goodwill if any demand is offering at the departure time — that is, if some people have boarded the bus. Moreover, there must be some probability that demand at that departure time on future days will be discouraged by the knowledge that the operator is prepared to cut buses out of the timetable if there are not enough people waiting at the starting point.

(b) He may withdraw the bus at that departure time but insert it into the timetable as an extra service at some other time of the day, in the hope that its revenue will exceed marginal cost then. Since by assumption, and quite realistically on many such services, demand is effective only at timetabled departure times, the operator can only offer this extra service by making it more or less simultaneous with the departure time of another scheduled bus. Since the time distribution of demand is assumed unpredictable, he can only choose at random when to run. Therefore the probability that he can make a profit on such an intervention in an already optimised bus timetable must also be low.

(c) This leaves the alternative of waiting at the bus stop to try to pick up traffic arriving for the bus due at the next departure time. Even if we assume that he keeps all the passengers on the bus who did arrive at his scheduled departure time, and that he then manages to take from the next bus enough fares to cover the marginal costs of his operation, he should allow for the fact that some furious passengers will arrange to depart at another time in future, possibly transferring their custom to an operator who does not make a practice of waiting till the bus is fuller. The profitability of the delay is likely to be even less if assumption (iv) is relaxed. As, again, the demand offering at successive bus stops is unknown to the driver, waiting will make the bus late at some subsequent bus stops. This at least makes it more likely that some passengers will give up waiting, or that they will learn from experience and alter their departure times on subsequent occasions, or that the bus behind will decide to leap-frog. Therefore “hanging back” does not promise to be a profitable course of action.

If the assumption of independent probabilities of demand is dropped, one can imagine circumstances in which the discovery of less demand at one time tells operators that there is likely to be more demand at another time. If so, they will have the opportunity of putting on buses at that other time.

Thus one draws the conclusion that, even with a random element in demand, stable equilibrium is likely, with buses keeping to their timetables. Therefore, in most circumstances the requirement in the Transport Act that buses keep to published timetables is unlikely to be necessary for stable equilibrium if bus operators are profit-maximising. There is no reason to suppose it will do any harm if the timetables can easily be altered to reflect daily variations in demand. However, the more important consideration is likely to be that passengers in the long run must be assured that a bus will arrive on time if they are to have enough certainty to use the service.
Dense and unpredictable demand

The crucial change in the characteristics of a bus route occurs when demand is sufficiently dense and unpredictable for would-be passengers not to plan their arrivals at the bus stop to match the timetable, but to arrive at random in order to catch the next bus. We will continue to suppose there is no congestion and that assumptions (ii) and (iii) in this section hold. A bus which then finds it picks up more traffic than expected and becomes full is then likely to race and miss out bus-stops, only stopping to set down passengers; it may then take up more, or stop thereafter to pick up more. In those circumstances it cannot be said that anyone is worse off for the “racing” and “missing” of bus-stops. Those on the bus are gainers from shorter journey times. Those passed at the bus stops by full buses may be angry, but they are no worse off, since there is no room for them on the bus.

Buses on the same route may find they are picking up less traffic than the expected level. They could chase after the bus in front to overtake it. But in general the profitability of doing so is likely to be less than that of crawling, hanging back or waiting. If a bus chases it will have a low probability of attracting extra traffic till it has overtaken the bus in front. If it crawls, then on average the slower it moves the more extra traffic it will pick up, encroaching on to what would otherwise be the traffic of the bus behind. In such dense circumstances the ideal position is for a bus to hang back as far as possible so as to increase the effective headway between it and the bus ahead, possibly to the point where it is just invisible to the bus behind (so as not to encourage it to retaliatory action). Even this may not be important, because, as has already been argued, the bus behind is unlikely to overtake, especially if the driver believes deficient demand is the reason why the bus in front is hanging back. Instead he will hang back in his turn. At least on average, if the size of the buses is well tailored to demand, no one should lose by this process. This is clearest if we assume that demand is predictable for the day as a whole but its distribution over the day is unpredictable. The buses will hang back till a peak in the traffic distribution is reached, when hanging back will lessen or stop. In these circumstances, hanging back will be an efficient way of adjusting to changes in the patterns of demand, though if buses generally have excess capacity there will be less tendency for the process to be self-correcting. This is a reason for leaving operators free to choose the size of bus they believe to be best (as the Act does). Perhaps it is also worth pointing out that the more stops there are on a route the less profitable it will be to hang back, provided there is not much interdependence between demand at the different bus stops.

It might, however, be argued that, even if demand is assumed continuous, buses may gain from hanging back as near as possible to the bus behind. So every bus, except possibly the last, would increase its effective headway (or not reduce it). For example, let us assume that each bus falls back to the time of the bus behind. Then in effect all but the first and last buses keep the same headway (but are delayed by one headway). The first bus doubles its headway. The last bus loses its headway. But this is unlikely to be an equilibrium position, for two reasons. First, it would pay another bus to enter at the beginning of the day.
Secondly, the ability of a bus at the beginning of the day to pick up so much extra demand suggests that it is too large. (That need not be so, because the optimal size may be determined by more dense demand later in the day for the same bus.) Thus the practice of hanging back is unlikely except in reaction to the actual distribution of demand, and in that case it should result in a more efficient service. (Individuals may lose through having to wait longer than they planned, but more on balance will wait a short time than if the bus service had not adjusted.) The conclusion to be drawn is that in such circumstances to insist on a fixed timetable may be inefficient. Thus, if it were practicable, the authorities should try to distinguish between routes where it is sensible to insist on fixed timetables and those where this is not sensible because demand is both dense and unpredictable in its distribution over time. However, such dense routes must be far less common than they were in the 1920s; and, where they exist, there almost certainly will be congestion.

Congestion

Without congestion bus drivers can stick to their planned schedules, or the drivers can depart from them by chasing, or hanging back. With congestion, bunching of vehicles is caused principally by intersections and the differing speeds at which traffic attempts to travel. Congestion occurs when vehicles attempting to travel at different speeds cannot pass, so that queues form with the slower vehicles at the front. Buses are quite likely to be at the front of the queues, particularly on narrow roads or on roads where they cannot pull out of the traffic flow at stops. Congestion is also caused by intersections, where traffic lights or other obstacles to free flow will cause bunching, by delaying some vehicles and allowing others through. By a well known theorem and by common observation, there is a tendency for buses to bunch in crowded traffic. The main cause of their bunching is however variations in demand at bus-stops, as one bus is delayed by having to pick up a large number of people while the bus behind is speeded up by having fewer to take on. Congestion accentuates this (Newell and Potts, 1964). Moreover, the congestion may hinder buses from responding by hanging back or chasing.

It remains rational for buses to react by trying to avoid bunching as far as they can, if by doing so they can better match the pattern of demand. After deregulation, provided bus crews share in profits, they will have an incentive to avoid being bunched, for the reasons which were argued earlier in this section. At present crews in the rear buses in a bunch benefit from a quieter life and do not lose from the drop in carrying. The difference between the effects on rational behaviour of stochastic demand and congestion is that, while the benefits of matching demand are the same, the costs of countering congestion are positive, and therefore the matching of demand and supply over the day will be less efficient. (Congestion has a similar effect whether demand is predictable or stochastic.)

There is another contingency if congestion forces buses into a bunch. Drivers may decide to compete in another way by treating competition for traffic as a game. They are much more likely to compete for traffic at each and every bus stop by cutting in or by other such practices. The accumulation of buses at a
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

stop will further increase congestion for all vehicles; in these circumstances the social costs of competition may well be considerable and may outweigh its efficiency. This is a zero sum game for the bus drivers as a whole, but the superior skill of particular drivers and conductors may increase their share of the market.

In congestion there need not be any tendency for long-run equilibrium to be reached, since the essence of the bunching caused by urban congestion is that the actual bunchings produced are themselves unpredictable, varying greatly from day to day. The extra congestion caused may well lead to longer journey times (and higher rescheduling costs) for passengers on average, as well as higher costs for other traffic and lower profits to all bus operators taken together. But these may not be the only social costs: more erratic and uncomfortable rides and more accidents must be expected.

The Act provides for this also (in Sections 7 to 9) by allowing licensing authorities to act in such circumstances to prevent dangerous driving. This must be enforced if gaming behaviour is to be prevented. If gaming is prevented, rational behaviour will tend to discourage bunching.

If gaming behaviour can be prevented when bunching occurs under conditions of congestion, neither stochastic demand nor congestion affects our main conclusion that a spaced timetable will be more profitable than action which leads operators to “nurse” or otherwise schedule-match.

5. CONCLUSION

Which then was right – the younger or the older Ponsonby? It is gratifying for one of us, who would particularly like to believe that one does not become more foolish as one becomes older, that the older Ponsonby would seem to have the better case. It would seem that under almost any circumstances one should not expect the bad old practices to be rational; and some of them, that may have been rational, need not have been bad. Indeed, they could make stable equilibrium more likely.

But what then did the younger Ponsonby, and those others in the 1920s and earlier, observe? One will never be able to recall that past, but four observations may be sensible:

(i) Buses were far denser on many routes than they are ever likely to be again. Volumes of traffic were not so great, but traffic control was poor, traffic behaviour was far worse, and mixes of many different speeds made for bad congestion.

(ii) Tempers may often have been lost when buses came together, or the fun of it may have made some drivers indulge in the more extreme and dangerous behaviour, even unprofitably.

(iii) The records of the time suggest that what may have led to bad and sometimes reckless behaviour was predatory action by new or established operators deliberately aimed at driving the buses of others off the road, or at paving the way for a merger. But that issue has nothing to do with the stability of competition; it depends on the effectiveness
of competition law and the future structure of the bus industry. Predatory behaviour is rational if one operator believes he can drive the other out or force a merger or takeover, so that there will be a resumption of monopoly. The more fragmented the bus industry to prevent the possibility of cross-subsidisation, and the more effective competition law to prevent predation, the less common it will be.

(iv) There is one other case where schedule-matching may be rational for a new market entrant. Suppose there are only a very few buses a day on a route — perhaps no more than one each way serving the peak. Then it is probable that there is no information at all on demand latent at other times of day. A potential entrant sees the peak demand and decides to enter at the same time as an existing bus but at a lower price, simply because he does not know whether there would be demand offering at an earlier or a later time. A quarter of an hour later and most of the passengers would be late for work or school. A quarter of an hour earlier and they would have to rush breakfast without any advantage. That could be a substantial cost of entry for a small operator. Government funds to provide such information may be helpful here in bringing about real competition.

What implications may this analysis have for policy? Five would seem to stand out:

(a) On uncongested routes where demand is reasonably predictable, timetables should be published as the Bill requires. The timetables should be achieved by an iterative procedure, as the government intends, so that all operators have some idea of others' intentions. It should be easy for operators to alter the timetable so as to maintain freedom of entry, but only after another iterative procedure. It might be hazarded that the consumer's interest would be best met by requiring that timetables and changes in these timetables be published not only in newspapers but by use of the telephone and, as it develops, Prestel.

(b) Where demand is more variable, but where there is no appreciable congestion, it may be more efficient and in the consumer's interest for operators to be able to depart from timetables on a daily basis to match demand as it arises. But this will probably need careful consideration, since the only departures from the timetable should be by hanging back or chasing.

(c) Where there is congestion, rigid sticking to a timetable is unenforceable, and deregulation is more likely to be beneficial after legislative provisions against dangerous gaming behaviour have been provided and strictly enforced.

(d) It needs to be made absolutely clear that predatory behaviour is unlawful and that the law is to be enforced.

(e) There may be a case for government-financed market research on routes where the present timetable is sparse and little is known about the underlying pattern of demand.
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

![Diagram of fare over time with shaded area representing fare F_A between Bus 1 and Bus 2.]

FIGURE 1

APPENDIX

Diagrammatic Exposition

For the purpose of the diagrammatic exposition, we assume that time is measured on the horizontal axis and is continuous. Given that a bus departs at time \( t \), the function on the diagram represents, for time \( x \), the total fare which could be collected by a perfectly discriminating bus operator whose service is used by the people who prefer to travel at time \( x \) but who have rescheduled this travel to time \( t \).

The case of one bus operator running several evenly spaced buses at an identical fare \( F_A \) is presented in Figure 1. Figure 2 demonstrates the flexibility of the model: bus services run as a monopoly operation are compatible with a variety of service qualities and fares.

Let us define \( m \) as the total time between two bus departures and \( z \) as the catchment area of bus 1. The two catchment areas meet. A traveller is indifferent between the two buses when:

\[
F_1 + uz = F_2 + u(m - z)
\]

Solving for \( z \) and substituting in the fare equation (2), we obtain:

\[
F_1 = \frac{4a}{7b} + \frac{4k}{7} - \frac{F_2}{7} - \frac{um}{7}
\]

When \( F_1 = F_2 = F_A \), the situation is that in Figure 1, and the usual fare equation (2) still obtains. When \( F_1 \neq F_2 \) the two fares are inversely related: this reflects our assumption that the catchment areas of the two buses meet. Indeed, if \( F_1 \) goes up, the catchment area of bus 1 is reduced and bus 2 extends its own catchment area by lowering its fare. We observe a transfer of consumer surplus from
the consumers to the bus operator, for the consumers still using bus 1, and from
the operator to the consumers, for those using bus 2.

Bus 1 can carry on reducing its catchment area — at the limit, it will turn into
a taxi — till its net total fare (shaded area in Figure 1) does not cover its total
fixed cost, including the running cost of trip at time t.

Since the fare equation describes the behaviour of a profit-maximising mono-
opolist, it cannot apply to a situation of free competition. We can nevertheless
use Figure 3 to describe the potential benefit brought about by free competition,
SOME CURIOUS OLD BUS PRACTICES

C. Foster and J. Golay

which (we assume here for the purpose of diagrammatic exposition) doubles bus frequencies.

In the initial situation, buses run at times A and B, and the uniform fare is set at $F_0$. The total gross benefit to society — or potential consumer surplus — is the area under the thick line. The net benefit is the gross benefit minus fixed and running costs of bus operations. If frequencies are doubled, with buses running at times C and D, the gross benefit to society is increased by the shaded area, which must be at least equal to the fixed and running costs of the extra buses if the improved service is to bring net benefits. Fares rise to $F_1$. Free competition brings the same net benefits to society when it brings about a doubling of the bus frequencies, but the distribution of the net benefits falls more to the consumers than to the operator, with fares probably somewhere between $F_0$ and $F_1$, or even below $F_0$, depending not only on the size of the monopoly profits at $F_0$ and $F_1$, but also on the cost savings arising from greater efficiency of operation and better tailored bus sizes.

REFERENCES