A Theory of Consumption Norms and Implications for Environmental Policy

Alistair Ulph

(School of Social Sciences and Sustainable Consumption Institute, University of Manchester)

and

David Ulph

(School of Economics, University of St. Andrews)

Abstract

In this paper we assume that for some commodities individuals may wish to adjust their levels of consumption from their normal Marshallian levels in order to conform to the consumption norms for a group of people to which they wish to belong. Unlike conspicuous consumption this can mean that some individuals may reduce their consumption of the relevant commodities. We first model the decisions of an individual using a three-stage game in which individuals first decide whether or not they wish to adhere to a norm, then decide which norm they wish to adhere to, and finally decide their actual consumption. We then assume there is a population of individuals with differing tastes and analyse which norms constitute an equilibrium norm, and how many equilibrium norms might exist. Finally we study the implications of our model for redistributive policies, environmental policies and econometric analysis of consumer demand.

March 2017

JEL Classification: D11, D69

Keywords: desire for conformity, participation-consistent consumption interval, distribution existence of equilibrium consumption norms, policy implications.

Preliminary Version. Please do not quote.

Acknowledgements We are grateful to Partha Dasgupta, Dale Southerton and seminar participants in the University of St. Andrews for comments on an earlier version of this paper.
1. **Introduction**

In this paper we examine the implications for understanding consumer behaviour and the design of public policy of assuming that individual consumption behaviour is influenced by the consumption decisions of other individuals through the existence of *consumption norms*. We begin by setting out how our treatment of consumption norms relates to the broader literature on consumer behaviour and social norms.

First, we distinguish such consumption norms from the interaction between individual consumption decisions through the Veblen effect (Veblen (1924)), whereby individuals’ consumption decisions are influenced by those of others in a competitive manner as individuals seek to match their consumption to that of an aspirational group (and differentiate it from that of a distinction group)\(^1\). The Veblen effect is an externality which can sustain overconsumption and a market distortion that needs to be corrected by a policy such as a tax on goods prone to conspicuous consumption. We consider a different route by which individuals’ consumption decisions may be influenced by those of others, namely through a desire to be seen to belong to a group of similar-minded individuals, thereby establishing consumption norms\(^2\). A key distinction from the Veblen effect is that such a proclivity to conform to a consumption norm can lead some individuals to *reduce* their consumption of a good relative to what they would have consumed in the standard economists’ model where consumers take no account of the consumption of others.

Second, we distinguish consumption norms from the broader concept of social norms. Social norms play a number of roles of which we highlight two. As Young (2014) notes a key function of social norms is to coordinate people’s expectations in interactions which are characterised by multiple equilibria, for example public good games. Analysis of social norms often involves using evolutionary game theory to predict which of a multiplicity of possible outcomes emerge as stable equilibria and a focus on the design of punishment

---

\(^1\)For recent analyses of the Veblen effect see Arrow and Dasgupta (2010), Dasgupta, Southerton, Ulph and Ulph (2016) and Ulph (2014). The Veblen effect is invoked to explain the Easterlin Paradox (Easterlin (1974, 2001) ) whereby, after a certain level of per capita income, further growth in income per capita seems to have no effect on measures of well-being as captured by surveys of happiness (see for example Blanchflower and Oswald (2004)).

\(^2\) The most influential sociological theories of consumption – especially Bourdieu’s (1984) account of taste and distinction and Bauman’s (1990) account of neo-tribal lifestyles – both present social norms and belonging as the fundamental mechanisms underpinning its contemporary social patterning (see Southerton (2002) for a full discussion). In our use of the term consumption norms should be interpreted as a subset of the much broader category of social norms which can affect behaviour.
strategies by other players (e.g. Axelrod (1986)). Typically there are multiple stable equilibria, and these often involve discrete choices, such as whether or not to smoke in public (Nyborg and Rege, 2003) whether or not to recycle household waste (Brekke, Kipperberg and Nyborg, 2010).

Another aspect of social norms (dating back to Festinger, 1954) arises from people’s uncertainty about their identity or opinions. For activities like provision of public goods, voting, or charitable giving evidence suggests that individuals are more willing to contribute if they know members of their norm group have contributed or think others might match their contributions (referred to as conditional cooperation) – see for example Ledyard (1995), Azar (2004), Frey and Meier (2004), Tan and Bolle (2007), Gerber and Rogers (2009), Chaudhuri (2011), Bucholz, Falkinger and Rubbelke (2012), Abbott, Nandeibam and O’Shea (2013).

Applying such concepts to the consumption of private goods, Hargreaves-Heap (2013) and Hargreaves-Heap and Zizzo (2009)) identify a number of benefits from social norms, including (a) observing members of a norm group consuming a product an individual has not experienced can give implicit information about the quality of that product; (b) in a related manner, giving people information about what similar people achieve in saving energy, or retirement savings can significantly increase levels of savings (Allcott (2011)); (c) by developing trust between members of a norm group, consumption norms can reduce transactions costs; (d) for a number of consumption activities, such as reading a book or attending a concert, the benefits are not just the private experience but the subsequent opportunity to share thoughts about such experiences (the ‘water cooler’ effect) and this requires individuals to have overlapping sets of cultural interests.

In our analysis of consumption norms we assume that individuals are perfectly informed about the characteristics of products. Our concept of consumption norms is closer to that of Akerlof and Kranton (2000), who argued that an ability to identify with a group of people is a

3 Axelrod’s analysis also differs from ours in that he uses an evolutionary game approach, while we assume that individuals are conventional utility-maximisers, albeit with non-standard utility functions.

4 See Bennett et al (2009) for a comprehensive analysis of the clustering of consumption activities based on overlapping cultural interests in the UK.

5 This is linked to notions of social capital. It is important to distinguish between group membership developing greater trust between insiders – a positive social benefit – and developing a greater distrust of outsiders – a reduction in social benefit (see Putnam (2000) and Dasgupta (2000) for a recognition that social capital may have negative as well as positive effects). Hargreaves-Heap and Zizzo (2009) construct a measure to test this distinction, and in their experiments they find it is the negative effect which predominates.
key part of self-identity and yields an important psychological benefit of belonging to a group, what Adam Smith referred to as the ‘special pleasure of mutual sympathy’. It is this pure psychological benefit of belonging to a group that we have in mind in this paper. An important implication is that it is the potential internal loss of such a benefit that provides the incentive to adhere to the consumption norm, rather than the use of punishment strategies by other players.

Much of the literature on consumption norms does not provide a formal model of how consumption norms might influence consumers’ behaviour. The paper that is closest to the model reported here is the study by Bernheim (1994) of conformity. In his model people differ in terms of their types (measured by a single index distributed over some interval). Society has a pre-specified notion of an ideal type and people suffer a loss of self-esteem the further their type is from the ideal. Individual’s well-being depends on the utility they get from their actions, and the esteem in which they are held by others. If an individual’s type was public information, all an individual could do is to act to maximise utility. But an individual’s type is private information, and has to be inferred from one’s actions, so individuals have an incentive to bias their actions towards that which an ideal person would perform; this leads some individuals to do more than they would do to maximise utility and others to do less. There are two possible equilibria: a fully-revealing equilibrium and a pooling equilibrium in which a group of individuals whose types are closer to the ideal type carry out the same level of action – so the equilibrium specifies a common action norm and the group of people who adhere to this common norm.

In this paper we focus directly on consumption behaviour and consumption norms, and we examine how behaviour influenced by such norms relates to traditional analysis of consumer demand captured by Marshallian demand curves. Like Bernheim we want to explain endogenously how consumption norms change individual consumer behaviour, which consumption norms can emerge as equilibrium norms, and how many norms there might be. All behaviour is assumed to be individual – there is no process for communication or coordination. Unlike Bernheim all information is public. In particular, to rule out other channels of interactions, we assume consumers are perfectly informed about the quality of the commodities being consumed and consumption is a private good. The crucial difference is

---

6 Hargreaves-Heap and Zizzo (2009) also develop a test to measure this psychological benefit of belonging to a group; they find that it balances out the negative effect of group membership noted in the previous footnote.
that there is no concept of an ideal type of consumption, and the motivation to belong to a group is the pure psychological benefit discussed above.

A final important way in which we seek to distinguish our treatment of consumption norms from that found in the literature is that it is frequently assumed that an individual’s utility loss in shifting consumption from the Marshallian demand towards a norm takes a simple quadratic form. This has the implication that people adjust their consumption towards the norm, but the only person who consumes at the norm level for that good is the individual whose Marshallian demand is the norm. This raises the question, often noted in the literature (Manski, 2000), that it can be difficult to identify the effect of consumption norms empirically. In contrast we assume that the utility loss suffered by an individual deviating from Marshallian demand depends on the absolute value of the loss. As we will see this implies that there will always be a significant group of individuals who consume the norm exactly; there may exist a second group whose Marshallian demands are from the norm than the first group and who adjust their consumption towards the norm (increasing their consumption if their Marshallian demand is below the norm, decreasing their consumption if their Marshallian demand is above the norm; and there may exist a third group whose Marshallian demands are even further from the norm who just consume their Marshallian demands. Indeed we will see that there are equilibria of our model where, although Marshallian demands vary systematically across the population, everyone consumes the norm level of consumption exactly so there is a striking difference between the pattern of consumption with and without such a consumption norm.

In the next section we set out our model of consumption norms and in section 3 analyse its implications for individual behaviour. In section 4 we determine what norms can emerge as stable equilibria. In section 5 we analyse some public policy implications from our model, in particular that for some parameter values conventional environmental policy recommendations may be ineffective or even welfare-reducing. Section 5 concludes.

2. The Model

There are two consumer goods, the individual consumption of which is denoted by the variables \( x \geq 0, z \geq 0 \), where \( x \) is a commodity the level of whose consumption might be a

---

7 In Dasgupta, Southerton, Ulph and Ulph (2016) we presented a brief summary of the model developed in the next section and illustrated its implication for environmental policy in a simple special case. In this paper we set out the model in greater detail and seek to draw more general public policy implications.
norm\(^8\) and \(z\) is expenditure on all other goods and serves as numeraire so its price is 1. The price of \(x\) is \(p\).

We take it that what identifies a particular level of \(x\), say \(x^*\), as a consumption norm, and gives individuals a sense of group identity by adjusting their individual consumption of \(x\) towards \(x^*\) - what we will refer to as **adhering to the norm**; \(n^*\) is the fraction of the population that adheres to the norm. So a consumption norm is characterised by the pair \((x^*, n^*)\), where \(x^*\) is the level of consumption to which individuals may seek to adhere and \(n^*, 0 \leq n^* \leq 1\) is the fraction of the population adhering to that norm.

There is a population of individuals each of whom has some income \(y\), and a utility function

\[
\bar{u}(x, z; \delta; x^*, n^*) = ax - \frac{x^2}{2} + z + \delta \left[ \sigma(n^*) - \alpha|x^* - x| \right],
\]

where \(\delta \in [0,1]\) is a choice variable that takes the value 1 if the individual chooses to adhere to a norm, and 0 if the individual chooses not to adhere and so behaves in a traditional Marshallian fashion.

Substitute in the budget constraint, and we can express utility in terms of the norm parameters \((x^*, n^*)\) and the two individual decision variables of interest - \(\delta\), whether or to adhere to the norm and \(x\) consumption of the norm-influenced good – as

\[
u(x, \delta; x^*, n^*) = y + ax - px^2 - \frac{x^2}{2} + \delta \left[ \sigma(n^*) - \alpha|x^* - x| \right]
\]

(1)

For an individual who has chosen to adhere to the norm:

- \(\alpha|x^* - x|\) is what we call the **strength of attraction** of the norm level of consumption since, it measures the rate at which utility falls as individual consumption of \(x\) deviates in either direction from the norm level. As mentioned in the introduction we have chosen to reflect the strength of attraction by using the absolute deviation rather than the more conventional square of the deviation, since, as we will see, this implies that there will be a mass of individuals who will consume **exactly** the norm level of consumption, whereas under the alternative specification the only individuals who

---

\(^8\) In principle this could be an aggregate of goods which act as norms.
will choose exactly the norm are those whose Marshallian level of consumption would have been $\bar{x}$. In this way the norm consumption level becomes more perceptible.

- $\sigma(n^*)$ is what we call the strength of desire for conformity with a group that makes up a fraction $n^*$ of the population. We assume that

$$\sigma(n^*) = -\chi + \varphi n^*, \quad (2)$$

where $\chi > 0$ is a fixed cost (i.e. unrelated to consumption decisions) of adhering to a norm group, which can be thought of as a cost of giving up individuality; while $\varphi > 0$ is the rate at which the strength of the desire to conform grows with the fraction of the population that choose to conform. The variable $\varphi n^*$ can therefore be thought of as measuring the benefit derived from establishing a sense of identity with a fraction $n^*$ of the population. So the strength of the desire for conformity captures a tension between a psychic cost of giving up one’s sense of individuality and a psychic benefit/comfort from being part of a larger group.

Obviously for an individual to adhere to any norm it has to be the case that the strength of desire for conformity is positive when the entire population adheres to it – i.e. when $n = 1$ - so we assume that $\varphi > \chi > 0$, and let $n = \frac{\chi}{\varphi}$, $0 < n < 1$ be the minimum fraction of the population that need to adhere to a norm for that norm to have a positive strength of desire for conformity. We can therefore re-write (1) as:

$$u(x, \delta; x^*, n^*) = y + ax - px - \frac{x^2}{2} + \delta \left[ \varphi(n^* - n) - \alpha \left| x^* - x \right| \right], \quad (3)$$

We initially make the stronger assumption on $n$, namely that

$$0.5 < n < 1. \quad (4)$$

This rules out the possibility of there being a multiplicity of co-existent norms to which individuals would have to consider adhering. In section 4.2 we weaken the restriction in (4) and consider under what conditions there may exist two equilibrium norms.
Finally we assume that the taste parameter, $a$, is uniformly distributed in the population on the interval $[a, \bar{a}]$, $0 < p < a < \bar{a}$. We denote by $\omega = 0.5(\bar{a} - a)$ the width of the distribution of preferences, or the degree of diversity of preferences.

3. Individual Decisions

In this section we take as given the existence of some norm $\left(x^*, n^*\right)$ and determine which individuals will choose to adhere to this, and then, in the following section we determine which norms could merge as equilibria. In order to determine which individuals will choose to adhere to a given norm, we first need to determine an individual’s consumption-maximising choices conditional on the adherence decision.

3.1 Consumption decisions

3.1.1 Marshallian Consumption

If an individual has chosen not to adhere to a norm - $\delta = 0$ - then from (3) the utility-maximising choice of $x$ is:

$$x^0 = \text{ArgMax}_{x \geq 0} u(x, 0, x^*, n^*) = \text{ArgMax}_{x \geq 0} ax - \frac{x^2}{2} - px = a - p . \quad (5)$$

For notational simplicity, in what follows we identify individuals in terms of their Marshallian consumption. Given our assumption above about the distribution of $a$ we take these Marshallian consumptions to be uniformly distributed on the interval $[a - p, \bar{a} - p] = \left[X, \bar{X}\right]$, $0 < X < \bar{X}$, with mean $\mu = \frac{X + \bar{X}}{2}$, and $\omega = \frac{\bar{X} - X}{2} = \frac{\bar{a} - a}{2}$ the width of the spread of tastes in the population. This will turn out to be a crucial variable in what follows.

For an individual of type $x^0$ who has chosen not to adhere to the norm, the level of indirect utility associated with their Marshallian consumption is:

---

9 Given the quasi-linear structure of preferences the utility maximising choices of $x$ are independent of income, and so the precise distribution of income, $y$, plays no role in our analysis. All we require is that its distribution is sufficiently positively correlated with that of $a$ such that for all individuals $y > p(a - p + \alpha)$, and so individuals always buy a positive amount of the numeraire good.
$$v_0(x_0, x^*, n^*) = \text{Max}_{x \geq 0} u(x, 0, x^*, n^*) \equiv \text{Max}_{x \geq 0} \left( y + ax - \frac{x^2}{2} - px \right) = y + \left( \frac{x_0}{2} \right)^2. \quad (6)$$

### 3.1.2 Consumption of individuals who adhere to the norm

If $\delta = \mathbf{1}$ then, taking account of (3) and the budget constraint the utility-maximising consumption is:

$$\hat{x}(x^*, n^*) = \text{ArgMax}_{x \geq 0} u(x, 1, x^*, n^*) \equiv \text{ArgMax}_{x \geq 0} ax - \frac{x^2}{2} - px - \alpha |x^* - x|. \quad (7)$$

Carrying out the maximisation it is easy to see that for an individual of type $x^0$ who has chosen to adhere to a norm $(x^*, n^*)$ chosen consumption is:

$$\hat{x}(x^0, x^*) = \begin{cases} 
  x^0 - \alpha & \iff x^* < x^0 - \alpha \\
  x^0 & \iff x^0 - \alpha \leq x^* \leq x^0 + \alpha \\
  x^0 + \alpha & \iff x^* > x^0 + \alpha 
\end{cases} \quad (8)$$

So when an individual of type $x^0$ adheres to any norm their chosen level of consumption lies within what we call their norm-consistent interval of consumption $\left[x^0 - \alpha, x^0 + \alpha\right]$. This illustrates what can be thought of as the gravitational pull of consumption norms:

- if the norm level of consumption is sufficiently close to an individual’s Marshallian level of consumption – specifically if it lies inside the individual’s norm-consistent interval of consumption - the individual will consume the norm level exactly;
- if the norm level of consumption is outside an individual’s norm-consistent interval of consumption then the individual’s consumption will lie at the boundary of the norm-consistent interval of consumption that is closest to the norm level.

It follows from the first bullet point that there will be a range of individuals whose Marshallian demands differ from the norm, but nevertheless choose to consume exactly at the level specified by the norm, thus making this norm level of consumption highly perceptible.

For an individual of type $x^0$ who has chosen to adhere to the norm $(x^*, n^*)$ the indirect utility utility is
\[ v_1(x^0, x^*, n^*) = \max_{x \geq 0} u(x, 1, x^*, n^*) = \max_{x \geq 0} y + \varphi(n^*-n) - px - \alpha |x^*-x|. \quad (9) \]

Substitute (8) into (4) to get:

\[
v_1(x^0, x^*, n^*) = y + \varphi(n^*-n) + \begin{cases} \frac{(x^0 - \alpha)^2}{2} + \alpha x^*, & x^* < x^0 - \alpha \\ x^0 - \alpha x^*, & x^0 - \alpha \leq x^* \leq x^0 + \alpha \\ \frac{(x^0 + \alpha)^2}{2} - \alpha x^*, & x^* > x^0 + \alpha \end{cases} \quad (10)\]

3.2 The decision to adhere to a norm

The net benefit to an individual of type \(x_0\) from choosing to adhere to the norm \((x^*, n^*)\) is

\[ \beta(x^0, x^*, n^*) = v_1(x^0, x^*, n^*) - v_0(x^0, x^*, n^*). \quad (11) \]

Substitute in (6) and (10) and, after a little re-arranging, we get

\[ \beta(x^0, x^*, n^*) = \varphi(n^*-n) - L(x^0, x^*), \quad (12) \]

where

\[ L(x^0, x^*) = \begin{cases} \frac{\alpha^2}{2} + \alpha [x^0 - (x^0 + \alpha)] - x^*, & x^* < x^0 - \alpha \\ \frac{(x^0 - x^*)^2}{2}, & |x^0 - x^*| \leq \alpha \\ \frac{\alpha^2}{2} + \alpha [x^* - (x_0 + \alpha)], & x^* > x^0 + \alpha \end{cases} \quad (13) \]

captures the loss of utility suffered by an individual from making the “wrong” consumption, and shows that there are TWO sources of this welfare loss:

(i) consumption is potentially different from the Marshallian amount;
(ii) chosen consumption may also be different from the norm.

We can write the utility loss in a more compact form as:
\[ L(x^*, x^0) = \begin{cases} \frac{-\alpha^2}{2} + \alpha |x^0 - x^*|, & |x^0 - x^*| > \alpha \\ \frac{|x^0 - x^*|^2}{2}, & |x^0 - x^*| \leq \alpha \end{cases} \]  

(14)

Notice that for all individuals this loss is non-negative and is zero only for an individual whose Marshallian demand coincides with the consumption norm. It is easy to see that the way this loss varies across individuals of different types – i.e. with different levels of Marshallian demand -is as illustrated in Figure 1\(^{10}\).

Figure 1 here

It follows that an individual will adhere to a norm \((x^*, n^*)\) iff the net benefit from doing so is positive. This certainly requires that the fraction of consumers adhering to it is greater than the minimum threshold \(\pi\) – i.e. it requires \(\beta(x^0, x^*, n^*) > 0 \implies n^* > \pi\);

Finally we characterise which individuals in the population would adhere to a given norm \((x^*, n^*)\), i.e. for which types \(x^0\), \(\beta(x^0, x^*, n^*) > 0\). So define by \(\chi^0(x^*, n^*), \bar{x}^0(x^*, n^*)\) the range of values of \(x^0\) of individuals who would adhere to the norm \((x^*, n^*)\), ignoring for the moment the need for \(\chi^0(x^*, n^*), \bar{x}^0(x^*, n^*)\) to lie in the range \([X, \bar{X}]\). There are 2 cases, which are differentiated by whether the gains from adhering to a norm \(x^*\) are greater or less than the costs of adhering to the norm when \(x^0 = x^* \pm \alpha\).

**Case A.** \(\varphi(n^* - \pi) \leq 0.5\alpha^2\)

NOTE: a sufficient condition for this case to arise is \(\varphi(1 - \pi) \leq 0.5\alpha^2\);

Then:

\[
\chi^0(x^*, n^*) = x^* - \sqrt{2\varphi(n^* - \pi)} \geq x^* - \alpha; \quad \bar{x}^0(x^*, n^*) = x^* + \sqrt{2\varphi(n^* - \pi)} \leq x^* + \alpha
\]

\[x^0 < \chi^0(x^*, n^*) \implies \hat{x}(x^0, x^*) = x^0\]

\[\chi^0(x^*, n^*) \leq x^0 < \bar{x}^0(x^*, n^*) \implies \hat{x}(x^0, x^*) = x^*\]

\[x^0 > \bar{x}^0(x^*, n^*) \implies \hat{x}(x^0, x^*) = x^0\]

\[\sqrt{2\varphi(n^* - \pi)}, \sqrt{2\varphi(n^* - \pi)}\]  

(15)

\(^{10}\) The figures are at the end of this paper.
See Figure 2:

Figure 2 here

Case B: \( \varphi(n^* - n) > 0.5\alpha^2 \)

Then:

\[
\begin{align*}
\hat{x}^0(x^*,n^*) &= x^* - \frac{\alpha}{2} - \frac{\varphi}{\alpha} (n^* - n) < x^* - \alpha; \\
\bar{x}^0(x^*,n^*) &= x^* + \frac{\alpha}{2} + \frac{\varphi}{\alpha} (n^* - n) > x^* + \alpha
\end{align*}
\]

\begin{align*}
x^0 < \hat{x}^0(x^*,n^*) &\Rightarrow \hat{x}(x^0,x^*) = x^0 \\
\hat{x}^0(x^*,n^*) &\leq x^0 < x^* - \alpha \Rightarrow \hat{x}(x^0,x^*) = x^0 + \alpha \\
x^* - \alpha &\leq x^0 \leq x^* + \alpha \Rightarrow \hat{x}(x^0,x^*) = x^* \\
x^* + \alpha &< x^0 \leq \bar{x}^0(x^*,n^*) \Rightarrow \hat{x}(x^0,x^*) = x^0 - \alpha \\
x^0 > \bar{x}^0(x^*,n^*) &\Rightarrow \hat{x}(x^0,x^*) = x^0
\end{align*}

See Figure 3:

Figure 3 here

For both Case A and Case B we now take account of the need for \( \hat{x}(x^*,x^0) \) to satisfy the condition: \( \bar{X} \leq \hat{x}(x^*,x^0) \leq \bar{X} \). So define:

\[
\begin{align*}
\hat{X}^0(x^*,n^*) &\equiv \max[\bar{X},\hat{x}^0(x^*,n^*)], \\
\bar{X}^0(x^*,n^*) &\equiv \min[\bar{X},\bar{x}^0(x^*,n^*)]
\end{align*}
\]

Then the two conditions a norm \( (x^*,n^*) \) must satisfy to be a *Nash equilibrium norm* are:

\[
\begin{align*}
x^* &= \left[ \int_{\hat{X}^0(x^*,n^*)}^{\bar{X}^0(x^*,n^*)} \hat{x}(x^0,x^*)dx^0 \right] / [\bar{x}^0(x^*,n^*) - \hat{x}^0(x^*,n^*)] \\
n^* &= [\bar{x}^0(x^*,n^*) - \hat{x}^0(x^*,n^*)] / 2\omega \geq n \geq 0.5
\end{align*}
\]

Condition (18) is just the requirement that average consumption of those adhering to a norm equals the norm (where in constructing the average we need to scale the density function to reflect the fact that we are taking the average over the range of values \( [\hat{x}^0(x^*,n^*),\bar{x}^0(x^*,n^*)] \), which may be a subset of the range \( [\bar{X},\bar{X}] \)). Condition (19) is just the requirement that the fraction of the population adhering to the norm must be at least \( n \).
4. Existence of Equilibrium Norms

For any set of parameters \((\alpha, \phi, \eta, \omega)\) we investigate in section 4.1 whether a single equilibrium norm exists, and if so whether there is a unique value for a single equilibrium norm or there is a range of possible values a single equilibrium norm might take. In section 4.2 we relax somewhat the assumption that \(0.5 < \eta\) and consider whether there might exist two equilibrium norms.

4.1 Existence of a Single Equilibrium Norm

To provide some intuition for the results that follow, suppose we fix the parameters \((\alpha, \phi, \eta)\) and focus on values of \(\omega\) for which \(x^* = \mu\) might be an equilibrium norm with an associated value of \(n^*\). As we have already noted, \(x^* = \mu\) gives the greatest space for people to adhere to a norm.

Suppose initially \(\omega\) is sufficiently small that:

\[
\bar{x}^0(\mu, 1) < \bar{X} < \bar{X} < \bar{x}^0(\mu, 1)
\]  \hspace{1cm} \text{(20)}

and \((\mu, 1)\) is an equilibrium norm. Now suppose \(\mu\) stays fixed but \(\omega\) increases, so \((\bar{X} - \bar{X})\) widens. The benefit for all those adhering to the norm \((\mu, 1), \phi(1 - \eta)\), does not change, but the cost to the marginal person adhering to the norm \((\mu, 1)\):

\[
L(\mu, X) = L(\mu, \bar{X}) = \begin{cases} 
\frac{-\alpha^2}{2} + \alpha \omega, & \omega > \alpha \\
\frac{\omega^2}{2}, & \omega \leq \alpha
\end{cases}
\]  \hspace{1cm} \text{(21)}

now increases. So the range \([\bar{x}^0(\mu, 1) - \bar{x}^0(\mu, 1)]\) shrinks. As long as (20) continues to hold \((\mu, 1)\) remains an equilibrium norm.

Suppose \(\omega\) continues to increase to an intermediate value such that the inequalities in (20) no longer hold, so:

\[
n^* = \frac{\bar{x}^0(\mu, n^*) - \bar{x}^0(\mu, n^*)}{2\omega} < 1
\]  \hspace{1cm} \text{(22)}
There are now two effects of further increases in $\omega$. First, as above, for a given $n^*$, the benefit of adhering to a norm $\varphi(n^*-n)$ does not depend on $\omega$, while the cost of adhering to a norm $(\mu,n^*)$:

$$L(\mu, x^0(\mu,n^*)) = L(\mu, \bar{x}^0(\mu,n^*)) = \begin{cases} \frac{\alpha^2}{2} + \alpha \varphi n^*, & \omega > \alpha / n^* \\ \frac{(\omega n^*)^2}{2}, & \omega \leq \alpha / n^* \end{cases}$$

(23)

increases with $\omega$. So again the range $[\bar{x}^0(\mu,n^*) - x^0(\mu,n^*)]$ shrinks.

But there is a second effect, because, from (22), as $\omega$ increases the numerator of (23) decreases while the denominator increases, so $n^*$ decreases.

So as $\omega$ increases, $n^*$ shrinks toward $\bar{n}$, so the benefit of adhering to the norm shrinks to zero while, again, the costs increase with $\omega$. So for large enough values of $\omega$, there cannot exist an equilibrium norm $(\mu, n^*)$. This intuition is confirmed in Results 1 to 3 below. Proofs of all results are in the Appendix.

We note that in setting out the results below we will again need to take account whether any equilibrium norm takes the form of Case A or Case B, recognising that the definition of these Cases given in section 3.2 above depends on $n^*$, which is an endogenous variable. In setting out Results 1 and 2 it will be useful to define the following variables, which will define ranges of values within which $x^*$ must lie for $(x^*, n^*)$ to be an equilibrium norm.

Define: $\xi = \max[\bar{X}, \bar{X} - \sqrt{2\varphi(1-n)}]$, $\bar{\xi} = \min[\bar{X}, \bar{X} + \sqrt{2\varphi(1-n)}]$, $\psi = \max[\bar{X}, \bar{X} - \alpha]$ $\bar{\psi} = \min[\bar{X}, \bar{X} + \alpha]$ $\eta = \frac{\varphi(\omega n - 0.5\alpha)}{\varphi - \alpha \omega} + \alpha / 2$, $\bar{\eta} = \bar{X} - \frac{\varphi(\omega n - 0.5\alpha)}{\varphi - \alpha \omega} - \alpha / 2$. Note all these ranges are symmetric around the mean, $\mu$

**Result 1:** If EITHER (i) $\omega^2 < 2\varphi(1-n) \leq \alpha^2$ OR (ii) $\omega^2 \leq \alpha^2 < 2\varphi(1-n)$ then for any $x^*$ such that (i) holds and $x^* \in [\xi, \bar{\xi}]$ or (ii) holds and $x^* \in [\psi, \bar{\psi}]$, $(x^*, 1)$ is an equilibrium norm.

As noted above, the intuition is that when $\omega$ is small, specifically $\omega \leq \alpha$, so the width of the interval of types is not greater than the width of the band within which people adhering to the norm adhere exactly, then there exists a range of values for $x^*$, symmetric around $\mu$, such that
everyone adheres to the norm \((x^*, 1)\). The range of values within which \(x^*\) must lie depends on whether we are dealing with Case A or Case B.

We now turn to the intermediate result.

**Result 2:** If \(\frac{\varphi(1-n)}{\alpha} + \frac{\alpha}{2} \geq \omega > \alpha\) then (i) for any \(x^* \in [\eta, \bar{\eta}], (x^*, n^*)\) is an equilibrium norm where \(n^* = \frac{\varphi n - 0.5\alpha^2}{\varphi - \alpha\omega}\); (ii) \((\mu, 1)\) is also an equilibrium norm, but it is unstable.

Result 2(a) is shown in Figure 4. The interesting feature of Result 2(a) is that because \(\omega\) is sufficiently greater than \(\alpha\) this allows an equilibrium norm in which some people adhere exactly to that equilibrium norm, some people adhere to the norm by adjusting their consumption upwards or downwards from their Marshallian demands by an amount \(\alpha\), while yet others do not adhere to this norm and just consume their Marshallian demands \((n^* < 1)\).

As in Result 1 there is a range of possible values for \(x^*\) such that \((x^*, n^*)\) is the single equilibrium norm, so the equilibrium norm can differ significantly from the mean value of Marshallian demands, \(\mu\).

Result 2(b) is shown in Figure 5.

As in Result 2(a), some people adhere exactly to the single equilibrium norm level of consumption \(\mu\) while others adjust their consumption towards the norm by consuming more or less than their Marshallian demands by an amount \(\alpha\). However, unlike Result 2(a), there are no individuals who do not adhere to the norm and consume only their Marshallian demands. The implication of this latter point, and the important aspect of Result 2(b), is that there is a unique single equilibrium norm, which has to be the mean value of Marshallian demands. The rationale for this result is that if we considered a possible value of an equilibrium norm, \(x^*\) which differed from \(\mu\) this would imply that more weight is given to one of the two groups of consumers who adhere to the norm, but do not adhere exactly to the norm either raising or lowering their consumption from their Marshallian demands by an amount \(\alpha\). Giving one of these two groups more weight than the other unbalances average consumption away from the norm, violating condition (18) which requires that for an equilibrium norm the average value of consumption of those adhering to the norm must be the norm itself. This argument is even stronger if a possible norm is sufficiently different.
from $\mu$ that one of groups who do not adhere exactly to the norm no longer lies in the range $[X, \bar{X}]$.

An important feature of Result 2 is that as in some of the literature on social norms we can get multiple possible equilibrium norms, and one of them, $(\mu, 1)$, is unstable. The interesting policy implication of Result 2(b) is that even a slight shift in the parameters of this problem, due to a policy change say, which caused $[X, \bar{X}]$ to shift slightly, creating a new mean, $\bar{\mu} \neq \mu$, would cause the previous equilibrium norm $\mu$ to no longer be a norm.

Finally we present the case where $\omega$ is sufficiently large that no equilibrium norm exists.

**Result 3:** If EITHER (i) $2\varphi(1-n) \leq \alpha^2$ and $\omega^2 > 2\varphi(1-n)$; OR (ii) $2\varphi(1-n) \geq \alpha^2$ and $\omega > 0.5\alpha + \varphi(1-n)/\alpha$ then no equilibrium norm exists.

The intuition, as noted, is that as $\omega$ increases the cost of adhering to the norm increases, but, because $n^*$ falls, the benefit of adhering to the norm decreases towards zero, so the net benefit of adhering to a norm becomes negative.

### 4.2 Two Equilibrium Norms

We now relax the assumption that $n > 0.5$ so that we have the possibility of more than one equilibrium norm. We consider Result 4(a), and suppose now that $0.5 > n > 1/3$ and that parameters are such that $1/3 < n < n^* < 0.5$, which requires $\varphi n > 0.5\alpha^2$. Then it is clear that we can have 1 or 2 equilibrium norms. If a norm lies in an interval around $X + 0.5\omega$ then there is sufficient space for a second norm in an interval around $\bar{X} - 0.5\omega$. However if a norm lies in an interval around $X + \omega = \bar{X} - \omega$ there will be insufficient space for a second norm. Since individuals will only adhere to a norm if they are at least as well off as they would be consuming their Marshallian demands then wellbeing must be higher with two equilibrium norms than with one equilibrium norm. Of course this conclusion depends strongly on our assumption of a uniform distribution of preferences.

This completes our analysis of what consumption norms can be considered equilibrium norms.
5. Policy Implications

In this section we consider the policy implications of our analysis of consumption norms. In Section 5.1 we assume again that \(0.5 < n < 1.0\) so only one equilibrium norm \((x^*, n^*)\) can exist, though there can be range of possible values for that single norm. Because there may multiple possible values for an equilibrium norm, and individuals simply take such a norm as given, we ask whether from a welfare perspective there is an optimal consumption norm and if so what are the policy implications of how such an optimal norm might be brought about. In Section 5.2 we consider a situation where consumption generates environmental damage and ask what are the implications for the design of environmental policies of the fact that individuals wish to adhere to a consumption norm.

5.1 Optimal Consumption Norms

In Results 1 and 2(i) we have shown that there is a range of possible values of \(x^*\) for which a single equilibrium norm, \((x^*, n^*)\) exists, where the associated value for \(n^*\) is either 1 or the constant \(\frac{\varphi n - 0.5\alpha^2}{\varphi - \alpha\delta}\) independent of \(x^*\). We now consider the following thought experiment: if a policy maker was able to choose one of these values for \(x^*\), which one would be chosen? From (14) the net benefit to an individual with Marshallian demand \(x^0\) of adhering to a norm \(x^*\) is given by \(\varphi(n^* - n) - L(x^0, x^*)\), where \(L(x^0, x^*)\) is given by (13). So if we ask which value of \(x^*\) maximises the expected net benefit of adhering to that norm, that is equivalent to choosing \(x^*\) to minimise the expected value of \(L(x^0, x^*)\), denoted \(E_{x^0}[L(x^0, x^*)]\).

We have the following result:

**Result 4.** In Results 1 and 2(i) there is a range of possible values for an equilibrium norm \(x^*\) (with an associated value of \(n^*\) which is either 1 or a constant independent of \(x^*\)). In Result 1 the value of \(x^*\) which minimises the expected value of the utility loss \(E_{x^0}[L(x^0, x^*)]\) is \(x^* = \mu\). In Result 2(i) there is no optimal value of \(x^*\) - the value of \(E_{x^0}[L(x^0, x^*)]\) is the same for all possible equilibrium values of \(x^*\).

The intuition behind these results is that in Result 1, because everyone adheres to a norm, if \(x^* \neq \mu\) the loss of utility from those furthest from \(x^*\) outweigh losses from those closer to the norm, so centring the norm on the mean value of Marshallian demand reduces these
extreme losses. On the other hand in Result 2(i) because not everyone adheres to the norm the losses of those adhering to the norm are the same no matter which value of \((x^*,n^*)\) constitutes the norm.

What does this imply for policy in Result 1? It is inherent in our analysis that we have treated the norm as exogenously determined outside the model, so policy makers are not able to manipulate the norm. But we assume that policy makers are able to use taxes and lump-sum transfers to shift the Marshallian demand for the norm good so that Marshallian demand of an individual with the mean value of the taste parameter for the norm good equals the norm. So if \(x^* < \mu\) the optimal policy is to tax the norm good to reduce mean demand to the norm, while if \(x^* > \mu\) the optimal policy is to subsidise the norm good to raise mean demand to the norm. In Result 2(i) there is no role for policy.

5.2 Implications for Environmental Policy.

We now suppose that each unit of consumption of the norm good generates environmental damage with a constant unit damage cost \(\gamma\), and the only way of reducing this environmental damage is to reduce the consumption of the norm good. Suppose that parameter values are such that in Results 1 and 2(i), there is a single equilibrium norm, \(x^*\). For simplicity, assume that in the absence of any environmental policy, but perhaps as a result of the redistributive policies we analysed in the previous section, initially \(x^* = \mu^0\).

Suppose now the government imposes the standard Pigovian tax \(\tau = \gamma\). As in the standard model of consumption without norms this Pigovian tax will reduce mean Marshallian demand for the norm good to its optimal level \(\mu^1 = \mu^0 - \gamma\); similarly it will reduce by \(\gamma\) the range of values \([X, \bar{X}]\) within which Marshallian demand must lie, and the ranges of values \([\underline{x}, \overline{x}]\), \([\underline{\eta}, \overline{\eta}]\) and \([\underline{\psi}, \overline{\psi}]\) for \(x^*\) within which a single equilibrium norm \((x^*, n^*)\) can lie in Results 1 and 2(i); for ease of notation we will just refer generically to the pre-policy limits within which \(x^*\) must lie for \((x^*, n^*)\) to be an equilibrium norm to lie by \([\underline{x}, \overline{x}]\), and the post-policy range by \([\underline{x}, \overline{x}]\).

There are three possible outcomes.
• First, if \( \gamma \) is sufficiently small that \( x^* \) still lies in the new interval \([\zeta^1, \bar{\zeta}^1]\) then consumers adhering to the norm will not change their behaviour. So society will suffer from unabated pollution damage.

• Second, if \( \gamma \) is sufficiently large that \( x^* \) no longer lies in the interval \([\zeta^1, \bar{\zeta}^1]\) then, in the first instance, all consumers just revert to consuming at their Marshallian level, taking account of the Pigovian tax \( \gamma \). So consumption by every individual reduces by \( \gamma \), generating a saving per individual in damage costs of \( \gamma^2 \). But each individual adhering to the norm also suffers a loss in the benefit of adhering to the norm given by \( \varphi(n^* - n) \). So for large enough values of \( \varphi \) welfare from implementing what is normally considered to be optimal environmental policy will fall.

• Third, suppose that, after a while, a new norm emerges. We have not specified how this might come about, but it is plausible that the new norm might be established at the new upper limit of equilibrium norms, \( \bar{\zeta}^1 \), as close as possible to the original norm \( x^* \). This would mean that while consumers would now experience again the benefit of adhering to a norm, the level of consumption will be too high. So it may be necessary to increase the pollution tax rate beyond \( \gamma \) to stop consumption of the norm good rising again.

5.3 Empirical Analysis of Consumption Norms

We briefly address the issue we raised in the introduction that much of the analysis of consumption norms assumes that the loss of well-being from deviating from Marshallian demand towards a norm is quadratic in the distance between the Marshallian demand and norm-influenced level of consumption. This means that individuals adjust their consumption away from Marshallian demand towards the norm, but only an individual whose Marshallian demand is the norm will actually consume the norm exactly. This makes it difficult to distinguish the effects of a norm unless one knows the distribution of Marshallian demand. Given that our analysis in the previous section suggests some important implications for environmental policy arising from the existence of consumption norms, it would be problematic if policy makers were not able to know whether consumption was being influenced by norms.

By contrast, using our assumption the welfare loss from adhering to a norm depends on the absolute value of the distance between the Marshallian demand and the demand with a norm,
in Results 1 and 2 in this paper, in the equilibrium a significant number of people consume the norm exactly; indeed in Result 1, *everyone* consumes at the norm level of consumption. Of course our model has a number of simplifications, but it will remain the case more generally that an equilibrium norm will have a range of individuals who exactly consume the norm – i.e. there will be a range of individuals for whom the income elasticity is zero. This suggests that it may be possible for empirical analysis of demand and hence for policy makers to identify the existence of consumption norms.

**6. Conclusions**

In this paper we have presented a model of consumption norms in which, for some goods, there may exist a level of consumption to which individuals wish to conform because they benefit from being identified as belonging to a group of like-minded individuals. Unlike the Veblen notion of conspicuous consumption this desire for conformity will lead some individuals to reduce their consumption from the Marshallian level to the norm. Our modelling of the welfare cost of adhering to the norm also has the important implication that a significant number of people, in some cases the entire population, will consume the norm exactly. This means that in terms of both econometric analysis of demand and policy design, it may be easier to know when consumption norms are influencing demand. The implications for policies, such as the standard Pigovian taxation of goods which cause environmental damage, is that such policies may be ineffective, if the original, pre-policy, norm remains an equilibrium norm, or even counter-productive in terms of lowering welfare if the consumption norm is no longer an equilibrium norm and the loss of benefits from conformity outweigh the gains in reducing pollution damage costs.

Of course, although we emphasise the difference between consumption norms and the notion of conspicuous consumption, it has to be the case that it must be possible for others to observe whether or not one is conforming to a norm, so the actual act of consumption must be conspicuous even if the motivation for adhering to a consumption norm is different from that driving conspicuous consumption.

Our model is clearly extremely simple in at least three aspects.

- Our model of consumer demand uses a simple quadratic utility function (and hence linear demand) for the norm good and linear utility for all other consumption. In earlier work (summarised in Dasgupta, Southerton, Ulph and Ulph (2016)) used more
general utility functions, and showed that the implications for individual behaviour (as summarised in Section 2 of this paper) carry over to that more general treatment of utility functions.

- Our model of the costs and benefits of conforming to a norm also rely heavily of particular functional forms. However, in terms of the costs of adhering to a norm, we have argued that our assumption that costs depend on the absolute value of the difference in consumption has the important implication that a number of individuals will adhere exactly to the norm, which we have argued is perhaps a more plausible way of identifying a consumption norm.

- Our model of a uniform density function of Marshallian demands is also clearly a very special model. Again, in our earlier work (summarised in Dasgupta, Southerton, Ulph and Ulph (2016)) we went to the other extreme of assuming that either all consumers were identical or existed in 2 or 3 groups of identical individuals. Our argument for using the uniform density function is that it does not provide a fairly obvious candidate for a norm, namely that which corresponds to a peak in a density function.

However while we believe there are arguments for using our particular assumptions that go beyond just the benefit of simplifying the analysis. We think it will be important for future work to explore the implications of more general assumptions about these three key features of our model.
Figure 1: Loss of utility from adhering to a norm $(x^*, n^*)$
Figure 2: Consumption choices for different Marshallian demands – Case A
Figure 3: Consumption choices for different Marshallian demands – Case B
Figure 4: Equilibrium norms and associated consumption choices: Result 2 (a)
Figure 5: Equilibrium norm and associated consumption choices: Result 2(b)
References


Ulph, D. (2014), “Keeping up with the Jones: Who loses out?” mimeo, School of Economics & Finance, University of St Andrews

Appendix: Proofs of Results

Result 1: If EITHER (i) $\omega^2 < 2\varphi(1-n) \leq \alpha^2$ OR (ii) $\omega^2 < \alpha^2 < 2\varphi(1-n)$, then for any $x^*$ such that (i) $x^*$ lies in the range $[\xi, \bar{\xi}]$, or (ii) $x^*$ lies in the range $[\underline{v}, \bar{v}]$, $(x^*, 1)$ is an equilibrium norm.

Proof:

(i)

$$x^0(x^*, 1) = x^* - \sqrt{2\varphi(1-n)} \leq X; \quad \hat{x}^0(x^*, 1) = x^* + \sqrt{2\varphi(1-n)} \geq \bar{X}$$

$$\Rightarrow x^0(x^*, 1) = X; \quad \hat{x}^0(x^*, 1) = \bar{X}$$

so $n^* = 1$, $\hat{x}(x^0, x^*) = x^* \quad \forall x^0: X \leq x^0 \leq \bar{X}$, which satisfies the condition

$$\frac{1}{2\omega} \int_{\xi}^{\bar{\xi}} \hat{x}(x^0, x^*) dx^0 = x^*.$$ So $(x^*, 1)$ is a norm.

Now suppose that $\omega < \sqrt{2\varphi(1-n)} < 2\omega$ and that $X \leq x^* < \xi$ (the case $\bar{\xi} < x^* \leq \bar{X}$ is the same by symmetry). Then $x^* + \sqrt{2\varphi(1-n)} < \bar{X}$ so it cannot be the case that $(x^*, 1)$ is an equilibrium norm. We therefore check whether there is a value of $n^*$ ($n < n^* < 1$) such that $(x^*, n^*)$ is an equilibrium norm. So we look for $n^*$ such that all individuals with Marshallian demands $x^0 \in [X, x^* + \sqrt{2\varphi(n^*-n)}]$ adhere to $(x^*, n^*)$. So:

$$n^* = \frac{x^* + \sqrt{2\varphi(n^*-n)}}{2\omega}. \quad \text{(A1)}$$

Clearly $n^* < 1$. We need to check whether $n^* > n$. Solving (A1) for $n^*$ we have:

$$2\omega n^* - x^* = \sqrt{2\varphi(n^*-n)}$$

$$\Leftrightarrow (4\omega^2)n^* - 2x^* + 2\varphi n + (x^* + 2\varphi n) = 0$$

$$\Leftrightarrow n^* = \frac{2x^* + 2\varphi n + \sqrt{(2x^* + 2\varphi n)^2 - 4\omega^2(x^* + 2\varphi n)}}{4\omega^2}$$

$$\Rightarrow n^* = \frac{2x^* + 2\varphi n + \sqrt{4\varphi x^* + 2\varphi^2 - 8\omega^2\varphi n}}{4\omega^2}$$

So:

$$n^* > n \Leftrightarrow \sqrt{4\varphi x^* + 2\varphi^2 - 8\omega^2\varphi n} > 4\omega^2n - 2x^* - \varphi$$

$$\Leftrightarrow 4\varphi x^* + 2\varphi^2 - 8\omega^2\varphi n > 16\omega^2n^2 + 4\omega^2x^* + \varphi^2 - 8\omega^2\varphi n + 4\varphi x^* - 16\omega^2nx^*$$

$$\Leftrightarrow 4\omega^2n^2 - 4\omega n x^* + x^* < 0$$

$$\Leftrightarrow (x^* - 2\omega n)^2 < 0$$

$$30$$
which cannot be true.

(ii)

Since \( \hat{x}(x^0, x^*) = x^* \) \( \forall x^0 \leq x^0 \leq \bar{X} \) it is clear that \( n^* = 1 \) and that the average value of \( \hat{x}(x^0, x^*) = x^* \).

It is also clear that if \( X \leq x^* < \bar{X} - \alpha \) then for \( x^0 \) s.t. \( X \leq x^0 \leq x^* + \alpha \) \( \hat{x}(x^0, x^*) = x^* \) while for \( x^0 \) s.t. \( x^* + \alpha < x^0 \leq \bar{X} \) \( \hat{x}(x^0, x^0) = x^0 - \alpha \), so this will result in an outcome where average value of consumption in the norm consistent interval \( [X, \bar{X}] \) is not equal to the norm \( x^* \). So \( x^* \) cannot lie below \( \psi \). Similar arguments show that \( x^* \) cannot lie above \( \bar{\psi} \). QED

**Result 2.** Suppose \( \frac{\phi(1-n)}{\alpha} + \frac{\alpha}{2} \geq \omega > \alpha \). Then: (a) for any \( x^* \in [\eta, \bar{\eta}] \), \((x^*, n^*)\) is an equilibrium norm where \( n^* = \frac{\phi n - 0.5\alpha^2}{(\phi - \alpha \omega)} \), \( n < n^* < 1 \); (b) there is a second possible equilibrium norm \((\mu, 1)\), but this is unstable.

**Proof:**

(a)

\[
\bar{x}^0(x^*, n^*) = x^* - 0.5\alpha \frac{\phi}{\alpha} (n^* - n) \quad \text{and assume}
\]

\[
\bar{x}^0(x^*, n^*) = x^* + 0.5\alpha \frac{\phi}{\alpha} (n^* - n)
\]

and assume \( X < \bar{x}^0(x^*, n^*) < \bar{x}^0(x^*, n^*) < \bar{X} \) (we will prove below in (b) that this has to be the case if \( x^* \in [\eta, \bar{\eta}] \)).

Then:

\[
n^* = \frac{\bar{x}^0(x^*, n^*) - \bar{x}^0(x^*, n^*)}{2\omega} = \frac{\alpha^2 + 2\phi(n^* - n)}{2\alpha \omega}
\]

(A2)

We first show that \( n^* < 1 \). So

\[
n^* < 1 \iff \phi(1-n) > 0.5\alpha(2\omega - \alpha) \quad \text{(A3)}
\]

which is true by the assumption on parameter values for Result 2. We now show that \( n^* > n \).

From (A3) \( \phi > \frac{0.5\alpha(2\omega - \alpha)}{1 - n} \); furthermore \( \frac{0.5\alpha(2\omega - \alpha)}{1 - n} > \alpha \omega \iff n > \frac{\alpha}{2\omega} \) which is true since \( n \geq 0.5 \). So \( \phi > \alpha \lambda \).
Then:

\[ n^* > n \iff \varphi n - 0.5\alpha^2 > \varphi n - \alpha \omega n \iff n > \frac{\alpha}{2\omega} \quad \text{which is true since } \frac{n > 0.5 > \frac{\alpha}{2\omega}}{2} \quad \text{(A5)} \]

So we have proved \( n < n^* < 1 \).

Finally we show that \( \omega > \frac{\varphi(n^*-n)}{\alpha} + \frac{\alpha}{2} \quad \text{(A6)} \)

Note that \( n^*-n = \frac{\varphi n - 0.5\alpha^2}{(\varphi - \alpha \omega)} - n = \frac{\alpha(\omega n - 0.5\alpha)}{\varphi - \alpha \omega} > 0 \); So:

\[ \omega > \frac{\varphi(n^*-n)}{\alpha} + \frac{\alpha}{2} \iff \omega > \frac{\omega(\varphi n - 0.5\alpha^2)}{\varphi - \alpha \omega} \iff \omega < \frac{\varphi(1-n)}{\alpha} + \frac{\alpha}{2} \quad \text{(A7)} \]

which is true by the assumption on parameter values for Result 2.

So we have established that \( n < n^* < 1 \) and \( \omega > \frac{\varphi(n^*-n)}{\alpha} + \frac{\alpha}{2} \).

To establish that \((x^*, n^*)\) is a norm we need to show that \( x^* \) is the average of \( \hat{x}(x^0, x^*) \) over the range \([x^0(x^*, n^*), \bar{x}^0(x^*, n^*)] \). It is fairly clear that this must be the case by symmetry. But to show this formally, for ease of notation define \( \lambda \equiv \frac{\varphi(n^*-n)}{\alpha} + \frac{\alpha}{2} \) so that:

\( \bar{x}^0(x^*, n^*) = x^*+\lambda; \quad \bar{x}^0(x^*, n^*) = x^* - \lambda; \quad \bar{x}^0(x^*, n^*) - \bar{x}^0(x^*, n^*) = 2\lambda \). Then the average value of \( \hat{x}(x^0, x^*) \) is:

\[ \frac{1}{2\lambda} \left\{ \int_{x^*-\lambda}^{x^*+\alpha} (x^0 + \alpha)dx^0 + \int_{x^*-\alpha}^{x^*} x^* dx^0 + \int_{x^*+\lambda}^{x^*+\alpha} (x^0 - \alpha)dx^0 \right\} \]. Carrying out the integration, the average is:

\[ \frac{1}{2\lambda} \{0.5(x^* - \alpha)^2 + \alpha(x^* - \alpha) - 0.5(x^* - \lambda)^2 - \alpha(x^* - \lambda) + 2\alpha x^* + 0.5(x^* + \lambda)^2 - \alpha(x^* + \lambda) - 0.5(x^* + \alpha)^2 + \alpha(x^* + \alpha)\} = \frac{1}{2\lambda} \{2\lambda x^*\} = x^* \]

So \((x^*, n^*)\) is an equilibrium norm.

Finally we show that \( \underline{X} < \eta < \mu < \bar{X} \)

\( \eta \) is defined by as the smallest value of \( x^* \) such that the lower bound of demand of those just adhering to the norm \( x^* \) is at least \( \underline{X} \). So
\[
\eta - \frac{\alpha - \varphi}{2} (n^* - \bar{n}) = X \Rightarrow \eta = X + \frac{\alpha + \varphi(\omega n - 0.5\alpha)}{\varphi - \alpha \omega}.
\]

Hence by Result (A7): \(X < \eta < X + \omega = \mu < \bar{n} < \bar{X}\), where the last two inequalities follow by symmetry.

(b) a second possible equilibrium norm is \((\mu, 1)\).

From the assumption on parameter values for Result 2

\[
\hat{x}(\mu, 1) = \mu - 0.5\alpha - \frac{\varphi}{\alpha} (1 - n) < X \Leftrightarrow \varphi(1 - n) > \alpha(\omega - 0.5\alpha) \tag{A8}
\]

Similarly \(\hat{x}(\mu, 1) > \bar{X}\). Clearly, \(\forall x^0, \mu - \omega = \hat{x} \leq x^0 \leq \bar{X} = \mu + \omega\) so every consumer abides by the norm \((\mu, 1)\); hence \(n^* = 1\). Average consumption is given by:

\[
\frac{1}{2\omega} \left\{ \int_{\mu - \alpha}^{\mu + \alpha} (x^0 + \alpha)dx^0 + \int_{\mu - \alpha}^{\mu + \alpha} \mu dx^0 + \int_{\mu - \omega}^{\mu + \omega} (x^0 - \alpha)dx^0 \right\}
\]

\[
= \frac{1}{2\omega} \{0.5(\mu - \alpha)^2 + \alpha(\mu - \alpha) - 0.5(\mu - \omega)^2 - \alpha(\mu - \omega) + 2\mu \omega
\]

\[
+0.5(\mu + \omega)^2 - \alpha(\mu + \omega) - 0.5(\mu + \alpha)^2 + \alpha(\mu + \alpha)\} \tag{A9}
\]

\[
= \frac{2\mu \omega}{2\omega} = \mu
\]

So average consumption equals the norm. So \((\lambda, 1)\) is an equilibrium norm.

Could \((x^*, 1)\) be an equilibrium norm with \(x^* \neq \mu\). Suppose now that \(x^*\) shifts slightly away from \(\mu\) (the same argument applies for more extreme changes in norm, as we shall argue).

Then everyone adheres to the norm, and average consumption is now

\[
\frac{1}{2\omega} \left\{ \int_{\mu - \alpha}^{x^* - \alpha} (x^0 + \alpha)dx^0 + \int_{x^* - \alpha}^{x^* + \alpha} x^* dx^0 + \int_{x^* + \alpha}^{\mu + \alpha} (x^0 - \alpha)dx^0 \right\}
\]

\[
= \frac{1}{2\omega} \{0.5(x^* - \alpha)^2 + \alpha(x^* - \alpha) - 0.5(\mu - \omega)^2 - \alpha(\mu - \omega) + 2\alpha x^* \}
\]

\[
+0.5(\mu + \omega)^2 - \alpha(\mu + \omega) - 0.5(x^* + \alpha)^2 + \alpha(x^* + \alpha)\}
\]

\[
= \frac{\mu \omega + \alpha(x^* - \mu)}{\omega} \neq x^*
\]

So \((x^*, 1)\) cannot be a stable norm. This argument is even stronger if a possible norm is sufficiently different from \(\mu\) that one of groups who do not adhere exactly to the norm no longer lies in the range \([X, \bar{X}]\). So \((\mu, 1)\) is a possible equilibrium norm. But any change in parameter values which led to a different value for \(\mu\), call it \(\tilde{\mu}\), would mean the original norm \((\mu, 1)\) is no longer an equilibrium norm.

QED
Result 3. If EITHER (i) $2\varphi(1-n) \leq \alpha^2$ and $\omega^2 > 2\varphi(1-n)$ OR (ii) $2\varphi(1-n) \geq \alpha^2$ and $\omega > \frac{\varphi(1-n)}{\alpha} + \frac{\alpha}{2} > \alpha$ then no norm exists.

Proof:

(i) Suppose $x^* = \mu$. Then $2\varphi(1-n) \leq \omega^2$ implies that

$$X \leq \bar{x}^0(\mu, n^*) = \mu - \sqrt{2\varphi(n^*-n)} < \bar{x}^0(\mu, n^*) = \mu + \sqrt{2\varphi(n^*-n)} \leq \bar{X}$$

So $n^* = \frac{\bar{x}^0(\mu, n^*) - \bar{x}^0(\mu, n^*)}{2\omega} = \frac{\sqrt{2\varphi(n^*-n)}}{\omega}$ (A10)

Solving (A10) for $n^*$ yields

$$n^* - \frac{2\varphi}{\omega^2} n^* + \frac{2\varphi}{\omega^2} n = n^* - \psi n^* + \psi n = 0 \quad (A11)$$

where $\psi = \frac{2\varphi}{\omega^2}$. Hence, solving (A11) yields:

$$n^* = \frac{\psi \pm \sqrt{\psi^2 - 4\psi \cdot n}}{2}.$$ Since we want to show that $n^* < n$ we take the positive root: $n^* = \frac{\psi + \sqrt{\psi^2 - 4\psi \cdot n}}{2}$. We check first that $n^* = \frac{\psi + \sqrt{\psi^2 - 4\psi \cdot n}}{2} \leq 1$.

$$\frac{\psi + \sqrt{\psi^2 - 4\psi \cdot n}}{2} \leq 1 \Leftrightarrow \sqrt{\psi^2 - 4\psi \cdot n} \leq 2 - \psi \Leftrightarrow \psi(1-n) \leq 1 \Leftrightarrow 2\varphi(1-n) \leq \omega^2 \quad (A12)$$

which follows from the assumption about parameters for Result 3. We now show that it cannot be the case that $n^* \geq n$. So:

$$n^* \geq n \Leftrightarrow \sqrt{\psi^2 - 4\psi \cdot n} \geq 2n - \psi \Leftrightarrow \psi^2 - 4\psi \cdot n \geq \psi^2 - 4\psi \cdot n + 4n^2 \quad (A13)$$

which is not the case.

So $x^* = \mu$ cannot be a norm.

Since (A11) onwards does not depend on the norm being $x^* = \mu$, any other norm which preserves the inequalities in (A10) cannot be a norm either. If the norm is such that either $\bar{x}^0(x^*, n^*) \leq X$ or $\bar{x}^0(x^*, n^*) \geq \bar{X}$ then this will just shrink the norm consistent interval around the norm, making $n^*$ even smaller, and hence reinforcing the outcome that $n^* < n$. So no single norm can exist with these parameter values.
(ii) We consider first whether \( x^* = \mu \) could be a norm with associated \( n^* : 0.5 < n < n^* \leq 1 \).

Since \( \frac{\varphi(n^* - n)}{\alpha} + \frac{\alpha}{2} \leq \frac{\varphi(1 - n)}{\alpha} + \frac{\alpha}{2} < \omega \) by the assumption for this result, we must have:

\[
X = \mu - \omega < x^0(\mu, n^*) = \mu - \frac{\varphi(n^* - n)}{\alpha} - \frac{\alpha}{2} < x^0(\mu, n^*) = \mu + \frac{\varphi(n^* - n)}{\alpha} + \frac{\alpha}{2} < \mu + \omega = \overline{X}
\]

So:

\[
n^* = \frac{x^0(\mu, n^*) - x^0(\mu, n^*)}{2\omega} = \frac{\varphi(n^* - n) + 0.5\alpha^2}{\alpha\omega}
\]

\[
\Rightarrow n^* = \frac{\varphi n - 0.5\alpha^2}{\varphi - \alpha\omega} \quad (A14)
\]

It must be the case that the numerator of (A14) is positive because \( \varphi n \geq \varphi(1 - n) > 0.5\alpha^2 \) by the assumption that \( n \geq 0.5 \) and the assumption for Result 3 (ii), and hence so is the denominator. But then

\[
n^* \leq 1 \Leftrightarrow \varphi n - 0.5\alpha^2 \leq \varphi - \alpha\omega \Leftrightarrow \frac{\varphi(1 - n)}{\alpha} + \frac{\alpha}{2} \geq \omega \quad (A15)
\]

which contradicts the assumption for Result 3 (ii). So \((\mu, n^*)\) cannot be a norm for any \(0 < n^* \leq 1\). The same argument would apply \textit{a fortiori} to any other value of \( x^* \). QED.

**Result 4:** In Results 1 and 2(i) there is a range of possible values for an equilibrium norm \( x^* \) (with an associated value of \( n^* \) which is either 1 or a constant independent of \( x^* \)). In Result 1 the value of \( x^* \) which minimises the expected value of the utility loss \( E_\phi(L(x^0, x^*)) \) is \( x^* = \mu \).

In Result 2(i) there is no optimal value of \( x^* \) - the value of \( E_\phi[L(x^0, x^*)] \) is the same for all possible equilibrium values of \( x^* \).

**Proof:**

**A. Result 1.**

The equilibrium norm \( x^* \) lies in a range of possible values(\([\xi, \overline{\xi}], [\psi, \overline{\psi}]\) respectively) with \( n^* = 1 \), and everyone adheres exactly to the norm \( x^* \). So from (13):
\[
E[L(x^*, x^0)] = \frac{0.5}{(\bar{X} - \underline{X})_x^*} \int (x^0 - x^*)^2 \, dx_0 = \frac{0.5}{(\bar{X} - \underline{X})_x^*} \left[ \frac{x^0 - x^*}{3} + x^* x^0 + x^* x^0 \right]_{x^*}^{x^0}
\]

\[
= \frac{(\bar{X} - \underline{X})}{6(\bar{X} - \underline{X})} - \frac{x^*(\bar{X} - \underline{X})}{2(\bar{X} - \underline{X})} + \frac{x^*}{2}
\]

Hence:

\[
\frac{\partial E[L(x^*, x^0)]}{\partial x^*} = -\frac{\bar{X} + x^*}{2} + x^* = 0 \Rightarrow x^* = \frac{\bar{X} + x^*}{2} = \mu
\]

B. Result 2(i)

The equilibrium norm \( x^* \) lies in a range of possible values \([\underline{n}, \bar{n}]\) with associated

\[
n^* = \frac{\phi n - 0.5\alpha^2}{\phi - \alpha \delta}.\]

Define \( \zeta = \frac{2}{\phi} + \frac{\phi(n^* - \bar{n})}{\alpha} > \alpha \); then for those adhering to the norm \((x^*, n^*)\)

the chosen levels of demand associated welfare losses are:

\[
x^* - \zeta < x^0 < x^* - \alpha \quad \hat{x} = x^0 + \alpha \quad L(x^*, x^0) = -0.5\alpha^2 + \alpha(x^* - x^0)
\]

\[
x^* - \alpha < x^0 < x^* + \alpha \quad \hat{x} = x^* \quad L(x^*, x^0) = 0.5(x^0 - x^*)^2
\]

\[
x^* + \alpha < x^0 \quad \hat{x} = x^0 - \alpha \quad L(x^*, x^0) = -0.5\alpha^2 + \alpha(x^0 - x^*)
\]

Hence:

\[
\Omega = (\bar{X} - \underline{X}) E[L(x^*, x^0)] = \int_{x^* - \zeta}^{x^* + \alpha} [-0.5\alpha^2 + \alpha(x^* - x^0)] \, dx^0 + \int_{x^* - \alpha}^{x^* + \zeta} [0.5(x^0 - x^*)^2] \, dx^0
\]

\[
= \left[ -0.5\alpha^2 + \alpha x^* \right]_{x^*-\zeta}^{x^*+\alpha} - 0.5\alpha \left[ x^0 \right]_{x^*+\zeta}^{x^*+\alpha}
\]

\[
+ 0.5 \left[ \frac{x^0}{3} - x^* x^0 + x^* x^0 \right]_{x^*-\alpha}^{x^*+\alpha} + [-0.5\alpha^2 - \alpha x^*]_{x^*+\alpha}^{x^*+\zeta} + 0.5 \alpha \left[ x^0 \right]_{x^*+\alpha}^{x^*+\zeta}
\]

\[
\Omega = [-0.5\alpha^2 + \alpha x^*](\zeta - \alpha) - 0.5\alpha[(x^* - \alpha)^2 - (x^* - \zeta)^2]
\]

\[
+ 0.5 \left[ \frac{(x^* + \alpha)^3}{3} - x^*(x^* + \alpha)^2 + x^2 (x^* + \alpha) - \frac{(x^* - \alpha)^3}{3} + x^*(x^* - \alpha)^2 - x^2 (x^* - \alpha) \right]
\]

\[
+ [-0.5\alpha^2 - \alpha x^*](\zeta - \alpha) + 0.5 \alpha [(x^* + \zeta)^2 - (x^* + \alpha)^2]
\]

Hence:
\[ \frac{\partial \Omega}{\partial x^*} = \alpha(\zeta - \alpha) - \alpha[(x^* - \alpha) - (x^* - \zeta)] + 0.5[(x^* + \alpha)^2 - (x^* + \alpha)^2 - 2x^*(x^* + \alpha) \\
+ 2x^*(x^* + \alpha) + x^2 - (x^* - \alpha) + (x^* - \alpha)^2 + (x^* - \alpha)^2 + 2x^*(x^* - \alpha) - 2x^*(x^* - \alpha) - x^2] \\
- \alpha(\zeta - \alpha) + \alpha[(x^* + \zeta) - (x^* + \zeta)] \\
= 0. \]

QED