Fiscal Policy in a Liquidity-Constrained New Keynesian Economy*

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Abstract

We study the effects of fiscal policy on the macroeconomy using a New Keynesian model in which private financial assets are regarded as only partially liquid and government bonds as liquid. We find that the fiscal multipliers in this economic environment are large enough for fiscal policy to be highly effective. Fiscal policy works by improving liquidity in the economy.

Keywords: DSGE Models, Monetary Policy, Fiscal Policy, Liquidity Trap, Credit Constraints

JEL: E32, E52, E58, E62

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1 Introduction

Over the last decade, in many if not all developed countries (including the Euro-Area, the US and the UK), monetary policy has been the main instrument for managing the level and rate of growth of aggregate demand and inflationary pressure. The chief monetary policy tool has been short-term interest rates. The response to the recent financial crisis has typically been to lower the short-term nominal interest rate to its zero lower bound (i.e. generating a liquidity trap). Monetary policy loses its power at the zero lower bound, so the conventional policy option of reducing interest rates is no longer available. The ineffectiveness of monetary policy at the zero lower bound raises the question of whether fiscal policy can be used to mitigate the effects of the financial crisis.

Answering this question requires a model that can capture, at the very least, the key aspects of the crisis. As many observers have noted, the realisation at the onset of the crisis that many private financial assets were of a lower quality and therefore accompanied by higher default risks than previously assumed led to a flight to liquid assets. The asset markets experienced a severe shortage of liquid assets. At the height of the crisis, the financial markets for private assets essentially froze. The drop in the resaleability of private assets diminished firms’ ability to raise funds for investment and use their assets as collateral in borrowing. The consequent decrease in the funds available for investment led to substantial drops in investment, output and inflation. To combat the recession, central banks lowered nominal interest rates to their zero lower bound and implemented quantitative easing by purchasing private assets in the open market.

The aim of this paper is to study the effectiveness of fiscal policy using a model that is capable of capturing the scenario described above. To this end, we use the model proposed by Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) (henceforth “DEFK”). This model reformulates the state-of-the-art version of New Keynesian economics, as in Christiano, Eichenbaum
and Evans (2005) ("CEE") and Smets and Wouters (2007) ("SW"), by incor-
porating liquidity frictions as described by Kiyotaki and Moore (2008).

In the DEFK model, the economy is populated with a large number of
identical households. Each household has two types of members (entrepre-
neurs and workers) and two types of financial assets (government bonds and
private equity). The two types of financial assets differ in their degree of
liquidity. Government bonds are liquid, whereas private assets are not. The
sole role of entrepreneurs is to invest in new capital, which they rent out
to firms, while workers consume and work but do not undertake investment
projects. At the beginning of each period, all members of a household are
identical and share the household’s assets. During each period, a randomly
chosen fraction of household members becomes entrepreneurs. When they
invest, entrepreneurs face two types of financial frictions: resaleability con-
straints on their asset holdings and borrowing constraints. Entrepreneurs
can borrow by issuing new equity. However, the amount of equity that they
can issue in a given period is limited. In other words, they face borrowing
constraints. Compared with government bonds, private equity is illiquid in
the sense that households can sell only up to a certain proportion of their
equity holdings in a given period. Entrepreneurs thus also face resaleability
constraints. Another crucial assumption of the model is that the expected
return on new capital investment is always higher than that on government
bonds. The rest of the model’s assumptions are standard New Keynesian.
Firms and workers enjoy some degree of monopoly power; prices and wages
remain unchanged, on average, for several months. The central bank sets
monetary policy using a Taylor style rule and implements quantitative eas-
ing. Comparison of the empirical data with the model’s simulations shows
that the DEFK model performs well in explaining the responses of the key
macroeconomic variables to the recent credit crisis. We therefore believe that

\footnote{As noted by DEFK, private equity has a broad definition in this model. It can be interpreted as privately issued paper such as commercial paper, bank loans, mortgages, and so on.}
the model captures the essence of the recent crisis as well as central banks’ responses to it.\footnote{DEFK use their model to examine the effectiveness of quantitative easing and find this to be an effective policy. Ajello (2010), Driffield and Miller (2011) and Shi (2012) also use the KM framework to study the current financial crisis.} The presence of the resaleability constraints allows us to capture an important aspect of the recent financial crisis that many private financial assets become illiquid.

We introduce a role for government spending in the DEFK model and use it to determine the value of the fiscal multiplier. Since we examine crisis situations in which the economy may spend a long time away from the deterministic steady state, we numerically simulate the original non-linear model without log-linearisation. Consequently, the accuracy of our results does not depend on the economy’s vicinity to the steady state. In our experiments, we consider bond-financed fiscal expansion, in which the government issues bonds to households to be repaid by tax increases at a later date.

We carry out our analysis in two steps. First, we examine the value of the fiscal multiplier using the version of the DEFK model at normal times (i.e., without liquidity shocks) when the zero lower bound on the nominal interest rate does not bind. We find that the value of the fiscal multiplier is much greater than that suggested by the standard DSGE model without credit frictions. The cumulative multiplier obtained using the DEFK model is 1.6, compared with 0.55 in the standard model. We show that the multiplier effect on investment is the main reason for the larger multiplier. In response to a government spending shock, investment increases in a persistent hump-shaped manner, with a peak at about 10 quarters after the shock. The intuition for this result is as follows. In both the standard model and the DEFK model, an increase in government spending leads to a higher future tax burden and a rise in the real interest rate. Both of these factors cause households to postpone their consumption and increase their savings by holding liquid government bonds. In the standard model, investment de-
increases since the higher real interest rate increases the opportunity cost of investing in physical capital, thus crowding out investment. Unlike the standard model, however, the DEFK model shows that investment increases in response to a bond-financed government spending shock since fiscal policy improves liquidity in the economy by increasing the quantity of liquid government bonds, which in turn allows credit-starved entrepreneurs to make larger investments.

Second, we look at the fiscal multiplier in a credit crisis caused by the tightening of resaleability constraints, in which case the zero lower bound on the nominal interest rate becomes binding. We find that, in both the DEFK and the standard models, the multiplier is much larger in a liquidity trap than at normal times. Moreover, we find that in the liquidity-trap case, the multiplier in the DEFK model is still larger than that in the standard model. Under this scenario, the cumulative multiplier suggested by the DEFK model is around 2. The fiscal multiplier is larger in a liquidity trap because an increase in government spending creates inflationary pressures which reduce the real interest rate and stimulate consumption and output. In the DEFK model, fiscal expansion generates an even larger multiplier in a liquidity trap because the multiplier effect applies to both consumption and investment.

Before describing the model, we briefly review the literature on this topic. Most of the theoretical discussions of this issue have been based on

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3 Erceg and Linde (2012) criticise the assumption of an exogenous zero-bound condition in the study of the fiscal multiplier. They point out that as an increase in government expenditure may help push the economy out of a liquidity trap, the multiplier will be smaller if the zero-bound condition is endogenous. Mertens and Ravn (2010) warn that the value of the multiplier is sensitive to the type of shock that drives the economy into a liquidity trap. To address these issues, we examine the fiscal multipliers using a model in which the liquidity trap is endogenously caused by a financial crisis.

4 The majority of empirical research in this area seems to suggest that fiscal policy is not effective and that an increase in government spending does not have a significant effect on the economy (see, for example, Hall (2009), Ramey (2011) and references therein). The government spending multiplier is typically estimated to lie between 0.6 and 1.2. However, some recent empirical studies show that the fiscal multiplier is much larger during a recession (see, for example, Auerbach and Gorodnichenko (2012)).
the CEE/SW model (see, for example, Christiano, Eichenbaum and Rebelo (2011), Cogan et al. (2010) and Woodford (2011)). The CEE/SW model assumes frictionless financial markets and therefore cannot provide a detailed account of the crisis. Cogan et al. (2010) find that the fiscal multiplier is less than one and that this is true even when the economy stays at the zero lower bound for a prolonged period of time. Christiano et al. (2011) and Woodford (2011) show that the conclusion reached by Cogan et al. (2010) - in brief, that fiscal policy is ineffective in a liquidity trap - changes if the fiscal expansion lasts exactly as long as the zero-bound state. They find a fiscal multiplier considerably in excess of one. As we show later in this paper, the fiscal multiplier in the DEFK model is much larger than that suggested by a standard model similar to the one employed in Christiano et al. (2011), both in times of crisis and at normal times. The main reason for this result is that in the standard model, the increase in output in response to a government spending shock is almost entirely driven by consumption and the multiplier effect on investment is negligible.

Our paper belongs to the recent literature that examines the effects of fiscal policy in the presence of financial frictions. Important papers in this literature include Carrillo and Poilly (2013), Eggertsson and Krugman (2012) and Fernandez-Villaverde (2010). Eggertsson and Krugman (2012) use a model in which some agents’ ability to borrow diminishes following a shock similar to a “Minsky moment”. They find that, under such a shock, the government spending multiplier can be much greater than one, though its exact value depends heavily on the share of debt-constrained borrowers in the economy. Carrillo and Poilly (2013) and Fernandez-Villaverde (2010), on the other hand, use models that accommodate the type of credit frictions suggested by Bernanke, Gertler and Gilchrist (1999) (“BGG”), in which firms’ ability to borrow is determined by the market value of their net worth. Fernandez-Villaverde (2010) considers the government spending multiplier at normal times, whereas Carrillo and Poilly (2013) focus on the case of a liquidity
trap. Fernandez-Villaverde (2010) finds that the value of the impact multiplier is around one following the government spending shock and decreases quickly thereafter. His multiplier is larger than that suggested by standard models but smaller than ours.  

Carrillo and Poilly (2013) find that financial frictions have a greater contribution to the value of the multiplier in a liquidity trap than at normal times. Indeed, their cumulative multiplier in the liquidity-trap case is 3.7, which is almost twice as large as ours. Our paper differs from previous studies in the way that financial frictions are introduced. While the BGG model focuses on borrowing constraints, the DEFK model incorporates the credit frictions proposed by Kiyotaki and Moore (2008). The Kiyotaki and Moore (2008) model accounts for both borrowing constraints and resaleability constraints. It is worth to note that to generate a liquidity trap, Carrillo and Poilly (2013) assume that the capital returns perceived by entrepreneurs are affected by a risk premium shock, as in Smets and Wouters (2007). Since the empirical relevance of this kind of shock is uncertain (see Chari, Kehoe and McGrattan (2008) for a detailed discussion), our approach offers an alternative way to generate a liquidity trap in a financial crisis. An additional benefit of applying the DEFK model is that it allows us to investigate the effects of fiscal policy in crisis situations where the central bank also carries out unconventional monetary policy (i.e., quantitative easing). All in all, despite the difference in our approach, our findings are consistent with those of the above-mentioned papers, further strengthening their conclusions.

The remainder of this paper is structured as follows: Section 2 describes

\[5\] As shown later in our results, although our post-shock impact multiplier at normal times is smaller than 1, it increases gradually over time. As a result, the cumulative multiplier we obtain (1.6) is substantially larger than 1.

\[6\] See Table 1 in the online appendix that can be found as supplementary material at http://dx.doi.org/10.1016/j.red.2013.01.004.

\[7\] Although the DEFK model focuses mainly on resaleability constraints, borrowing constraints also play a significant role in generating large multipliers in this model. If there are no borrowing constraints, as discussed in Kiyotaki and Moore (2008), new investment could be wholly financed by issuing new equity. As a result, shocks to resaleability constraints would be negligible.
the credit-constraint features of the DEFK model. Section 3 focuses on the
calibration of parameters. Section 4 compares the fiscal multipliers produced
by the DEFK model to those obtained using a standard DSGE model in
various scenarios. In Section 4, we also discuss the results of the sensitivity
analysis of several parameters. Section 5 concludes our findings.

2 The Model with Credit Frictions

The model that we use in our analysis is proposed by DEFK (2011), which
incorporates a specific form of credit frictions as discussed in Kiyotaki and
Moore (2008). As mentioned earlier, households in this model are credit-
constrained and face stochastic shocks that further tighten their liquidity.
When a credit shock arrives, the central bank implements quantitative easing
to increase households’ liquidity through the purchase of private equity in
the open market. Government expenditure is absent in DEFK (2011). We
introduce the role of exogenous government expenditure to the model for
the study of the fiscal multiplier. Other aspects of the model are standard
New Keynesian (see, for example, CEE and SW). The special features of the
model are discussed below.

2.1 Households

The economy consists of a continuum of identical households. Each household
consists of a continuum of members \( j \in [0, 1] \). In each period, members have
an i.i.d. opportunity \( \zeta \) to invest in capital. Household members \( (j \in [0, \zeta]) \)
who receive the opportunity to invest are “entrepreneurs”, whereas those
who do not \( (j \in [\zeta, 1]) \) are “workers”. Entrepreneurs invest and do not work.
Workers work to earn labour income. Each household’s assets are divided
equally among its own members at the beginning of each period. After mem-
bers find out whether they are entrepreneurs or workers, households cannot
reallocate their assets. If any household member needs extra funds, they
need to obtain them from external sources. This assumption is important as it gives rise to liquidity constraints. At the end of each period, household members return all their assets plus any income they earn during the period to the asset pool.\(^8\)

The representative household’s utility depends on the aggregate consumption \(C_t \equiv \int_0^1 C_t (j) \, dj\) as consumption goods are jointly utilised by its members. Each member seeks to maximise the utility of the household as a whole, which is given by:

\[
E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu} \int_0^1 H_s (j)^{1+\nu} \, dj \right], \tag{1}
\]

where \(\beta\) is the discount factor, \(\sigma\) is the coefficient of relative risk aversion, and \(\nu\) is the inverse Frisch elasticity of labour supply. Labour supply \(H_t (j) = 0\) for entrepreneurs. Each period, household members choose optimally among non-durable consumption, saving in bonds or equity and, if they are entrepreneurs, investment in capital. Details of their saving and investment options are as follows: (i) **Investment in new capital.** Entrepreneurs have the opportunity to invest in new capital \((I_t)\) which costs \(p_t^I\) per unit. Each unit of capital goods generates a rental income of \(r_t^k\), depreciates at a rate of \(\delta\) and has a market value of \(q_t\). The return on new capital is therefore \(\frac{r_t^k + (1-\delta)q_{t+1}}{p_t}\). Entrepreneurs can borrow to invest. Borrowing is in the form of issuing equity, \(N_t^I\), that entitles the holder to claim the future returns on the underlying capital goods. (ii) **Saving in government bonds.** Household members can save in risk-free government bonds, \(L_t\), which have a unit face value and pay a gross nominal interest rate, \(R_t\), over the period \(t\) to \(t + 1\). (iii) **Saving in private equity.** Household members can also purchase the eq-

\(^8\)The assumption that entrepreneurs and workers belong to the same household is based on Shi (2011). This is different from the setting in KM (2008), in which entrepreneurs and workers are two separate entities. As noted by DEFK (2012), adopting this assumption increases the flexibility of the model to incorporate various modifications for sensitivity analysis.
uity issued by other households, \( N_t^O \), at the market price of \( q_t \). As equity holders receive income from the underlying capital goods, the return on equity over \( t \) to \( t+1 \) is \( \frac{r_{t+1}^{K} + (1-\delta)q_{t+1}}{q_t} \). The household’s net equity is defined as its equity holdings plus its capital stocks minus any equity issued by it: \( N_t = N_t^O + K_t - N_t^I \).

At the beginning of each period, the household also receive dividends from intermediate-goods and capital-goods firms amounting to \( D_t \) and \( D_t^K \) respectively. The household pay lump-sum taxes, \( \tau_t \), to the government. Taxes are lump-sum so that they are non-distortive. The intertemporal budget constraint is:\(^{10}\)

\[
C_t + p_t^I I_t + q_t [N_t - I_t] + L_t = \left[ r^K_t + (1-\delta) q_t \right] N_{t-1}^0 + \frac{R_{t-1} L_{t-1}}{\pi_t} + \int_{\pi}^1 W_t(j) H_t(j) \, dj + D_t + D_t^K - \tau_t
\]

where \( \pi_t \equiv \frac{P_t}{P_{t-1}} \) is the gross inflation rate at \( t \) and \( W_t(j) \) is the nominal wage earned by type-\( j \) workers. Entrepreneurs and workers face different problems as explained below.

### 2.1.1 Entrepreneurs

In the steady state and the post-shock equilibria, the market price of equity \( q_t \) is always greater than the investment cost of new capital \( p^I_t \). Hence, the return on new capital \( \left( \frac{r^K_{t+1} + (1-\delta)q_{t+1}}{p^I_t} \right) \) is strictly greater than the return on equity \( \left( \frac{r^K_{t+1} + (1-\delta)q_{t+1}}{q_t} \right) \) which is the same as the real return on government bonds due to the anti-arbitrage condition. Entrepreneurs are rational, so they would invest all their available resources in new capital. To spare more

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\(^{9}\)The implicit assumption is that holding the equity issued by other households has the same risk level as holding the capital goods directly.  
\(^{10}\)In this paper, stock variables at \( t \) show the amounts of stocks at the end of the period. This is different from the timing convention of stock variables in DEFK (2011). In their paper, stock variables at \( t \) are defined as the amounts at the beginning of the period.
funds for investment, entrepreneurs do not spend on consumption goods, i.e., $C_t(j) = 0$ for $j \in [0, \infty)$. They would also sell all their bond holdings so that $L_t(j) = 0$ for $j \in [0, \infty)$. There are, however, constraints if entrepreneurs want to obtain funds through equity: (i) Borrowing constraint. Entrepreneurs can borrow by issuing equity of only up to $\theta \in (0, 1)$ fraction of their new investment. (ii) Resaleability constraint. In each period, entrepreneurs can sell only up to $\phi_t \in (0, 1)$ fraction of their net equity holdings. Liquidity shocks, as explained later, are modelled as sudden drops in $\phi_t$. Since borrowing and resaleability constraints are both binding, entrepreneurs’ net equity evolves according to $N_t(j) = (1 - \phi_t) (1 - \delta) N_{t-1}(j) + (1 - \theta) I_t(j)$. Combining entrepreneurs’ first order conditions for $C_t(j), L_t(j)$ and $N_t(j)$ with the intertemporal budget constraint (2) gives the aggregate investment:

$$I_t = \int_0^\infty I_t(j) \, dj = \frac{\left[ r_k^t + (1-\delta) q_t \phi_t \right] N_{t-1} + \frac{R_{t-1} L_{t-1}}{\pi_t} + D_t + D^K_t - \tau_t}{p_t^t - \theta q_t}$$

(3)

Investment expenditure depends on the abundance of the household’s liquidity. By contrast, in a standard DSGE model without credit frictions, investment opportunity is not scarce. Investment in new capital simply provide the same rate of return as other forms of assets. In such models, investment expenditure is unaffected by credit conditions.

### 2.1.2 Workers

After solving for entrepreneurs’ problem, the workers’ consumption and saving decisions can be derived by considering the household as a whole. Workers choose $C_t, L_t$ and $N_t$ to maximise the household’s utility (1) subject to the intertemporal budget constraint (2) and the investment decision of entrepreneurs (3). The first-order conditions give the respective Euler equations for
bonds and equity:

\[
C_t^\sigma = \beta E_t \left\{ C_t^{\sigma} \left[ \frac{R_t}{\pi_t+1} + \frac{\pi (q_{t+1} - p_{t+1})}{\pi_t+1} \right] \right\}
\]

\[
C_t^\sigma = \beta E_t \left\{ C_t^{\sigma} \left[ \frac{rk_{t+1} + (1-\delta)q_{t+1}}{q_t} + \frac{\pi (q_{t+1} - p_{t+1})}{p_{t+1}^{l+1}} \frac{rk_{t+1} + (1-\delta)q_{t+1}^{l+1}}{q_t} \right] \right\}
\]

These Euler equations reduce to the standard ones when \( \pi = 0 \). In the DEFK model, there is a premium on top of the standard returns on bonds and equity because households are credit-constrained. By choosing to buy one extra unit of government bonds at \( t \) instead of consumption, the bond-holder gains \( \frac{R_t}{\pi_t+1} \) extra units of liquidity at \( t+1 \). Similarly, by choosing to purchase one extra unit of equity at \( t \) instead of spending, the equity-holder receives \( \frac{rk_{t+1} + (1-\delta)q_{t+1}}{q_t} \) extra units of liquidity at \( t+1 \). The extra liquidity allows them to profit from the investment opportunity if it arrives at \( t+1 \).

The wage- and price-setting assumptions in this model are standard New Keynesian. Workers supply differentiated labour to the production sector through the arrangement of employment agencies, who bundle differentiated labour supply into homogeneous units for firms to hire. Wages are negotiated by labour unions representing each specific type of workers. Labour unions enjoy some degree of monopoly power which allows them to set wages in a staggered basis. Intermediate-goods firms choose the optimal amounts of labour and capital inputs that maximise their expected profits, taking wages and capital rent as given. They set prices for their differentiated products according to the Calvo-pricing scheme, and sell them to final-goods firms for the production of homogeneous final goods. Capital-goods producers convert final goods into physical capital, incurring an adjustment cost. Details of these standard features are included in the Appendix, together with some equilibrium equations.
2.2 Government Policies

In this model, the government carries out quantitative easing in the event of a credit crisis. A credit crisis occurs due to a sudden worsening of equity’s resaleability, expressed by a 60% drop in the resaleability parameter \( \phi_t \) from its steady-state value, \( \phi \). Evolution of \( \phi_t \) thus follows \( \dot{\phi}_t = \epsilon_t^\phi < 0 \), where \( \dot{\phi}_t \equiv \frac{\phi_t - \phi}{\phi} \). Unlike DEFK who assume that \( \dot{\phi}_t \) follows a two-state Markov process, we assume that \( \dot{\phi}_t \) stays below zero following a credit shock for a deterministic number of periods depending on the public’s expected duration of the crisis.\(^{11}\) During a credit crisis, the government buys equities \( N_t^g \) from households mainly by selling bonds. Unlike private equity, government bonds are not subject to resaleability constraint, households’ liquidity improves as a result of the quantitative easing. The size of the open market intervention is proportional to the magnitude of the credit shock:

\[
N_t^g = \frac{\psi^g k_t}{K} \left( \frac{\phi_t}{\phi} - 1 \right),
\]

where \( \psi^g k < 0 \) is the policy parameter.

We introduce exogenous government spending to the model to study the value of the fiscal multiplier. A government spending shock is measured as a percentage of GDP so that \( \ddot{G}_t \equiv \frac{G_t - G}{Y} \), where \( G \) and \( Y \) are the respective steady-state values of government spending and output. Evolution of \( \ddot{G}_t \) follows \( \ddot{G}_t = \rho_G \ddot{G}_{t-1} + \epsilon_t^G \), where \( \rho_G \) governs the persistence of government spending. The government’s budget constraint is:

\[
G_t + q_t N_t^g + \frac{R_{t-1} L_{t-1}}{\pi_t} = \tau_t + \left[ r_t^k + (1 - \delta) q_t \right] N_{t-1}^g + L_t
\]

In addition, taxes are proportional to the government’s net liability at the

\(^{11}\text{Carlstrom, Fuerst and Paustian (2012) find that an interest rate peg with stochastic exit tends to exaggerate the value of the fiscal multiplier. In this model, although the duration of a zero-bound interest rate is endogenous, it depends heavily on the expected duration of the credit crisis that causes the zero-bound condition. Therefore, we assume that a credit crisis lasts for a deterministic number of periods.}
beginning of the period:

\[ \tau_t - \tau = \psi_\tau \left[ \left( \frac{R_{t-1}L_{t-1}}{\pi_t} - \frac{RL}{\pi} \right) - q_t N_{t-1}^g \right], \quad (8) \]

where \( \psi_\tau > 0 \), \( \tau \) and \( \frac{RL}{\pi} \) are the respective steady-state values of taxes and government debt; \( N_{t-1}^g \) is zero at steady state by assumption. The value of \( \psi_\tau \) is low to reflect that the adjustment on taxes is slow compared to bond issue, so the government has to obtain funds for fiscal expansion or quantitative easing mainly by issuing bonds.

We assume that the central bank adopts a generalised Taylor rule similar to the one in SW, which targets both inflation and output. The nominal interest rate also follows a short-run feedback from the change in output:

\[ R_t = \max \left\{ R_{t-1}^\rho R \left( \frac{Y_t}{Y} \right)^{\psi_Y} \left( \frac{Y_t}{Y_{t-1}} \right)^{\psi_{\Delta Y}}, 1 \right\} \quad (9) \]

where \( R \) is the steady-state gross nominal interest rate, \( \rho_R \) is the interest rate smoothing parameter, \( \psi_\pi > 1 \), and \( \psi_Y \) and \( \psi_{\Delta Y} \) are both between zero and one. The zero lower bound on the nominal interest rate requires that \( R_t \) cannot be lower than 1. The gross real interest rate is obtained by \( r_t = \frac{R_t}{E_t(\pi_{t+1})} \).

### 2.3 Equilibrium and Solution Strategy

The resource constraint of the economy requires that:

\[ Y_t = C_t + \left[ 1 + S\left( \frac{I}{T} \right) \right] I_t + G_t \quad (10) \]

Other aggregate equilibrium equations are included in the Appendix. In DEFK, equations are approximated around a steady state by log-linearisation. Since in our simulation experiments the competitive equilibria achieved following a credit shock can stay far away from the steady state for a long
time, applying log-linearisation may lead to misleading results. For this reason, we instead carry out deterministic simulations with the exact, nonlinear equations using Dynare. Under the deterministic setting, the model assumes that agents have perfect foresight on the future paths of shocks and they expect with certainty no subsequent shocks to follow in the future. The deterministic simulations generated by Dynare are in fact the exact paths of the endogenous variables that evolve according to the model’s equilibrium equations and shock structure. The solutions obtained in this way are not linearised, therefore provide better estimates than the log-linear solutions when the economy is far away from the steady state.

3 Calibration

Most of the calibration in this paper is drawn from the estimations of SW, except for the parameters related to credit frictions which largely follow DEFK. The calibrated values are summarised in Table 1. Two important parameters, the borrowing constraint $\theta$ and the resaleability constraint $\phi_t$, jointly determine the amount of liquidity in the economy. We follow DEFK to set the steady-state values of both $\theta$ and $\phi$ to 0.185, which means that entrepreneurs can sell up to 56% ($= 1 - 0.815^4$) of their equity holdings in one year’s time. Also following DEFK, a credit shock is modelled as a 60% drop in the value of $\phi_t$ from 0.185 to 0.074 (i.e., $e_t^\phi = -60\%$). In DEFK, $\theta$ is fixed at its steady-state value even in a credit crisis. In our analysis, we also study the effects of a tightening of borrowing constraints by lowering the starting value of $\theta$ from 0.185 to 0.074.

Other parameters related to capital investment are $\zeta$, $\kappa$, $\gamma$ and $\delta$. Consistent with DEFK, we calibrate the i.i.d. opportunity to invest in each quarter ($\zeta$) to 0.05, which equals to a 20% opportunity to invest in one year.\footnote{As noted by DEFK, 5% is a conservative estimate of the investment opportunity in the literature. We thus carried out numerical experiments to increase the value of $\zeta$ and found that even a slight increase of $\zeta$ to 5.5% would cause the condition that $q_t > p^I_t$ not to
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<th>Structural parameters:</th>
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<tbody>
<tr>
<td>$\beta$ 0.99 Discount factor</td>
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<tr>
<td>$\sigma$ 1.39 Relative risk aversion</td>
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<tr>
<td>$\delta$ 0.025 Depreciation rate</td>
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<tr>
<td>$\gamma$ 0.36 Capital share</td>
</tr>
<tr>
<td>$\kappa$ 1 Capital goods adjustment cost parameter</td>
</tr>
<tr>
<td>$\nu$ 1.92 Inverse Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\lambda_f$ 0.11 Price mark-up</td>
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<tr>
<td>$\lambda_\omega$ 0.11 Wage mark-up</td>
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<tr>
<td>$\zeta_p$ 0.65 Price Calvo probability</td>
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<tr>
<td>$\zeta_\omega$ 0.73 Wage Calvo probability</td>
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<th>Parameters related to liquidity constraints:</th>
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</thead>
<tbody>
<tr>
<td>$\pi$ 0.05 Probability of investment opportunity</td>
</tr>
<tr>
<td>$\theta$ 0.185 Borrowing constraint at steady state</td>
</tr>
<tr>
<td>$\phi$ 0.185 Equity resaleability constraint at steady state</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_\pi$ 2.03 Taylor rule coefficient on inflation</td>
</tr>
<tr>
<td>$\psi_Y$ 0.08 Taylor rule coefficient on output</td>
</tr>
<tr>
<td>$\psi_\Delta Y$ 0.22 Taylor rule coefficient on change in output</td>
</tr>
<tr>
<td>$\rho_R$ 0.81 Interest rate smoothing</td>
</tr>
<tr>
<td>$\rho_G$ 0.80 Persistence of government spending</td>
</tr>
<tr>
<td>$\psi_k$ -0.063 Open-market intervention parameter</td>
</tr>
<tr>
<td>$\psi_\tau$ 0.1 Taxation rule parameter</td>
</tr>
</tbody>
</table>

Table 1: Calibration
<table>
<thead>
<tr>
<th>Endogenous Variable</th>
<th>Symbol</th>
<th>Steady-state Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption-to-GDP ratio</td>
<td>$C/Y$</td>
<td>0.60</td>
</tr>
<tr>
<td>Investment-to-GDP ratio</td>
<td>$I/Y$</td>
<td>0.22</td>
</tr>
<tr>
<td>Government spending share</td>
<td>$G/Y$</td>
<td>0.18</td>
</tr>
<tr>
<td>Quarterly GDP</td>
<td>$Y$</td>
<td>2.92</td>
</tr>
<tr>
<td>Quarterly labour</td>
<td>$H$</td>
<td>0.85</td>
</tr>
<tr>
<td>Capital stocks</td>
<td>$K$</td>
<td>25.84</td>
</tr>
<tr>
<td>Public debt-to-GDP ratio</td>
<td>$L/4Y$</td>
<td>0.40</td>
</tr>
<tr>
<td>Tax-to-GDP ratio</td>
<td>$\tau/Y$</td>
<td>0.19</td>
</tr>
<tr>
<td>Real wage</td>
<td>$w$</td>
<td>1.97</td>
</tr>
<tr>
<td>Capital rent</td>
<td>$r^k$</td>
<td>3.66%</td>
</tr>
<tr>
<td>Cost of new capital</td>
<td>$p^I$</td>
<td>1</td>
</tr>
<tr>
<td>Market price of equity</td>
<td>$q$</td>
<td>1.07</td>
</tr>
<tr>
<td>Real marginal cost</td>
<td>$mc$</td>
<td>0.90</td>
</tr>
<tr>
<td>Nominal interest rate (quarterly)</td>
<td>$R - 1$</td>
<td>0.57%</td>
</tr>
<tr>
<td>Real interest rate (quarterly)</td>
<td>$r - 1$</td>
<td>0.57%</td>
</tr>
</tbody>
</table>

Table 2: Steady-state values of endogenous variables

capital adjustment cost parameter ($\kappa$) is set to 1 as in DEFK. The capital share in the production function ($\gamma$) and the quarterly depreciation rate ($\delta$) takes on the conventional values of 0.36 and 0.025 respectively.

For the parameters that are standard in a DSGE model, we assign values mainly by referring to the mode of the posterior estimates obtained by SW. The coefficient of relative risk aversion ($\sigma$) is 1.39, and the inverse Frisch elasticity of labour supply ($\nu$) is 1.92. The Calvo probabilities for prices ($\zeta_p$) and wages ($\zeta_w$) are 0.65 and 0.73 respectively. Following Chari, Kehoe and McGrattan (2000), we assume the curvature parameters of the Dixit-Stiglitz aggregators in goods and labour markets to be 10, meaning a markup of 0.11 in both goods and labour markets. We set the discount factor ($\beta$) equal to 0.99 as in DEFK.

We also adopt the estimates of SW to the values of the parameters governing the conduction of monetary policy. The coefficients of inflation ($\psi_\pi$) hold. Since such condition is crucial in deriving the first order conditions of entrepreneurs, we stick with DEFK’s calibration to set $z$ at 5%.
and output \( (\psi_Y) \) in the monetary policy rule are 2.03 and 0.08 respectively; whereas the feedback coefficient on the change in output \( (\psi_{\Delta Y}) \) is 0.22. The degree of interest rate smoothing is calibrated at 0.81. Since the government’s quantitative easing policy is an invention by DEFK, we follow their calibration to set the parameter on open market intervention \( (\psi_k) \) to -0.063. As in DEFK, we assume the taxation rule parameter \( (\psi_r) \) to be 0.1, implying that the adjustment of taxes to the government’s debt position is gradual. As the baseline, we follow Christiano, Eichenbaum and Rebelo (2011) to set the persistence of government spending \( (\rho_G) \) at 0.8. In the next section, we also compare the results by changing \( \rho_G \) to 0.97, which is the estimate obtained by SW.

The steady-state values of the endogenous variables are reported in Table 2. Two steady-state ratios are exogenous: the public debt-to-GDP ratio \( (L/4Y) \) and the government spending share in GDP \( (G/Y) \). The former shows the amount of government bonds issued as a share of annual GDP. Following DEFK, we set it to 40%. The latter takes the average value of government spending share observed in the post-war United States of 18%. Inflation is zero at the steady state, i.e., \( \pi = 1 \).

4 How Large Is the Government Spending Multiplier?

In the literature, studies of the fiscal multiplier usually focus on the impact multiplier which is defined as \( \frac{dY_t}{dG_t} \), where \( dY_t \equiv Y_t - Y \) and \( dG_t \equiv G_t - G \) are the respective differences of output and government expenditure from steady state at certain period \( t \). As noted by Woodford (2011), this way of calculating the multiplier requires the output rise to follow the same shape of time path as that of the government spending rise for the multiplier to be meaningful. We recognise in our simulations that the effects of fiscal stimulus
on GDP are often delayed, so the time paths of the two can differ from each other substantially. For this reason, we instead focus on the cumulative multiplier, defined as \( \frac{E_t \sum_{i=0}^{\infty} dY_t}{E_t \sum_{i=0}^{\infty} dG_t} \). Under this definition, the multiplier measures the expected cumulative increase in output given a one-dollar cumulative increase in government expenditure. If it is greater than one, it implies that any change in government spending has a spillover effect on GDP. We study the fiscal multipliers under two scenarios: at normal times and in times of a credit crisis. We define normal times as the times when government spending shocks are the only source of disturbances; credit crisis times are when the economy is also struck by credit shocks.

### 4.1 The Multiplier at Normal Times

We use the DEFK model to calculate the fiscal multiplier at normal times by giving the steady state a positive government spending shock of 1% of GDP. Credit frictions are present in the DEFK model even at normal times, due to the borrowing and resaleability constraints faced by households. We assume that a government spending shock follows an AR(1) process with a persistence of 0.8 and that no subsequent shock is expected. We obtain a cumulative multiplier of 1.61.

How does this result compare with that obtained using a standard New Keynesian DSGE model? We carry out a control experiment by stripping all credit-constraint features from the DEFK model.\(^\text{13}\) With the same government spending shock, the model without credit frictions (henceforth the

\(^{13}\text{In this standard DSGE model, investment opportunities are not scarce, so } x = 1.\) Investing in capital is not more profitable than investing in other assets as \( q_t = p_t^I \) for all \( t \). The investment function (3) hence reverts to a standard Euler equation. There are no liquidity constraints, implying that \( \theta = \phi_t = 1 \). The government does not need to carry out quantitative easing, as this model does not incorporate credit shock. We use the calibration shown in Table 1 with the exception of \( \beta \), which is adjusted slightly to 0.9943 to keep steady-state interest rates in line with those in the DEFK model.
Fiscal multipliers

<table>
<thead>
<tr>
<th>Model</th>
<th>Impact</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard model</td>
<td>0.58</td>
<td>0.55</td>
</tr>
<tr>
<td>DEFK model</td>
<td>0.70</td>
<td>1.61</td>
</tr>
</tbody>
</table>

Table 3: Government spending multipliers in normal times in the DEFK and the standard models

“standard model”) predicts that the cumulative multiplier will take a value of 0.55. This result is consistent with the conclusions reached by Woodford (2011), who observes that the government spending multiplier is less than one in a simple New Keynesian DSGE model in which monetary policy follows a standard Taylor rule. In Table 3, we summarise the fiscal multipliers obtained using the two models under normal economic conditions.

To understand these results, we report in Figure 1 the impulse-response functions (IRFs) of some key macroeconomic variables to a government spending shock. The IRFs obtained using the DEFK model are shown by solid blue lines; those obtained using the standard model are shown by dotted black lines. The results show that the increase in government expenditures creates inflation pressures, leading the central bank to tighten its monetary policy by increasing the real interest rate. Using the model without credit frictions, both private investment and consumption are crowded out by the rising interest rate. In addition, as Ricardian equivalence holds in the standard model, forward-looking households anticipate the future tax increase and react by reducing their consumption. As a result, the increase in output in this model is moderate and short-lived. However, the IRFs generated by the DEFK model are very different for some variables, especially investment expenditure. Following the government spending shock, private investment falls slightly but then rises in a hump-shaped manner after two quarters. The positive effect on investment peaks at around ten quarters after the shock and disappears thirty quarters after the shock.\footnote{When the shock first hits the economy, investment decreases slightly. This is a con-
a similar hump-shaped pattern, rising above the steady state from quarter 10 onwards. It returns to the initial steady state after about 80 periods of the shock. Accordingly, the increase in output is larger and more persistent. As consumption and investment decrease in both models when the shock first hits the economy, the corresponding impact multipliers are not too different. However, the cumulative multiplier obtained using the DEFK model (1.61) is almost three times that obtained using the standard model (0.55).

Our impulse response analysis suggests that both the consumption multiplier and the investment multiplier are positive in the DEFK model. To confirm this suggestion, we compute the cumulative consumption and investment multipliers in both the standard and the DEFK models. These multipliers measure the expected cumulative increases in consumption and investment respectively, given a one-dollar cumulative increase in government expenditure. Indeed, as shown in Table 4, both the investment and consumption multipliers are positive in the DEFK model. The consumption multiplier is 0.27 and the investment multiplier is larger, at around 0.4. Both multipliers are negative in the standard model. Therefore, the output multiplier is less than one. The prediction by the standard model that consumption decreases in response to a government spending shock is inconsistent with empirical evidence provided by Blanchard and Perotti (2002), Gali, Lopez-Salido and Vallés (2007) and others. These studies suggest that an increase in government spending increases consumption. The consumption response in the DEFK model better fits this empirical evidence. As noted above, although consumption in the DEFK model declines when the shock hits the economy, it rises quickly and becomes positive after about 10 quarters of the shock. It remains above the steady state for a long time. Even though the positive deviation from the steady state is small, the increase is highly

sequence of our assumption that the stock variables in period $t$ show the amounts of the stocks at the end of the period. As a consequence, investment in period $t$ depends on entrepreneurs' bond holdings in period $t-1$. Thus, an increase in bond holdings in period $t$ has an effect on investment in period $t+1$. 

21
Cumulative multipliers

<table>
<thead>
<tr>
<th></th>
<th>Consumption multiplier</th>
<th>Investment multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard model</td>
<td>-0.35</td>
<td>-0.11</td>
</tr>
<tr>
<td>DEFK model</td>
<td>0.27</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 4: Consumption and investment multipliers in normal times

persistent. Therefore, as the above-mentioned empirical studies suggest, the consumption multiplier is positive.

To understand the mechanism that generates larger multipliers in the DEFK model, first note that after an increase in government spending, the increased real interest rate along with the higher future tax burden causes households to reduce their consumption and increase their savings. Therefore, households hold a larger amount of government bonds. As government bonds are more liquid than private equity, an increase in bond holdings improves households’ liquidity. Moreover, following DEFK, we focus on the constrained equilibria in which the market price of equity ($q_t$) is greater than the price of newly produced capital ($p_t^I$). As previously discussed, the implication of this assumption is that investing in new capital is more profitable than buying government bonds or private equity. Therefore, it is optimal for utility-maximising entrepreneurs to sell their bonds and invest all their resources in new capital. Investment thus increases.\footnote{Following Shi (2011), DEFK assume that entrepreneurs and workers in a household pool their assets at the beginning of each period. This asset pooling allows entrepreneurs to use the assets accumulated by others members in the current period. Thus, it relaxes entrepreneurs' financing constraints. When the pooling is not allowed, as in Kiyotaki and Moore (2008), the financing constraint would be tighter. However, an increase in government bonds would still provide entrepreneurs with more liquidity to invest, resulting in a large investment multiplier and hence a large fiscal multiplier.} Increased investment has a knock-on effect on consumption. The fact that consumption becomes positive later than investment reinforces this insight. The intuition for the positive consumption multiplier is as follows. Due to intertemporal substitution effects, workers initially respond by reducing consumption and increas-
ing labour supply. We assume that government spending follows an AR(1) process with a persistence parameter of 0.8, such that the increase in government spending dissipates over time. As government spending decreases, the real interest rate decreases. Therefore, workers increase their consumption and reduce the labour supply. As capital is still being produced, reflected by the persistently higher than usual level of investment, the demand for labour is greater than normal. A greater demand for labour translates into higher real wages. The resulting increase in real wages leads to an increase in consumption. Indeed, as the figure shows, consumption closely follows the dynamics of real wages.

We should note here that, due to the presence of credit constraints, Ricardian equivalence does not hold in the DEFK model. Therefore, changes in taxes do affect households’ decisions and the value of the fiscal multiplier is expected to be sensitive to the taxation rule. We carry out sensitivity analysis on the taxation rule parameter, \( \psi_r \), which measures how quickly the government increases taxes following bond issues. In the baseline, \( \psi_r \) is set to 0.1 following DEFK to reflect that a slow rise in taxes. If we increase \( \psi_r \) to 1, the cumulative multiplier in the DEFK model reduces to 0.67. As one would expect, this model indicates that the government should delay increasing taxes to ensure effective fiscal policy.

The stickiness of prices and wages also plays a role in generating a large fiscal multiplier. Table 5 presents the cumulative multipliers we obtain with different degrees of nominal rigidities given the same government spending shock. Column i (\( \zeta_{p,w} = 0 \)) shows the results under fully flexible prices and wages. Absent both price and wage stickiness, the standard model gives a very low cumulative multiplier of 0.09. The DEFK model suggests a much larger cumulative multiplier (0.90), although it is small compared to the baseline case (1.61). These results seem to suggest that, in the DEFK model, both credit frictions and nominal rigidities play a key role in generating large fiscal multipliers.
Cumulative multipliers

\begin{tabular}{lccc}
\hline
 & (i) $\zeta_{p,w} = 0$ & (ii) $\zeta_w = 0$ & (iii) $\zeta_p = 0$ \\
Standard model & 0.09 & 0.16 & 0.51 \\
DEFK model & 0.90 & 0.97 & 1.59 \\
\hline
\end{tabular}

Table 5: Government spending multipliers under (i) fully flexible prices and wages; (ii) sticky prices and flexible wages; and (iii) flexible prices and sticky wages

An impulse-response analysis (not reported here) reveals the reason why the fiscal multipliers are smaller without nominal rigidities. In the case with fully flexible prices and wages, government spending expansion causes inflation to quickly increase by more than it does in the case with sticky prices and wages. The nominal interest rate therefore increases by more, as the central bank adopts a Taylor rule that targets inflation. The high degree of interest rate smoothing implies that the nominal interest rate stays high for a long time. Since inflation is not persistent under flexible prices and wages, inflation expectations fall quickly after the shock, causing the real interest rate to stay persistently high along with the nominal interest rate. As a consequence, the crowding out effects of fiscal policy are amplified, giving smaller fiscal multipliers in this case.

Table 5 also reports the results with only one kind of nominal rigidity. Column ii ($\zeta_w = 0$) shows the results obtained with fully flexible wages but sticky prices; Column iii ($\zeta_p = 0$) shows those obtained with sticky wages and fully flexible prices. With price stickiness alone, the multipliers are not too different from those obtained absent nominal rigidities ($\zeta_{p,w} = 0$). With wage stickiness alone, on the other hand, we are able to obtain multipliers very similar to those in the baseline case, both in the DEFK and the standard models. These results imply that wage stickiness is more important than price stickiness in producing a large fiscal multiplier, a finding which is consistent with previous studies that suggest wage stickiness causes...
more output persistence than price stickiness.\footnote{See, e.g., Andersen (1998), Huang and Liu (1998) and Woodford (2003, Chapter 3) for a discussion.}

The findings reported in Christiano et al. (2011) and Woodford (2011) suggest that the fiscal multiplier is smaller as the parameter governing the persistence of government spending ($\rho_G$) increases. We repeat our experiments to find out the multipliers in both models by increasing $\rho_G$ from 0.8 to 0.97, which is the estimate suggested by SW. The cumulative multiplier in the DEFK model reduces to 1.04, whereas the one in the standard model falls to only 0.27. The reason is that as government spending rises are more persistent, the present value of the associated tax rises also increases, causing larger negative wealth impacts on consumption. The rises in output are therefore much smaller, resulting in much smaller multipliers. However, our conclusion that the multiplier is larger in the DEFK model than in the standard model remains unchanged.

We also carry out sensitivity analysis on the monetary policy rule. In this experiment, we assume that, instead of (9), the central bank follows a standard Taylor rule with $\psi_\pi = 1.5$, $\psi_Y = 0.125$ and no interest rate inertia. In this case, the cumulative multiplier on output in the DEFK model is slightly higher at 1.8, whereas the one in the standard model (0.6) is almost the same as the baseline. These results seem to confirm that the fiscal multiplier is larger the DEFK model regardless of the monetary policy rule.

### 4.2 The Multiplier at Times of Crisis

We have shown that the government spending multiplier is large in the presence of credit frictions, even without a credit shock. We now examine the value of this multiplier at times of credit crisis. As mentioned earlier, a credit crisis occurs when the value of the resaleability constraint parameter, $\phi_t$, falls by 60%. If the government decides to increase its spending during a credit crisis, the output response will be even larger in the DEFK model.
crisis, we assume that this happens in the same quarter as the arrival of the credit shock, i.e., \( t = 1 \). The cumulative fiscal multiplier in a credit crisis is obtained by

\[
E_t \sum_{t=0}^{\infty} (dY_t - dY_{t}^*)
\]

where \( dY_t \) denotes the change in output from steady state due to the combined effects of the credit shock and the government spending shock, and \( dY_{t}^* \) denotes the same due to the credit shock alone by holding \( G_t \) constant. The difference between these two measures of output change is specifically due to the increase in government spending. The impact multiplier is calculated in a similar way by focusing on \( t = 1 \).

As we show later, the credit crisis brings about a liquidity trap. Using the DEFK model, we simulate credit crises of various expected durations, and compute the output multipliers in response to a government spending shock of 1% of GDP with \( \rho_G = 0.8 \). This exercise cannot be carried out using the standard model as this model does not allow for financial frictions. The results are summarised in Table 6. The number of periods in a credit crisis in which the nominal interest rate falls to its zero bound is also reported.

Table 6 shows that in the DEFK model, the longer is the credit crisis, the longer the liquidity trap is. In addition, the longer is the liquidity trap, the larger the fiscal multiplier is. This table also shows that the DEFK model implies values of the output multiplier between 2.00 and 2.28. These numbers suggest that the multiplier is much larger in the crisis state than at normal times.

To determine the cause of a larger multiplier in the crisis state, we report the IRFs of a credit crisis with an expected duration of three years, both

\[17\text{The size of the government spending shock is the same as that in the first section of Cogan et al. (2010). Erceg and Linde (2012) find that the value of the multiplier can be affected by the size of the fiscal stimulus when the liquidity trap is endogenous. The larger is the fiscal stimulus, the faster the economy exits the liquidity trap, causing a smaller multiplier. We repeat our experiments by increasing the size of the shock to 2\% of GDP. We find that at normal times, the multipliers are unaffected; at times of crisis, the multipliers decrease only slightly (by around 0.1 on average).} \]
Table 6: Fiscal multipliers at times of credit crisis in the DEFK model: Baseline case

<table>
<thead>
<tr>
<th>Duration of credit crisis</th>
<th>Duration of liquidity trap</th>
<th>Impact multiplier</th>
<th>Cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>1q</td>
<td>1.27</td>
<td>2.00</td>
</tr>
<tr>
<td>4q</td>
<td>1q</td>
<td>1.26</td>
<td>2.00</td>
</tr>
<tr>
<td>8q</td>
<td>3q</td>
<td>1.31</td>
<td>2.09</td>
</tr>
<tr>
<td>12q</td>
<td>8q</td>
<td>1.42</td>
<td>2.20</td>
</tr>
<tr>
<td>16q</td>
<td>12q</td>
<td>1.42</td>
<td>2.25</td>
</tr>
<tr>
<td>20q</td>
<td>16q</td>
<td>1.41</td>
<td>2.28</td>
</tr>
</tbody>
</table>

with and without fiscal stimulus.\textsuperscript{18} Figure 2 displays the IRFs for capital, government bonds and investment; Figure 3 plots the remainder of the key variables. We first discuss the effects of the liquidity shock in the case without fiscal expansion. The shock leads to a large decrease in the resaleability of equity, which means that entrepreneurs can obtain less funds for investment by selling their equity. As it is clear from Equation (3), the reduction in equity resaleability leads to a large fall in investment. Figure 2 shows that the fall in investment in period $t$, when the shock first hits the economy, is as large as 17%. This substantial fall in investment seems to suggest that in the DEFK model, most new investment is financed by the sales of entrepreneurs’ asset holdings, rather than the issues of new equity. Employment decreases with investment. Consumption and output fall by significant amounts when the credit shock hits.\textsuperscript{19} In period $t$, output falls by around 7%, consumption by around 5% and labour by around 10%. Reflecting the flight to liquidity, households’ bond holdings increase by around 20%. As the figure shows, the nominal interest rate hits its zero lower bound in response to a credit shock. The nominal interest rate is zero-bound for eight quarters. Inflation

\textsuperscript{18}Note that the IRFs are not smooth in this case. Most of the lines bend upwards after 12 quarters from the shock, when the economy is expected to exit from the credit crisis.

\textsuperscript{19}In the absence of quantitative easing, as shown by DEFK, output decreases more than it does in the presence of quantitative easing. However, the fiscal multipliers do not change significantly.
decreases by around 2%. Given the zero lower bound, the real interest rate increases by around 1.5%.

These responses describe the initial effects of the shock. Figure 2 also shows that investment closely follows the equity resaleability. As the reduction in the equity resaleability is assumed to be persistent, its effect on investment is also persistent. Despite investment increases from its trough in period \( t + 1 \), it stays more or less constant thereafter until the equity resaleability returns to its steady state value. The rebound of investment in period \( t + 1 \) is the result of the increase in households’ bond holdings. However, it appears that the improvement of entrepreneurs’ liquidity due to the increased bond holdings is not enough to offset the adverse effects of the substantial fall in the equity resaleability. As a result, investment stays below the steady state until the equity resaleability returns to normal. Unlike investment, consumption responds gradually to the credit shock as its dynamics are mainly governed by the real interest rate, which adjusts gradually. As the real interest rate declines over time, consumption increases.

In general and with the exception of consumption, these responses are in line with the empirical data reported by Christiano et al. (2011) (see Figure 6, p. 113). However, the fall in consumption predicted using this model (5%) is larger than that indicated by the data (2%).

We now consider the case with fiscal expansion. For the same reasons as at normal times, fiscal expansion in times of crisis increases households’ bond holdings, leading to increases in investment and output. As a result, inflation shows a smaller decrease. Given the zero nominal interest rate and the smaller deflation, the real interest rate increases by less relative to the case without fiscal stimulus, leading to smaller falls in consumption and hence in output. A natural question arises: why is the multiplier larger in the crisis state than at normal times? It is larger since the multiplier effect on consumption is larger in a liquidity trap than at normal times. To confirm this, we report in Table 7 the investment and consumption multipliers in
Table 7: Cumulative consumption and investment multipliers in the DEFK model at times of credit crisis

<table>
<thead>
<tr>
<th>Duration of credit crisis</th>
<th>Duration of liquidity trap</th>
<th>Consumption multiplier</th>
<th>Investment multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>1q</td>
<td>0.68</td>
<td>0.32</td>
</tr>
<tr>
<td>4q</td>
<td>1q</td>
<td>0.68</td>
<td>0.33</td>
</tr>
<tr>
<td>8q</td>
<td>3q</td>
<td>0.76</td>
<td>0.34</td>
</tr>
<tr>
<td>12q</td>
<td>8q</td>
<td>0.89</td>
<td>0.33</td>
</tr>
<tr>
<td>16q</td>
<td>12q</td>
<td>0.93</td>
<td>0.33</td>
</tr>
<tr>
<td>20q</td>
<td>16q</td>
<td>0.97</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 8: Cumulative consumption, investment and fiscal multipliers in the standard model when the zero lower bound binds

<table>
<thead>
<tr>
<th>Duration of liquidity trap</th>
<th>Consumption multiplier</th>
<th>Investment multiplier</th>
<th>Fiscal multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>0.47</td>
<td>0.18</td>
<td>1.65</td>
</tr>
<tr>
<td>4q</td>
<td>0.61</td>
<td>0.23</td>
<td>1.84</td>
</tr>
<tr>
<td>8q</td>
<td>0.48</td>
<td>0.18</td>
<td>1.66</td>
</tr>
<tr>
<td>12q</td>
<td>0.30</td>
<td>0.12</td>
<td>1.42</td>
</tr>
<tr>
<td>16q</td>
<td>0.17</td>
<td>0.08</td>
<td>1.24</td>
</tr>
<tr>
<td>20q</td>
<td>0.09</td>
<td>0.05</td>
<td>1.13</td>
</tr>
</tbody>
</table>

To gain an insight into the role of credit constraints in generating large multipliers when the zero lower bound is binding, we also calculate the consumption, investment and fiscal multipliers in a liquidity trap using the standard model. We follow Cogan et al. (2010) to assume that the nominal interest rate in the standard model remains constant at its steady-state value for various durations. The multipliers obtained are reported in Table 8.

A comparison of Tables 6 and 8 suggests that the fiscal multiplier is larger in the DEFK model than in the standard model. Comparing Tables
7 and 8 reveals the source of the larger multiplier. The fiscal multiplier is larger in the DEFK model since the investment multiplier is larger. In the standard model, the fiscal multiplier increases in a hump-shaped manner, reaching its peak value when the liquidity trap lasts for 4 quarters. In all cases, its value is driven largely by the consumption multiplier. The finding that the fiscal multiplier rises in a hump-shaped manner is related to the observation made by Christiano et al. (2011) and Woodford (2011) that the fiscal multiplier is largest if the fiscal expansion lasts exactly as long as the zero bound state. We assume that government spending evolves according to an AR(1) process with a persistent parameter of 0.8. In our experiments, therefore, the majority of the increases in public spending happens within four quarters after the shock. So the fiscal multiplier peaks when the liquidity trap lasts for the same duration. As the liquidity trap lengthens, fiscal policy becomes less effective and the multiplier decreases.

4.3 Borrowing Constraints

So far in our analysis, we have kept the borrowing constraint parameter, \( \theta \), constant at its steady-state value even in times of credit crisis. Recall that \( \theta \) represents the maximum amount entrepreneurs can borrow in each period to fund their new investments. In reality, the difficulty to borrow varies across economies as well as across industries. In light of these variations, we seek to determine whether a change in the steady-state value of \( \theta \) affects the value of the fiscal multiplier. In the previous cases, we follow DEFK to set \( \theta \) equals to 0.185. Here, we lower its value to 0.074 to simulate an economic environment with tougher borrowing conditions. This value is chosen to be equal to the resaleability constraint parameter \( \phi_i \) in the credit crisis.\(^\text{20}\) We calculate the fiscal multipliers in both normal and crisis times using the

\(^{20}\text{Changing the value of } \theta \text{ from 0.185 to 0.074 requires us to also change the capital share in the production function, } \gamma, \text{ from 0.36 to 0.275 in order to keep the steady-state real interest rate the same as in the baseline case for fair comparison.}\)
**Normal times:**

<table>
<thead>
<tr>
<th></th>
<th>Impact multiplier</th>
<th>Cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEFK model</td>
<td>0.73</td>
<td>1.76</td>
</tr>
</tbody>
</table>

**Crisis times:**

<table>
<thead>
<tr>
<th>Duration of credit crisis</th>
<th>Duration of liquidity trap</th>
<th>Impact multiplier</th>
<th>Cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>1q</td>
<td>0.76</td>
<td>1.78</td>
</tr>
<tr>
<td>4q</td>
<td>1q</td>
<td>1.21</td>
<td>2.02</td>
</tr>
<tr>
<td>8q</td>
<td>1q</td>
<td>1.28</td>
<td>2.08</td>
</tr>
<tr>
<td>12q</td>
<td>6q</td>
<td>1.42</td>
<td>2.23</td>
</tr>
<tr>
<td>16q</td>
<td>11q</td>
<td>1.45</td>
<td>2.28</td>
</tr>
<tr>
<td>20q</td>
<td>15q</td>
<td>1.45</td>
<td>2.34</td>
</tr>
</tbody>
</table>

Table 9: Government spending multipliers in the DEFK model with a lower steady-state value of the borrowing constraint parameter (\(\theta = 0.074\))

DEFK model by assuming \(\rho_C = 0.8\). Table 9 reports the results. We find that the multipliers are not too different from those in the baseline case when \(\theta\) is lowered. Although the multipliers at normal times are slightly bigger (e.g. 1.76 as opposed to 1.61 for the cumulative multiplier), those obtained in the crisis cases are almost the same as in the baseline (see Table 6). This is probably because of the fact that, as noted above, the majority of new investment is financed by selling asset holdings, rather than issuing new equity.

One may argue that the borrowing conditions should be worsening instead of static in a credit crisis. In view of this argument, we redo the simulation experiments by assuming that the value of \(\theta_t\) is dynamic and decreases along with \(\phi_t\) in a credit crisis. A credit crisis is thus re-defined as when the borrowing and the resaleability constraints tighten simultaneously, in a way that \(\theta_t\) and \(\phi_t\) fall hand-in-hand from their steady-state value to their low value of 0.074. The fiscal multipliers obtained are shown in Table 10.
Table 10: Government spending multipliers in credit crises where borrowing and resaleability constraints tighten simultaneously

<table>
<thead>
<tr>
<th>Duration of credit crisis</th>
<th>Duration of liquidity trap</th>
<th>Impact multiplier</th>
<th>Cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>1q</td>
<td>1.27</td>
<td>2.00</td>
</tr>
<tr>
<td>4q</td>
<td>1q</td>
<td>1.24</td>
<td>1.99</td>
</tr>
<tr>
<td>8q</td>
<td>6q</td>
<td>1.38</td>
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</tr>
<tr>
<td>12q</td>
<td>10q</td>
<td>1.37</td>
<td>2.16</td>
</tr>
<tr>
<td>16q</td>
<td>14q</td>
<td>1.35</td>
<td>2.18</td>
</tr>
<tr>
<td>20q</td>
<td>18q</td>
<td>1.33</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Table 10 shows that the values of the multipliers seem to be very close to those in the baseline. We also calculate the investment and the consumption multipliers in this case (not reported here). Both multipliers are very similar to the baseline results so that our conclusion from the previous experiment remains unchanged.

5 Summary and Conclusions

In this paper, we have extended the DEFK model by introducing a role for government spending. We use the resulting model to study the effects of an increase in government expenditure on the macroeconomy. The DEFK model accounts for liquidity constraints and generates a liquidity trap in a credit crisis.

Our main finding is that fiscal policy can be highly effective in an economic environment in which private financial assets are partially liquid and government bonds are liquid. The main reason for this result is that bond-financed fiscal expansion increases households’ bond holdings by increasing the supply of liquid government bonds, thereby making the aggregate portfolio of households more liquid. As the return on new investment is higher than that on government bonds, credit-starved entrepreneurs sell all their bond...
holdings and channel the resulting liquidity into new capital investment, giving rise to a multiplier effect on investment. The increased investment increases output, generating a large fiscal multiplier.

We also study the effects of fiscal policy in a liquidity crisis. A negative shock to liquidity reduces the resaleability of private assets further and brings about a liquidity trap. As a multiplier effect on consumption also arises in a liquidity trap along with that on investment, the fiscal multiplier is even larger under a negative liquidity shock. This result is consistent with previous research findings which show that, relative to the case without fiscal expansion, an increase in public demand at the zero lower bound pushes up prices, thereby pushes down real interest rates and stimulates consumption and output (see, e.g., Christiano, Eichenbaum and Rebelo (2011)).

In sum, our findings suggest that government spending may be a powerful tool to stimulate output in the short-run in an economic environment in which private financial assets are only partially liquid.
A Appendix

A.1 Standard New Keynesian Features of the Model

Differentiated workers \( j \in [\zeta, 1] \) supply labour \( H_t(j) \) to the production sector through the arrangement of employment agencies as in Erceg, Henderson and Levin (2000). Competitive employment agencies choose their profit-maximising amount of \( H_t(j) \) to hire, taking nominal wages \( W_t(j) \) as given. They combine \( H_t(j) \) into homogeneous units of labour input, \( H_t \), according to:

\[
H_t = \left( \frac{1}{1-\zeta} \right)^{\frac{\lambda_\omega}{1+\lambda_\omega}} \int_\zeta^1 H_t(j) \frac{1}{1+\lambda_\omega} dj \right]^{1+\lambda_\omega}
\]

Accordingly, the demand for type-\( j \) labour is:

\[
H_t(j) = \frac{1}{1-\zeta} \left[ \frac{W_t(j)}{W_t} \right]^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_t,
\]

where \( \lambda_\omega \geq 0 \) and \( W_t \) is the aggregate wage index. Each type-\( j \) labour is represented by a labour union who sets their nominal wage \( W_t(j) \) optimally on a staggered basis. Each period, there is a history-independent probability of \( (1 - \zeta_\omega) \) for a union to reset their wage. Otherwise, they keep their nominal wage constant. The optimal wage-setting equation, which is the same across labour unions, in real terms is:

\[
E_t \sum_{s=t}^{\infty} (\beta \zeta_\omega)^{s-t} C^{-\sigma}_s \left\{ \frac{\tilde{w}_t}{\pi_{t,s}} - (1 + \lambda_\omega) \left[ \frac{\tilde{w}_t}{\pi_{t,s} w_s} \right]^{-\frac{1+\lambda_\omega}{\lambda_\omega}} \left[ \frac{1}{1-\zeta} \left( \frac{\tilde{w}_t}{\pi_{t,s} w_s} \right) \right]^{\sigma} H_s \right\} - \left( \frac{\tilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_s = 0,
\]

\( \quad (11) \)

\( ^{21}\) The term \( \frac{1}{1-\zeta} \) is added to the labour aggregate to simplify the notations without changing the substance.
where \( \tilde{w}_t(j) \equiv \frac{\tilde{W}_t(j)}{\tilde{P}_t} \) is the optimal wage chosen by a labour union at \( t \),

\[
w_t \equiv \frac{W_t}{P_t} \text{ and } \pi_{t,s} \equiv \begin{cases} 1, & \text{for } s = t \\ \pi_{t+1} \pi_{t+2} \ldots \pi_s, & \text{for } s \geq t + 1 \end{cases}.
\]

The zero-profit condition for employment agencies gives rise to the dynamics of \( w_t \):

\[
w_t^{-1 \lambda} = (1 - \zeta_\omega) \tilde{w}_t^{-1 \lambda} + \zeta_\omega \left( \frac{w_{t-1}}{\pi_t} \right)^{-1 \lambda}.
\]  

(12)

Firms are classified according to the goods they produce. Final-goods firms produce homogeneous final goods \( Y_t \) by combining heterogeneous intermediate goods \( Y_t(i) \) according to \( Y_t = \left( \int_0^1 Y_t(i) \frac{1}{1+\gamma} \, di \right)^{1+\lambda_f}, \) where \( \lambda_f \geq 0 \).

Their profit-maximising condition implies that the demand for type-\( i \) intermediate good is \( Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{1+\lambda_f} Y_t, \) where \( P_t(i) \) and \( P_t \) are the respective nominal prices for intermediate and final goods. Monopolistic competitive intermediate-goods firms produce according to the production function \( Y_t(i) = A_t K_t(i)^{\gamma} H_t(i)^{1-\gamma}, \) where \( A_t \) is productivity and \( \gamma \) is the capital share.

Intermediate-goods firms maximise their real profits \( D_t(i) \) by choosing the optimal capital and labour inputs, taking real wage and rental rate of capital as given. The cost-minimising conditions imply that \( \frac{K_t(i)}{H_t(i)} = \frac{\gamma \gamma}{(1-\gamma)^2}. \)

Accordingly, their marginal cost is:

\[
m_c = m_{c_t}(i) = \frac{1}{A_t} \left( \frac{w_t}{1-\gamma} \right)^{1-\gamma} \left( \frac{r^*_t}{\gamma} \right)^\gamma,
\]  

(13)

which is universal across firms. Intermediate-goods firms also set nominal prices for their heterogeneous goods. In each period, each firm has a constant probability of \( (1 - \zeta_p) \) to reset their price. They keep their price unchanged otherwise. Firms who reset their price choose the one that maximises their expected future profits, giving the following price-setting equation (in real terms):
\[ E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{p}_t}{\pi_{t,s}} - (1 + \lambda_f) mc_s \right\} \left( \frac{\tilde{p}_t}{\pi_{t,s}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} = 0, \quad (14) \]

where \( \tilde{p}_t(i) \equiv \frac{\tilde{p}_t(i)}{\tilde{p}_t} \) as the optimal price chosen at \( t \). Given the zero-profit condition for final-goods firms, we obtain the evolution of inflation:

\[ 1 = (1 - \zeta_p) \tilde{p}_t - \frac{1}{\lambda_f} + \zeta_p \left( \frac{1}{\pi_t} \right)^{-\frac{1}{\lambda_f}} \quad (15) \]

Capital-goods firms convert final goods into capital goods. The adjustment cost is quadratic in aggregate investment in a way that \( S(I_t) = \frac{\kappa}{2} \left( \frac{I_t}{I_t} - 1 \right)^2 \), where \( I \) is the steady-state investment and \( \kappa \) is the adjustment cost parameter. Under this equation, \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \). Capital-goods firms choose the amount of \( I_t \) to produce which maximises their profits \( D^K_t = [p^I_t - (1 + S(I_t))] I_t \). The first-order condition is:

\[ p^I_t = 1 + S\left( \frac{I_t}{I} \right) + S'(\frac{I_t}{I}) \frac{I_t}{I} \quad (16) \]

### A.2 Aggregate Equilibrium Conditions

Capital evolves according to:

\[ K_t = (1 - \delta) K_{t-1} + I_t \quad (17) \]

The market clears for both labour and capital such that \( H_t = \int_0^1 H_t(i)di \) and \( K_{t-1} = \int_0^1 K_t(i)di \). The aggregate capital-labour ratio becomes:

\[ \frac{K_{t-1}}{H_t} = \frac{\gamma}{1 - \gamma} \frac{w_t}{v^K_t}, \quad (18) \]

and the aggregate production function is:
\[ A_t K_{t-1} H_t^{1-\gamma} = \int_0^1 Y_t(i) di = Y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda}} di. \] (19)

Capital is owned either by households, or indirectly by the government through their private equity holdings:

\[ K_t = N_t + N_t^g \] (20)

The profits for intermediate-goods and capital-goods firms are wholly distributed to households as dividends. Substituting for \( D_t \) and \( D^K_t \), (3) becomes:

\[ I_t = \frac{[r^k_t + (1 - \delta_\ell) q_t \phi_t] N_{t-1} + r_{t-1} L_{t-1} + Y_t - w_t H_t - r^k_t K_{t-1} + p^I_t I_t - [1 + S(\frac{I_t}{I})] I_t - \tau_t}{p^I_t - \theta q_t} \] (21)
References


Figure 1: IRFs to a government spending shock with a persistence of 0.8 in normal times
Figure 2: IRFs of capital and investment a three-year credit crisis: Effects of fiscal stimulus
Figure 3: IRFs to a three-year credit crisis: Effects of fiscal stimulus