Treatment Interactions with Non-Experimental Data in Stata

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Abstract. Treatment effects may vary with the observed characteristics of the treated, often with important implications. In the context of experimental data, a growing literature deals with the problem of specifying treatment interaction terms that most effectively capture this variation. Some of the results of this literature are now implemented in Stata. With non-experimental (observational) data, and in particular when selection into treatment depends on unmeasured factors, treatment effects can be estimated using Stata’s `treatreg` command. Although not originally designed for this purpose, `treatreg` can be used to consistently estimate treatment interactions parameters. In the presence of interactions, however, adjustments are required to generate predicted values and estimate the Average Treatment Effect (ATE). This paper introduces commands that perform this adjustment for the case of multiplicative interactions and shows the adjustment that is required for more complicated interactions.

Keywords: treatment-effects models, interaction terms

1 Introduction

Treatment effects may vary with the observed characteristics of the treated, often with important implications (Royston and Sauerbrei 2008). In the context of experimental data, a growing literature deals with the problem of specifying treatment interaction terms that most effectively capture this variation (see Sauerbrei et al. 2007, for references). Some of the results of this literature are now implemented in Stata (Royston and Sauerbrei 2009). With non-experimental (observational) data, and in particular when selection into treatment depends on unmeasured factors, treatment effects can be estimated using the Stata `treatreg` ([R] `treatreg`) command. Although not originally designed for this purpose, `treatreg` can be used to consistently estimate treatment interactions parameters. In the presence of interactions, however, adjustments are required to generate predicted values and to estimate the Average Treatment Effect (ATE). This paper introduces commands that perform this adjustment for the case of multiplicative interactions and shows the adjustment that is required for more complicated interactions.1

1The command that accompanies this paper can be installed directly through Stata by typing `net install itreatreg`, from (http://people.bath.ac.uk/gkb22/stata) or downloaded from http://people.bath.ac.uk/gkb22/resources.html.
2 Treatment interactions and treatreg

Consider an example where selection into the treatment $Y_2$ is a function of $\epsilon_2$, which is correlated with $\epsilon_1$, the error term in the equation of the outcome of interest, $Y_1$:

\begin{align*}
Y_1 &= \beta_0 + \beta_1 X_1 + \beta_2 Y_2 X_1 + \delta Y_2 + \epsilon_1 \\
Y_2^* &= \gamma_0 + \gamma_1 X_2 + \epsilon_2 \\
Y_2 &= \begin{cases} 
1 & \text{if } Y_2^* > 0 \\
0 & \text{if } Y_2^* \leq 0
\end{cases}
\end{align*} (1)

We observe $X_1, X_2, Y_1$, and $Y_2$, $Var(\epsilon_i) = \sigma_i^2$ for $i = 1, 2$, and we assume that $\sigma_2^2 = 1$. Assuming that $\epsilon_1$ and $\epsilon_2$ follow a bivariate normal distribution with correlation $\rho$, the parameters $\beta_0, \beta_1, \beta_2, \delta, \gamma_0, \gamma_1, \sigma_1$, and $\rho$ can be consistently estimated using either the ML or the two-stage estimation procedure of treatreg. The use of treatreg to estimate models similar to (1) but with $\beta_2 = 0$ was first discussed in Cong and Drukker (2000). When $\beta_2 \neq 0$, we have an additional endogenous variable but this does not change the underlying random structure of the model; the identification conditions remain the same as when $\beta_2 = 0$ (Wooldridge 2002, p.234). For the purpose of estimating the above parameters, it is irrelevant whether treatreg recognizes the term $\beta_2 Y_2 X_1$ as an interaction term between the treatment and an exogenous variable or not. What matters is that the likelihood function (in the case of ML estimation) and the estimating equations (in the case of two-stage estimation) are correctly specified and therefore the estimates are consistent. Results computed with treatreg postestimation however must be corrected when it comes to estimating the average treatment effect (ATE). In the context of model (1), the ATE is given by $E(Y_1 \mid Y_2 = 1) - E(Y_1 \mid Y_2 = 0)$ (Wooldridge 2002, p.604). To estimate it, treatreg postestimation provides the command predict newvar, yctrt to estimate $E(Y_1 \mid X_1, X_2, Y_2 = 1)$ and predict newvar, ycntrt to estimate $E(Y_1 X_1, X_2, Y_2 = 0)$. These estimated conditional expectations are then averaged across the sample and differenced to obtain an estimate of the ATE. This is appropriate when there is no treatment interaction term. When a treatment interaction term is present, however, the predict commands do not condition the treatment interaction term according to the conditioning value of the treatment. The sample value of the treatment is used instead. It is instructive for what follows to derive the deviation between the two processes in the context of model (1). In the population, the conditional expectations of the outcome are given by:

\begin{align*}
E(Y_1 \mid X_1, X_2, Y_2 = 1) &= \beta_0 + (\beta_1 + \beta_2) X_1 + \delta + \sigma_1 \rho \frac{\phi(\gamma_0 + \gamma_1 X_2)}{\Phi(\gamma_0 + \gamma_1 X_2)} \\
E(Y_1 \mid X_1, X_2, Y_2 = 0) &= \beta_0 + \beta_1 X_1 - \sigma_1 \rho \frac{\phi(\gamma_0 + \gamma_1 X_2)}{1 - \Phi(\gamma_0 + \gamma_1 X_2)}
\end{align*} (2) (3)
where $\phi$ is the standard normal density and $\Phi(\cdot)$ is the standard normal cumulative distribution function. The effect of the treatment on a single observation is then just their difference:

$$E(Y_1 \mid X_1, X_2, Y_2 = 1) - E(Y_1 \mid X_1, X_2, Y_2 = 0) = \beta_2 X_1 + \delta + \sigma_1 \rho \left[ \frac{\phi(\gamma_0 + \gamma_1 X_2)}{\Phi(\gamma_0 + \gamma_1 X_2)[1 - \Phi(\gamma_0 + \gamma_1 X_2)]} \right]$$

(4)

The ATE, i.e. the treatment effect across the whole population, is then:

$$E(Y_1 \mid Y_2 = 1) - E(Y_1 \mid Y_2 = 0) = \beta_2 E(X_1) + \delta + \sigma_1 \rho E\left[ \frac{\phi(\gamma_0 + \gamma_1 X_2)}{\Phi(\gamma_0 + \gamma_1 X_2)[1 - \Phi(\gamma_0 + \gamma_1 X_2)]} \right]$$

(5)

Where (5) follows from (4) by the law of iterated expectations, and where the expectations of the RHS are over $X_1$ and $X_2$ respectively. An estimator of (5) is its sample analog:

$$\hat{\beta}_2 \bar{X}_1 + \delta + \sigma_1 \rho \left[ \frac{\phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)}{\Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)[1 - \Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)]} \right]$$

(6)

To derive the difference between (6) and the quantity produced on the basis of the \texttt{predict} commands note that the difference between the estimator of (2) and the output of the corresponding \texttt{predict} command is given by:

$$\left[ \hat{\beta}_0 + (\hat{\beta}_1 + \hat{\beta}_2) X_1 + \hat{\delta} + \hat{\sigma}_1 \rho \frac{\phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)}{\Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)} \right] - \left[ \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 Y_2 X_1 + \hat{\delta} + \hat{\sigma}_1 \rho \frac{\phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)}{\Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)} \right]$$

$$= \hat{\beta}_2 X_1 - \hat{\beta}_2 Y_2 X_1$$

(7)

Averaging across the sample, we have:

$$\hat{\beta}_2 \bar{X}_1 - \hat{\beta}_2 \bar{Y}_2 \bar{X}_1 = \hat{\beta}_2 \frac{1}{N} \left[ \sum_{i} X_{1i} - \sum_{i} Y_{2i} X_{1i} \right]$$

$$= \hat{\beta}_2 \frac{1}{N} \left[ \sum_{i: Y_2=1} X_{1i} + \sum_{i: Y_2=0} X_{1i} - \sum_{i: Y_2=1} Y_{2i} X_{1i} - \sum_{i: Y_2=0} Y_{2i} X_{1i} \right]$$

$$= \hat{\beta}_2 \frac{1}{N} \left[ \sum_{i: Y_2=1} X_{1i} + \sum_{i: Y_2=0} X_{1i} - \sum_{i: Y_2=1} X_{1i} \right]$$

$$= \hat{\beta}_2 \frac{1}{N} \sum_{i: Y_2=0} X_{1i}$$

(8)
Similarly, the difference between the estimator of (3) and the output of the corresponding \texttt{predict} command is:

\[
\left[ \hat{\beta}_0 + \hat{\beta}_1 X_1 - \hat{\sigma}_1 \rho \frac{\phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)}{1 - \Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)} \right] - \\
\left[ \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 Y_2 X_1 - \hat{\sigma}_1 \rho \frac{\phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)}{1 - \Phi(\hat{\gamma}_0 + \hat{\gamma}_1 X_2)} \right]
\]

\[= \hat{\beta}_2 Y_2 X_1 \quad (9)\]

Averaging across the sample gives:

\[-\hat{\beta}_2 Y_2 X_1 = -\hat{\beta}_2 \frac{1}{N} \sum_i Y_2 X_{1i} \]

\[= -\hat{\beta}_2 \frac{1}{N} \left[ \sum_{i: Y_2 = 1} Y_2 X_{1i} + \sum_{i: Y_2 = 0} Y_2 X_{1i} \right] \]

\[= -\hat{\beta}_2 \frac{1}{N} \sum_{i: Y_2 = 1} X_{1i} \quad (10)\]

Subtracting (10) from (8) gives the difference between the estimator in (6) and the quantity computed on the basis of the \texttt{predict} commands:

\[\hat{\beta}_2 \frac{1}{N} \sum_{i: Y_2 = 0} X_{1i} + \hat{\beta}_2 \frac{1}{N} \sum_{i: Y_2 = 1} X_{1i} = \hat{\beta}_2 \frac{1}{N} \left[ \sum_{i: Y_2 = 0} X_{1i} + \sum_{i: Y_2 = 1} X_{1i} \right] \]

\[= \hat{\beta}_2 \frac{1}{N} \sum_i X_{1i} = \hat{\beta}_2 X_1 \quad (11)\]

It is straightforward to extend this result to contexts of treatment interactions with more independent variables. In the case of a treatment interaction of the general form \(f(X_1, Y_2)\), where \(f(\cdot)\) is any function, the adjustment term corresponding to (11) is

\[\hat{\beta}_2 \left[ f(X_1, Y_2 = 1) - f(X_1, Y_2 = 0) \right] \quad (12)\]

### 3 The \texttt{itreatreg} command

The \texttt{itreatreg} command can be used when multiplicative treatment interactions enter the outcome equation in a model such as (1). In a model with non-experimental data and selection on the basis of unobservables, as in (1), multiplicative treatment interactions are interactions of the form \(Y_2 f(X_1)\), where
\( f(\cdot) \) can be any function of \( X_1 \). The \texttt{itreatreg} command produces the same parameter estimates of the model as \texttt{treatreg}. In addition to these estimates, it uses the adjustment described in the previous section to evaluate the estimator in (6). The computational heart of the commands calls \texttt{treatreg} internally and the adjustments are made from the estimates provided by \texttt{treatreg} and stored in two new variables. \texttt{itreatreg} also displays and returns the adjusted Average Treatment Effect and the standard deviation of the Treatment Effect.

### 3.1 Syntax

The syntax of the \texttt{itreatreg} command is:

\[
\texttt{itreat} \quad \texttt{reg} \quad \texttt{depvar} \quad [\texttt{indepvars} \_ni] \quad [\texttt{if}] \quad [\texttt{in}] \quad , \quad \texttt{treat} (\texttt{depvar} \_t=\texttt{indepvars} \_t \quad [\quad , \quad \texttt{noconstant}] \quad ) \quad \texttt{x} (\texttt{xvars} \quad [\quad =\texttt{indepvars} \_i] \quad ) \quad \texttt{gen} (\texttt{stubname}) \quad [\texttt{twostep}] \quad [\texttt{oos}]
\]

where \texttt{depvar} is the dependent variable of interest in the outcome equation. \texttt{indepvars} \_ni is the list of predictors in the outcome equation that are not interacted with the treatment variable. This is optional in so far as predictor variables that are interacted with the treatment variable are specified in the \texttt{x} option, so if all the predictor variables are included with interaction terms, then this list will be empty.

### 3.2 Options

\texttt{treat} (\texttt{depvar} \_t=\texttt{indepvars} \_t) specifies the equation for the treatment selection, where \texttt{depvar} \_t is the treatment variable itself and \texttt{indepvars} \_t is the list of predictor variables for the treatment, in a manner identical to the specification in the \texttt{treatreg} command itself. It is integral to the treatment estimation and is not optional. The \texttt{noconstant} option suppresses the constant in the treatment equation.

\texttt{x} (\texttt{xvars} \quad [\quad =\texttt{indepvars} \_i] \quad ) specifies the treatment interaction variables \texttt{xvars} and, optionally, the original variables \texttt{indepvars} \_i that were interacted with the treatment. It is required: the inclusion of \texttt{indepvars} \_t is optional in the sense that one may wish to include only the interaction term itself and not the original variable. At least one interaction term \texttt{xvar} must be specified, otherwise \texttt{treatreg} itself is appropriate. Moreover, if it is desired to include the original variables then it must be specified correctly in \texttt{x()} rather than included in the list of independent variables \texttt{indepvars} \_ni directly after the dependent variable. For example, \texttt{itreatreg y1, treat(y2=x1) x(y2x2) gen(pr)} would estimate a simple model in which an interaction between the treatment variable \texttt{y2} and an independent variable \texttt{x2}, \texttt{y2x2}, is the sole predictor of \texttt{y1}, aside from the treatment variable itself. Inclusion of the original independent variable \texttt{x2} in the model must be specified thus: \texttt{itreatreg y1 x2, treat(y2=x1) x(y2x2=x2) gen(pr)}.

\texttt{gen} (\texttt{stubname}) is required, and specifies the stubname for the new variables created by \texttt{itreatreg}. \texttt{itreatreg} creates two new variables \texttt{stubname}+ctr and
and stubname+cntrt which contain for each observation, respectively, the predicted value of the dependent variable depvar in the presence of the treatment, and the predicted value in the absence of the treatment. This is analogous to the predict varname, yctrt and predict varname, ycntrt postestimation commands for treatreg itself, but corrected for the effect of the interaction variables. Note that the predicted values are calculated only for those observations used for estimation (i.e. those included in any if/in clauses) unless the option oos is specified.

**oos** is optional and specifies that the predicted values generated by treatreg— and hence the calculation of the Average Treatment Effect—are applied to all observations in the dataset. By default, prediction is applied only to those observations included in the estimation of the coefficients. **oos** overrides this, and applies it to all observations.

**twostep** is optional and specifies that two-step consistent estimates of the parameters, standard errors, and covariance matrix of the model be produced, instead of the default maximum likelihood estimates.

### 3.3 Returned Results

It is important to remember that although itreatreg provides estimation of coefficients, it does so by calling the treatreg function internally. itreatreg is primarily a postestimation command that creates adjusted predictions for interaction terms. Hence, normal Stata postestimation commands such as predict run subsequent to itreatreg will act on the estimations provided by treatreg and will not take into account the adjustments for interaction made by itreatreg. In addition to the results returned by the treatreg function called internally, itreatreg returns the following additional results:

**Scalars**

- r(ate) Average Treatment Effect
- r(te_sd) standard deviation of the Treatment Tffect
- r(N_ate) number of observations used to generate ATE
- r(varctrt) Name of new variable containing predicted values in the presence of treatment
- r(varcntrt) Name of new variable containing predicted values in the absence of treatment

### 4 Examples

#### 4.1 Multiplicative interactions using itreatreg

This example uses the same data that Cong and Drukker (2000) used in their discussion of the treatreg command. It is the same data used in the Stata-Corp (2009) discussion of the treatreg command. The treatreg command is used with a dataset of women’s wages and other characteristics to explore the possibility that women’s college education is endogenous to wage determination (the hypothesis was rejected). Here the original model is modified to allow for
multiplicative interactions between the treatment (here college education) with the two exogenous variables in the wage equation, age and living in a large city.

.webuse labor, clear
.gen wc = 0
.replace wc = 1 if we > 12
69 real changes made
.gen wcXwa = wc * wa
.gen wcXcit = wc * cit
.itreatreg ww, treat(wc=wmed wfed) x(wcXwa=wa wcXcit=cit) gen(padjusted)
Iteration 0: log likelihood = -706.19914
Iteration 1: log likelihood = -706.19738
Iteration 2: log likelihood = -706.19738
Treatment-effects model -- MLE Number of obs = 250
Wald ch2(5) = 5.91
Log likelihood = -706.19738 Prob > ch2 = 0.3148

| Coef. Std. Err. | z  | P>|z| [95% Conf. Interval] |
|-----------------|----|--------|-------------------------|
| ww              |    |        |                         |
| wa              | 0.0057609 | 0.0236009 | 0.24 | 0.807 | -0.040496 | 0.0520178 |
| cit             | 0.0720367 | 0.3829244 | 0.19 | 0.861 | -0.678481 | 0.8225548 |
| wcXwa           | -0.0542976 | 0.0410126 | -1.32 | 0.186 | -0.1346807 | 0.0260855 |
| wcXcit          | 0.0980451 | 0.8044176 | 0.12 | 0.903 | -1.478584 | 1.674675  |
| wc              | 3.466534  | 1.900961  | 1.82 | 0.068 | -0.2592815 | 7.192349  |
| _cons           | 1.657002  | 1.059636  | 1.56 | 0.118 | -0.4198465 | 3.73385   |
| wc              |    |        |                         |
| wmed            | 0.1197113 | 0.032011  | 3.74 | 0.000 | 0.056971  | 0.1824517 |
| wfed            | 0.0964197 | 0.0291015 | 3.31 | 0.001 | 0.0393819 | 0.1534576 |
| _cons           | -2.633536 | 0.3310894 | -7.95 | 0.000 | -3.282459 | -1.984613 |
| /athrho          | 0.0435995 | 0.2000960 | 2.13 | 0.033 | -0.036068 | 0.1232664 |
| /lnsigma        | 0.9210499 | 0.0888913 | 10.68 | 0.000 | 0.748563 | 1.093535  |
| rho             | 0.0435719 | 0.190116  | 2.26 | 0.024 | -0.040971 | 0.127114  |
| sigma           | 2.511926  | 0.1127025 | 22.53 | 0.000 | 2.283527 | 2.739324  |
| lambda          | 1.094494  | 0.477908  | 2.27 | 0.023 | -0.615433 | 2.794422  |

LR test of indep. eqns. (rho = 0):  chi2(1) = 0.05 Prob > chi2 = 0.8191

Average Treatment Effect (ATE) = 1.3945965
Standard deviation of Treatment Effect = .44730832
.predict poriginalctrt, yctrt
.predict poriginalcntrt, ycntrt
.generate poriginaldiff = poriginalctrt - poriginalcntrt
.summarize poriginaldiff

LR test of indep. eqns. (rho = 0):  chi2(1) = 0.05 Prob > chi2 = 0.8191

Average Treatment Effect (ATE) = 1.3945965
Standard deviation of Treatment Effect = .44730832
.predict poriginalctrt, yctrt
.predict poriginalcntrt, ycntrt
.generate poriginaldiff = poriginalctrt - poriginalcntrt
.summarize poriginaldiff
This example first generates the necessary interaction terms that are not present in the original dataset and then calls `itreatreg` to estimate the parameters, generate predicted values and calculate the ATE. After calling `itreatreg`, the example then re-calculates the ATE and the standard deviation of the treatment effect on the basis on the unadjusted predicted values generated by the `treatreg` function. The unadjusted ATE is reported as the mean of the `poriginaldiff` variable in the summary table; the standard deviation of the Treatment Effect is the standard deviation of `poriginaldiff`. While the parameter estimates are the same, it can clearly be seen that there is a significant difference in the estimated treatment statistics. The ATE is almost three times higher in the unadjusted calculations than the correct ATE, while the standard deviation of the treatment effect is much smaller.

### 4.2 Non-multiplicative interactions

Non-multiplicative treatment interactions are rarely used. Here we modify the previous example to include a non-multiplicative interaction between age and the treatment, in addition to the multiplicative interaction between the treatment and living in a large city.

```
. webuse labor, clear
. gen wc = 0
. replace wc = 1 if we > 12
   (69 real changes made)
. gen wcxcit = wc*cit
. gen wc_wa = 1/(wa^wc)
. treatreg ww wa cit wc_wa wcxcit, treat(wc=wmed wfed)
Iteration 0:  log likelihood = -706.17482
Iteration 1:  log likelihood = -706.17325
Iteration 2:  log likelihood = -706.17325
Treatment-effects model -- MLE Number of obs = 250
Wald chi2(5) = 5.97
Log likelihood = -706.17325  Prob > chi2 = 0.3094

Coef.  Std. Err.    z     P>|z|     [95% Conf. Interval]

       WW
      wa  .005609   .0234476  0.24    0.811    -.0403474   .0515654
     cit  .0724072   .3828214  0.19    0.850   -.6779089   .822733
    wc_wa  94.45258   70.37642  1.34    0.180   -43.48267   232.3878
     wcxcit  .0996757   .8043637  0.12    0.901  -1.476848   1.676199
      wc  93.29493   68.59616  1.36    0.174  -41.15108   227.7409
     _cons -92.79207   70.87472 -1.31    0.190   -231.704    46.11982

       wc
     wmed  .1196906   .0320164  3.74    0.000     .0569394   .1824415
    wfed  .0964198   .0291069  3.31    0.001     .0393713   .1534683
     _cons -2.633293   .3310698 -7.95    0.000  -3.282178  -1.984408
```
The mean of the variable \texttt{wwhatdiff} is the estimate of the ATE produced on the basis of the predict commands without any adjustments. The mean of \texttt{wwatehat} is the estimate produced by computing the correct conditional expectations using the adjustments of equations (8) and (10) and following the generalization of (12). The model in this example has the non-multiplicative interaction term $X_2^{-1}Y_2$ but the results are similar to the previous model with the multiplicative interaction term. However, the absolute values of the estimated coefficients of age and its interaction term, and the constant of the outcome equation are much larger. The estimated ATE however is the same as in the previous example to the first decimal. The estimated ATE without the necessary adjustment—\texttt{wwhatdiff}—is very different.

5 Conclusion

The Stata \texttt{treatreg} command can be used to estimate models where selection into treatment depends on observed and non-observed factors. The \texttt{treatreg} command gives consistent estimates of the parameters whether treatment interactions are included or not. The \texttt{predict} command of \texttt{treatreg} postestimation however, gives the correct conditional predictions only when treatment interactions are not present. In this paper we derive the adjustments that are required to compute the correct conditional predictions and Average Treatment Effect (ATE). When the treatment interactions are multiplicative in the treatment, we introduce the \texttt{itreatreg} command which produces the appropriate estimate of
the ATE in addition to the usual output of the treatreg command. When treatment interactions are non-multiplicative in the treatment, we show the steps that are required to produce the appropriate estimates of the ATE.

6 References


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