Search Frictions, Efficiency Wages and Equilibrium Unemployment

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Search Frictions, Efficiency Wages and Equilibrium Unemployment

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Abstract

This paper analyses equilibrium unemployment in a model that combines efficiency wages with search and matching frictions in the labour market. We express equilibrium unemployment as the sum of a pure efficiency wage component and a component that reflects search frictions. Using standard values of calibrated parameters, we find that over 85% of equilibrium unemployment is due to efficiency wage effects.

Keywords: efficiency wages, search frictions, equilibrium unemployment

JEL Classification: J3, J6

1 Introduction

Economists distinguish between two types of equilibrium unemployment. The first is frictional unemployment, reflecting search and matching frictions in the labour market (e.g. Mortensen and Pissarides, 1999). The second type reflects wages set above the reservation wage of workers, typically explained by invoking either efficiency wage effects or union-firm bargaining.

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Although both types have been analysed extensively, their relative importance in explaining unemployment has not. This paper addresses this issue. We incorporate a generic representation of efficiency wage effects due to Solow (1979) into a search and matching model. Using this, we show that equilibrium unemployment can be expressed as the sum of a pure efficiency wage component and a component that reflects search frictions. Our work is related to Malcomson and Mavroeidis (2007) and Zaharieva (2010) who use an extension of the shirking model developed by Shapiro and Stiglitz (1984)\(^1\). In particular, our analytic decomposition of unemployment complements the graphical analysis of Malcomson and Mavroeidis (2007). In related work, Wesselbaum (2013) incorporates an efficiency wage mechanism into a search and matching model; he does not analyse the decomposition of unemployment.

The paper is structured as follows. We outline our model in section 2) and characterise optimal wage-setting by the firm. We obtain a simple generalisation of the Solow Condition that captures the impact of labour market frictions and efficiency wage effects on wage-setting. The wage is the sum of two components; the first due to efficiency wage effects and the second reflecting labour market frictions. Labour market frictions increase the wage. Intuitively, this is because output depends on effective labour input, the product of employment and effort per worker. An increase in hiring costs induces firms to adjust the composition of effective labour, increasing effort relative to employment. This is achieved through an increase in the wage. Section 3) analyses the aggregate equilibrium unemployment rate, showing how this can be decomposed into efficiency wage and frictional components. Section 4) presents a calibration of our model. Using standard values from the search-and-matching and efficiency wage literatures, calibrations of our model suggest that over 85% of equilibrium unemployment is due to efficiency wages. Section 5) summarises and concludes.

\(^1\)Equilibrium unemployment may also rise from wage rigidity and diminishing returns to scale in production, see Michaillat (2012).
2 The Model

Output at representative firm $i$ at time $t$ depends on employment and effort expended by workers

$$Y_{it} = A_t F(e_{it}, N_{it})$$

(1)

where $e$ is effort, $N$ is employment, $A_t$ is technology, given by $A_t = A_{st}$, where $s_t$ is a productivity shock and we assume $F'(.) > 0$ and $F''(.) \leq 0$. Effort is a function of the nominal wage paid by the firm and an external reference wage:

$$e_{it} = e(W_{it}, \bar{W}_t)$$

(2)

for $W_{it} > \bar{W}_t$, where $W$ is the wage, $\bar{W}$ is the reference wage and we assume, $e'(W) > 0$, and $e''(W) < 0$. Employment evolves according to

$$N_{it} = (1 - \tau)N_{it-1} + h_{it}$$

(3)

where $h$ is the number of workers hired and $\tau$ is the exogenous job separation rate. The labour market is characterised by search frictions and so firms must post vacancies in order to hire workers. We assume the aggregate matching function is $h_t = M(U_t, V_t)$ where $M'(.) > 0$, $M''(.) \leq 0$, $h$ are aggregate hires, $U$ is the number of job seekers and $V$ are aggregate vacancies. We also assume the matching function has constant returns to scale, so $h_t = V_t M(U_t, 1)$, hence the vacancy filling rate $q_t$ is given by $q_t = \frac{h_t}{V_t} = M(U_t) = M(U, 1)$. We further assume that the number of workers hired by firm $i$ is proportional to the relative number of vacancies it posts, so $h_{it} = \frac{V_{it}}{V_t} h_t$, where $V_{it}$ is the number of vacancies posted by the firm. As a result, $q_{it} = \frac{h_{it}}{V_{it}} = q_t$ and so the vacancy filling rate
is exogenous at the level of firm. We assume that firms are monopolistic competitors with demand function

\[ \frac{Y_{it}}{Y_t} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} \]  

(4)

where \( P_{it} \) is the price set by firm, \( Y_t \) is aggregate output and \( P_t \) is the aggregate price. We assume that firms seek to hire in every period and abstract from labour participation issues by assuming that workers supply labour inelastically. Finally, we follow the literature on search frictions in assuming the unit cost of a vacancy is \( \gamma_t = \gamma A_t \).

Per-period profit for firm \( i \) is

\[ \pi_{it} = P_{it}Y_{it} - W_{it}N_{it} - \gamma_t V_{it} \]  

(5)

In an efficiency wage context, the firm chooses employment and the wage. The value function for the firm is then

\[ V(N_{it-1}) = \max_{\{W_{it}, N_{it}\}} \{ \pi_{it} + \beta E_t V(N_{it}) \} \]  

(6)

since \( N_{it-1} \) is a state variable in period \( t \). Using (1)-(4) the first-order condition for the wage is

\[ \frac{P_{it}}{\mu} A_t F'(.) e'(W_{it}) = 1 \]  

(7)

where \( \mu = \frac{\epsilon}{\epsilon - 1} \) is the mark-up of price over marginal cost. The first-order condition for employment is

\[ \frac{P_{it}}{\mu} A_t F'(.) e''_{it} = W_{it} + \gamma \frac{A_t}{q_{it}} - \beta \gamma (1 - \tau) E_t \frac{A_{it+1}}{q_{it+1}} \]  

(8)
where the RHS of (8) is the marginal cost of a new hire, composed of the wage plus hiring costs, i.e. the cost of hiring an additional worker in the current period less the expected present value of the reduction in next period hiring costs implied by this.

These imply an extension to the Solow Condition, given by

\[
\frac{W_{it}e'(W_{it})}{e_{it}} = \frac{1}{1 + \frac{\lambda}{W_{it}}(\frac{A_{it}}{q_{it}} - \beta(1 - \tau)E_t\frac{A_{t+1}}{q_{it+1}})}
\]

(9)

At the optimum, the elasticity of effort w.r.t the wage equals the ratio of the wage to the present value of the cost of a new hire. In the absence of labour market frictions, this simplifies to the original Solow condition.

If the effort function is (Summers, 1988)

\[
e(W_{it}, \bar{W}_t) = \bar{e}(\frac{W_{it} - \bar{W}_t}{\bar{W}_t})^\sigma
\]

(10)

for \(W_{it} > \bar{W}_t\) and \(e(W_{it}, \bar{W}_t) = 0\) otherwise\(^2\), where \(0 < \sigma < 1\), then the optimal wage is

\[
W_{it} = \frac{1}{1 - \sigma} \bar{W}_t + \frac{\sigma}{1 - \sigma} \bar{e}(\frac{A_{it}}{q_{it}} - \beta(1 - \tau)E_t\frac{A_{t+1}}{q_{it+1}})
\]

(11)

The wage has two distinct components. The first is a pure efficiency wage effect in which the wage is a mark-up over the reference wage, where the mark-up reflects the strength of efficiency wages. The second reflects labour market frictions as the worker receives a proportion of hiring costs, where this proportion also reflects the strength of efficiency wage effects. The impact of hiring costs on wages arises from the fact that firms determine both the size and the composition of effective labour input \((e_{it}N_{it})\). In response to an increase in hiring costs, firms increase effort relative to employment, achieving this through an increase in the wage.

\(^2\)In Summers (1988), \(\bar{e} = 1\)
3 Equilibrium Unemployment

To analyse unemployment we aggregate the firm-level relationships derived above and consider steady-state outcomes. We assume that there is a continuum of firms and workers distributed along the unit interval. In order to obtain an explicit solution, we assume that the matching function is $M(U_t, V_t) = mU_t^a V_t^{1-a}$. Using $h = \tau N$, the vacancy filling rate is a function of unemployment: $q(U) = m \left( \frac{mU}{\tau(1-U)} \right)^{\alpha}$. We also assume $F(e_{it} N_{it}) = e_{it} N_{it}$ and that the reference wage is (e.g. Summers, 1988)

$$W_t = (1 - \phi U_t) W_t^a$$

where $W_t^a$ is the wage offered by other firms and $\phi$ measures the importance of unemployment in determining a worker’s outside opportunity ($\phi > 0$). At the aggregate level, effort is a function of unemployment: $e(U) = \tilde{e} \left( \frac{\phi U}{1 - \phi U} \right)^a$. Imposing the equilibrium conditions $W_t = W_t^a = W$ and $P_{it} = P$, the first-order conditions for the wage and employment become

$$\mu \frac{P A e(U)}{\phi U} = W$$

and

$$\frac{1}{\mu} P A e(U) = W + \gamma \frac{A (1 - \beta (1 - \tau))}{q(U)}$$

Solving, we obtain an implicit relationship for the equilibrium unemployment rate (denoted by $U^*$)

$$e(U^*) q(U^*) (1 - \frac{\sigma}{\phi U^*}) = \mu \gamma$$

6
where $\hat{\gamma} = (1 - \beta(1 - \tau))\frac{\gamma}{\beta}$ is the present value of marginal hiring costs. Defining $U^{ew} = \frac{\gamma}{\phi}$ as the equilibrium unemployment rate in a pure efficiency wage model, we have

$$e(U^*)q(U^*)(1 - \frac{U^{ew}}{U^*}) = \mu \hat{\gamma} \tag{16}$$

In the absence of labour market frictions ($\hat{\gamma} = 0$), this simplifies to $U^* = U^{ew}$. If there are labour market frictions, $U^* > U^{ew}$, giving an additional component to unemployment. Since $e(U)$ and $q(U)$ are increasing functions, the unemployment rate is an increasing function of the mark-up ($\mu$), the responsiveness of effort to the wage ($\sigma$), the separation rate ($\tau$), vacancy costs ($\gamma$); and a decreasing function of the efficiency of job matching ($m$) and the weight on unemployment in the reference wage ($\phi$).

Decomposing unemployment as

$$U^* = U^{ew} + U^{fr} \tag{17}$$

where $U^{fr}$ is the frictional component, (16) implies that

$$U^{fr} = \lambda(U^*)U^* \tag{18}$$

where $\lambda(U^*) = \frac{\mu \hat{\gamma}}{e(U^*)q(U^*)}$ and

$$U^{ew} = (1 - \lambda(U^*))U^* \tag{19}$$

### 4 Calibrations
In this section, we investigate the relative sizes of the components of equilibrium unemployment using a calibration of the model. Table 1) contains the calibrated values of our structural parameters.

Table 1) Calibration Parameters

<table>
<thead>
<tr>
<th>β</th>
<th>α</th>
<th>m</th>
<th>γ</th>
<th>τ</th>
<th>μ</th>
<th>σ</th>
<th>φ</th>
<th>ê</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.988</td>
<td>0.720</td>
<td>1.355</td>
<td>0.213</td>
<td>0.120</td>
<td>1.200</td>
<td>0.025</td>
<td>0.120</td>
<td>0.700</td>
</tr>
</tbody>
</table>

The first four parameters are consistent with Shimer (2005). The value of τ is consistent with Gali (2011). The values of μ and σ are respectively consistent with values assumed in the New-Keynesian literature and efficiency wage literature. The value φ implies that the unemployment compensation is 47% of wage income. These calibrations imply an equilibrium job finding rate of 68% and an equilibrium vacancy filling rate of 43%.

The implied values of unemployment are presented in Table 2)

Table 2) Equilibrium Unemployment

<table>
<thead>
<tr>
<th>Ue</th>
<th>Uew</th>
<th>Ufr</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.35%</td>
<td>4.70%</td>
<td>0.65%</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Equilibrium unemployment is 5.35%. The efficiency wage component is 4.70% and the frictional component is 0.65%. Thus λ = 0.122, implying that 12.2% of equilibrium unemployment is due to the frictional component; this compares to estimates by Malcomson and Mavroeidis (2007) of λ = 0.33 and λ = 0.02 for the US and UK respectively.

5 Conclusions

In this paper we have combined a generic representation of efficiency wage effects with a search and matching model in order to decompose equilibrium unemployment into a component due to efficiency wages and a component
reflecting frictional unemployment. We have also derived a simple generalisation of the Solow Condition, from which we used to express the wage as the sum of efficiency wage and search-and-matching components. Calibrations of our model suggest that over 85% of equilibrium unemployment is due to efficiency wages.

We would argue that our results are interesting but not definitive. We would wish to develop our work, in three main directions. First, more work is needed to establish the generality of our findings. Second, we should investigate why the contribution of frictional unemployment is so small. We suspect that this in part reflects the specific form for the effort function used in this paper. Thirdly, a natural extension of our work would analyse the decomposition of unemployment away from steady-state. We hope to address these issues in further work.

References


