Optimal size of majoritarian committees under persuasion

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Optimal size of majoritarian committees under persuasion*

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Abstract

We analyze the ‘optimal’ size of non-deliberating majoritarian committees with no conflict of interest among its members when committees can be persuaded by a biased and informed expert. We find that when this bias is small, the optimal size is one; when it is intermediate, the optimal size increases monotonically in the precision of members’ private information; when it is large this relation is non-monotonic. However the optimal committee-size never exceeds five. We also show that biased persuasion typically hurts a larger committee more severely. These results provide important implications on issues like universal enfranchisement, role of expert commentary in a democracy or size of governing boards in firms.

Keywords  Persuasion · Committee size · Information aggregation

JEL  D60 · D71 · D72 · D82

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1 Introduction

The Condorcet Jury Theorem (CJT) suggests that democratic societies are better off by delegating decision rights to larger committees as such committees make better collective judgements than smaller ones. As a result, if the cost of allowing an additional voter is insignificant, universal enfranchisement not only stands as a positive virtue of moral philosophy but also becomes an economically efficient practice. In today’s age of media capture however, where expert opinion is abundantly available and experts typically do not represent the preferences of an average member of the society, committees – or empowered voters – can hardly operate in isolation as has been assumed in the classical CJT framework. The literature around the CJT does not have much to offer on whether biased expert opinions can be an important modifier of the theoretical conclusions on committee size. In this paper we ask if a larger committee necessarily enhances the quality of collective decisions when it is persuaded by an expert whose preferences are not always aligned with those of the committee members. Furthermore, we ask whether expert commentary, albeit informative, can hurt committee decisions even if voters understand that it emanates from a biased source and accordingly make the committee optimally sized.

We address this question in the common interest voting model of [Austen-Smith and Banks (1996)] with an odd number of voters (or committee members) to whom the society has delegated all decision rights. We extend this classical framework to one where we allow for strategic information transmission by a single expert to a group of voters. While the expert’s preferences are not aligned with those of the voters and he does not participate in the collective decision (an ‘outsider’), he has free access to information. In particular, there are two possible alternatives that the voters must choose from collectively: X or Y. The expert always prefers X while the preference of the voters is state-dependent. There is a state variable \( \omega \) such that if \( \omega \) is small enough then the voters also prefer X, but if \( \omega \) is larger they instead prefer Y. Therefore, the ‘magnitude’ of \( \omega \) determines the extent to which the voters’ and the expert’s preferences are misaligned. Any amount of information about the state \( \omega \) is freely available to the expert, while the voters only receive limited information about \( \omega \). In particular, each voter receives privately an informative binary signal about the state \( \omega \) with precision \( p \). The expert chooses a persuasion strategy that generates a public message about \( \omega \) that is verifiable. Based on their signal and the expert’s message, a member votes in favor of the alternative that maximizes his expected utility and the committee is a priori unbiased so that the outcome voted for by the majority of voters is chosen. Our results are based on how voters behave in the most efficient Bayes-Nash equilibrium of the voting game.

The presence of expert persuasion is expected to provide additional public information and hence intuitively one would envisage that the usefulness of an additional committee member would get reduced when compared to the case without an expert. We find that while this is indeed the case, the effect is staggering: the presence of expert persuasion removes all benefits of large committees as we show that in no circumstance is it strictly beneficial for the society.

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1The literature on the CJT is large. [Feddersen and Pesendorfer (1998)] show that the unanimity rule can actually reverse the efficiency of large committees by increasing the probability of convicting an innocent. Nevertheless, even with strategic voting, certain environments yield equilibrium outcomes that converge to the efficient one as the number of voters goes to infinity. [Feddersen and Pesendorfer (1997)] show that in a setting with heterogeneous preferences where each voter receives a private signal about which alternative is best, there exists an equilibrium for each population size such that the outcome converges to the full information outcome as the number of voters goes to infinity. [Myerson (1998)] shows that asymptotic efficiency can be achieved even if there is population uncertainty.

2Thus we address the case of pure persuasion although all our results are qualitatively robust to state-dependent preferences of the expert as well.
to have a committee larger than five. When the expert’s bias is small compared to that of the members so that the probability of preference alignment is greater than 1/2, we prove that the optimal committee size is one, that is, the society is equally well-off by delegating the decision rights to a single decision-maker (Theorem [3]). When the expert’s bias increases so that the probability of preference alignment is below 1/2, optimal committees become larger depending on the precision \(p\) of the members’ private information (Theorem[4]). In particular, if the probability of preference alignment is larger than a threshold of approx. 1/4, then the optimal committee size increases monotonically in \(p\) with the largest required committee size being three. On the other hand, when the probability of preference alignment is lower than this threshold, the optimal committee size is non-monotonic in \(p\): starting from a one-member committee with a very low \(p\), as \(p\) increases, we obtain a three-member and then a five-member committee and as \(p\) rises further, the optimal size falls to three again.

As the presence of a biased expert restricts drastically the benefits from having large committees, we then ask if expert persuasion hurts or improves the quality of decision making of ‘optimally-sized’ committees, that is, committees that cannot improve by becoming larger in size. Among other things, we show that whenever society delegates all decision rights to a single voter (i.e. a one-member committee) the presence of a persuasive expert can never lower the quality of the decision (Theorems 2 and 4 part c). However, when the size of the optimal committee is larger, the results are nuanced. For each precision level \(p\) of private information, there exists a threshold value of the probability of preference alignment such that for all alignment probabilities below that threshold, expert persuasion enhances the probability of correct collective decision while for alignment probabilities above, expert persuasion is harmful. Moreover, this upper bound is non-increasing in \(p\). In general, we also show that the presence of expert persuasion is more harmful the larger the committee size. In particular we show that with seven or more voters, persuasion unambiguously reduces the probability of a correct collective decision relative to a scenario without expert persuasion. Another robust conclusion from this study is that more informed committees obtain more information from expert persuasion.

Given these findings, what can we say about the desirability of universal enfranchisement and the usefulness of expert commentary in a democracy? From the CJT literature, we know that as the number of voters increases so does the probability of correct decisions (the exact conditions for this in our model is provided in Proposition 1). On the other hand we find that under expert persuasion, the optimal committee-size is small (i.e. no larger than five) and in certain circumstances these committees perform better in the presence of the expert while in other circumstances expert persuasion is harmful. So whenever expert persuasion is harmful, it follows directly that the first-best scenario for the society is to impose universal voting rights and discourage expert commentary. On the other hand, when persuasion is useful, there will exist a threshold size of the society larger than the optimal committee size such that for all societies smaller than the threshold size, the first best outcome is to have expert persuasion but where decision rights are delegated to the optimal-sized committee, thereby violating universal enfranchisement. When the size of society is larger, the first best is to discourage expert persuasion and impose universal voting rights.

What if society considers delegating the decision rights to the informed expert? If the bias is small one may expect this to be desirable as this would minimize the variance of the action without distorting it too much from the optimal one. In our model with pure persuasion, the expert always takes the action \(X\) which is, a priori, the correct decision for society with probability \(F(\omega_v)\). Hence such a delegation is equivalent to taking that action with certainty. Our analysis on equilibrium behavior (see Prop. 3 Part 2) shows that when the likelihood of preference alignment between the expert and the voters is high and the precision of private information
of the individual voters is small, expert persuasion is fully uninformative and each committee member chooses alternative $X$. Hence in this case delegation to the expert is equivalent to having a committee of any size. However, in all other cases (see Prop. 2 and Prop. 3), expert persuasion is informative and influential but it is always the case that the committee chooses $Y$ for some message that arrives with strictly positive probability and whenever this happens the decision is correct with probability 1. Hence in all other cases, delegation of decision rights to the expert is strictly worse for society. To summarize, dictatorship of an informed and biased expert is never strictly desirable no matter how small is the bias.

1.1 Related literature

There is a literature concerning optimal size of committees that arise from a different problem faced by large committees. If individual members have to acquire private information through costly investment (unlike in our case), large committees may generate more stringent free rider problems: a single member has larger incentives to avoid this cost the larger is the committee size, particularly in a decision problem with no conflict of interest (like in our case). In view of this plausible source of inefficiency in large committees, Mukhopadhyay (2003) shows that with fixed cost of acquiring information, the optimal size of a committee is indeed bounded. Martinelli (2006) proves that with variable cost of acquiring private information where this cost is fully responsive to the ‘quality’ of information obtained, this problem can be avoided so that larger committees indeed perform better. Further, Koriyama and Szentes (2009) show that though the optimal size of committee remains bounded when this cost has a fixed and a variable component, the welfare loss from having a large committee is insignificant.

Austen-Smith and Banks (1996) also studies the impact of public information on committee decisions although the dissemination of public information is non-strategic in their framework. They show that with binary state-space where public information arrives non-strategically as a binary public signal, sincere voting (meaning votes are solely based upon private signals) cannot be informative (meaning votes mimic private signals) in equilibrium. However the study does not address the issue of optimal committee-size.

The social value of public information in general has been a well addressed subject since the work of Hirshleifer (1971). In a model with strategic complementarity, Morris and Shin (2002) show that public information can hurt social welfare only if agents also have access to independent sources of information. On the other hand, in the investment game of Angeletos and Pavan (2004) public information necessarily improves welfare. Also, Angeletos and Pavan (2007) show how welfare properties of public information depend not only on the form of strategic interaction but also on other external factors that determine the gap between equilibrium and efficient use of public information. However in these papers, public information is non-strategic.

With this paper, we also add to the literature on strategic persuasion by complementing the findings of Kamenica and Gentzkow (2011). In their framework, the receiver of expert information is a single uninformed decision-maker, rather than a group of voters. Kamenica and Gentzkow (2011) characterize sender-optimal persuasion strategies and show that (w.l.o.g.) these strategies take the form of an action recommendation by the expert, which is duly followed by the decision-maker. However, they only briefly discuss the possibility of extending...
their framework to either multiple receivers of information, or to a setting where the single decision-maker receives a private signal in addition to the expert’s communication. Our setting combines both these scenarios – multiple receivers who are privately informed – and shows that the expert’s equilibrium persuasion strategy is no longer always simply a recommendation as to how voters should vote that is then followed in equilibrium. Instead, in our model, there are states of the world in which the expert conveys a public signal so that voters vote in line with their private signals that are probabilistic. Ricardo and Câmaras (2015) also study how an outside expert can persuade voters who may have heterogeneous preferences over two alternatives. In their model, however, voters do not receive private information and their focus is on comparative statics with respect to the voting rule. Like us, they find that when there is a single voter, persuasion is never harmful. However, contrary to our results, they find that if voters are homogeneous, persuasion has no impact on the probability of correct decisions irrespective of the voting rule used. This is driven by the fact that voters who are homogeneous in preferences remain ‘informationally homogeneous’ at the time the expert releases information. In contrast, in our model the interim preferences of voters always remain unpredictable as they depend on voters’ private signals. This interim unpredictability of voters’ preferences has deeper consequences in our model because Ricardo and Câmaras (2015) find that when voters do not differ in their information but solely in their preferences, then a majority of voters is always weakly worse-off with persuasion when the collective decision is made using the simple majority rule. This stands in stark contrast with our findings whereby expert persuasion can hurt as well as help majoritarian committees. Therefore, there is a non-trivial difference between preference heterogeneity with informational homogeneity and preference homogeneity with informational heterogeneity. While both lead to ‘interim preference heterogeneity’, in our case it is incomplete information while in their case it is full information.

The remainder of the paper is organized as follows. In Section 2 we describe the model formally. Section 3 deals with our main results on optimal committee size for efficient information aggregation and compares the scenarios where expert advice is available versus when it is not. Sections 4 (high bias) and 5 (low bias) characterize the nature of the equilibrium persuasion strategy of the expert and the subsequent voting behavior induced by it. We draw our conclusions in Section 6. The appendix (Section 7) contains all proofs.

2 The Model

Basic setting. There is a committee of voters \( I = \{1, \cdots , n\} \) (where \( n \geq 1 \) and odd) who all have identical preferences over the two alternatives in \( A \equiv \{X, Y\} \). Each voter must cast a vote in order to arrive at a collective choice from \( A \). The voters’ preferences over \( A \) depend on the unknown state of the world \( \omega \in \Omega \equiv [0, 1] \). There is an expert with preferences different from those of the committee members who has free access to information about the unknown state.

Preferences. Voters all have the same state-dependent preference relation \( \succsim_\omega \) over \( A \) such that (abbreviated by s.t. from now on) for given \( \omega_v \in (0, 1) : X \succsim_\omega Y \) for all \( \omega \leq \omega_v \) and \( Y \succsim_\omega X \) for all \( \omega > \omega_v \). This preference relation is represented by a utility function \( u : A \times \Omega \rightarrow \mathbb{R} \) s.t. for \( y, \bar{u} \in \mathbb{R}, y < \bar{u}, \) we have:

\[
u(X, \omega) = \begin{cases} \bar{u} & \text{if } \omega \leq \omega_v, \\ u & \text{otherwise}, \end{cases} \quad \text{and} \quad u(Y, \omega) = \begin{cases} u & \text{if } \omega \leq \omega_v, \\ \bar{u} & \text{otherwise}. \end{cases}\]
The expert strictly prefers $X$ over $Y$ in all states. His preferences are represented by a utility function $u_m : A \times \Omega \to \mathbb{R}$ such that for $y_m$, $\tilde{u}_m \in \mathbb{R}$ with $y_m < \tilde{u}_m$ we have $u_m(X, \omega) = \tilde{u}_m$ and $u_m(Y, \omega) = y_m$ for all $\omega \in \Omega^t$.

**Information structure.** The state of the world is modeled as a random variable, and the voters’ common prior over $\Omega$ is assumed to be a probability distribution $F(\omega)$ with atomless density function $f(\omega)$. In any version of our model where $F(\omega_0) > 1/2$ we shall say that there is a high likelihood of preference alignment (or low bias) between voters and the expert, while a low likelihood of preference alignment (or high bias) corresponds to $F(\omega_0) < 1/2$. Each voter $i \in I$ receives an i.i.d. private signal $s_i \in \{X, Y\} \equiv S$ with common precision $p \in (1/2, 1)$: $\mathbb{P}[s_i = X|\omega \leq \omega_0] = \mathbb{P}[s_i = Y|\omega > \omega_0] = p$. Let $s = (s_1, \ldots, s_n) \in S^n$ denote a signal-profile.

**Communication.** The expert can costlessly disseminate information about the true state of the world through his choice of persuasion strategy, which he commits to and announces before learning the realization of the true state. Under any such strategy, voters publicly receive information about the true state in the form of a specific ‘range’ of states and thus the strategy takes the form of an interval partition of the state space $\Omega$ s.t. an interval from this partition is revealed to the voters if and only if the true state lies in this interval. For any integer $k \geq 1$, let $\Omega^t = \{\Omega^t_1, \ldots, \Omega^t_k\}$ denote a $k$-element interval partition of $\Omega$ chosen and announced by the expert. A public signal generated by a persuasion strategy is therefore an interval $\Omega^t_i \subseteq \Omega$, where $i \in \{1, \ldots, k\}$. Given a signal $\Omega^t_i$, the agents’ posterior density function is $f(\omega|\Omega^t_i)$, which is obtained using Bayes rule.

**Voting.** After observing the information revealed through the expert’s persuasion strategy and their respective private signal realizations, voters cast their votes simultaneously. Given a persuasion strategy $\Omega^t$, a (pure) voting strategy for voter $i \in I$ is a function $v_i : \Omega^t \times S \rightarrow A$, $(\Omega^t_i, s_i) \mapsto v_i(\Omega^t_i, s_i) \in A$, that maps the public and private signals to a vote. Let $V$ be the set of all possible voting strategies of a voter, and denote by $v(\Omega^t_i, s)$ the vote-profile $(v_1(\Omega^t_i, s_1), \ldots, v_n(\Omega^t_i, s_n))$. In order to capture the way that individual votes are aggregated into a collective decision, we introduce the notion of a majoritarian committee decision function $\delta : A^n \rightarrow A$ that maps a vote profile $v \in A^n$ to an outcome $\delta(v) \in A$ such that $\delta(v) = X$ if and only if $\{i \in I : v_i = X\} \geq \frac{n+1}{2}$.

**Equilibrium.** We assume here that the committee cannot deliberate and given that, we focus on the most efficient equilibrium (in the sense of maximizing the probability of the correct decision) among the class of symmetric (pure strategy) perfect Bayesian equilibria of the voting continuation game. Under such a strategy, a voter votes in favor of the alternative that maximizes his expected utility after having made full use of any available information. This consists of the public signal generated by the expert’s persuasion strategy, the voter’s private signal, and any inference about other voters’ signals that can be drawn from the fact that his vote affects

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5Our results remain qualitatively intact in a more general environment where in some states the expert prefers alternative $Y$.

6In general, this form of persuasion strategy is equivalent to an arbitrary Bayesian persuasion strategy where messages yield a probability distribution over the state space since for each such probability distribution, there will exist an interval of the state space in our model that will also yield that distribution while for each interval of the state space, there will exist a probability distribution that will carry the same information.
the outcome only in events where he is pivotal. Given a persuasion strategy $\Omega^k$, a vote-profile $v(\Omega^k, s)$ constitutes an efficient equilibrium of the voting continuation game if for every $i \in I$, $\Omega_i^k \in \Omega^k$, $s \in S^n$, and all $\hat{v}_i \in V$, we have:

$$
\int_{\Omega^k_i} \left( \sum_{s \in S^I} P[s_i] \mathbb{P}[s_{-i} | \omega, \Pi_i] u(\delta(v(\Omega^k_i, s)), \omega) \right) f(\omega | \Omega^k_i, s_i) d\omega \\
\geq \int_{\Omega^k_i} \left( \sum_{s \in S^I} P[s_{-i} | \omega, \Pi_i] u(\delta(v_{-i}(\hat{v}_i, s_{-i})), \omega) \right) f(\omega | \Omega^k_i, s_i) d\omega,
$$

where $\Pi_i$ denotes the event that voter $i$ is pivotal and $v_{-i}$ is the profile of votes across all voters other than $i$. We note here that for every persuasion strategy $\Omega^k$ and for each state $\omega$, there is always a unique symmetric equilibrium of the voting continuation game. Given this, we proceed to define the equilibrium of the full game. As in Kamenica and Gentzkow (2011), a persuasion strategy constitutes an equilibrium of the full game if and only if it maximizes the expert’s ex-ante expected payoff. Take a strategy-pair $(\Omega^k, v)$ s.t. the symmetric voting-strategy $v$ is the equilibrium of the continuation voting game given the persuasion strategy $\Omega^k$. Then $(\Omega^k, v)$ is an equilibrium of the full game if for all other pairs $(\Omega^k, \hat{v})$ such that $\hat{v}$ is the equilibrium of the continuation voting game given the persuasion strategy $\Omega^k$, we have:

$$
\int_{\Omega} \left( \sum_{s \in S^n} P[s] u_m(\delta(v(\Omega^k, s)), \omega) \right) f(\omega) d\omega \\
\geq \int_{\Omega} \left( \sum_{s \in S^n} P[s] u_m(\delta(\hat{v}(\Omega^k, s)), \omega) \right) f(\omega) d\omega.
$$

There may be multiple equilibria of the full game, but since all equilibria will be payoff-equivalent for the expert and the voters, we shall consider henceforth only the coarsest equilibrium persuasion strategy, which consists of the minimum number of partitions of $\Omega^k$ that elicit the same voting behavior for every state of the world when compared to any other persuasion strategy.

**Information aggregation.** We evaluate the performance of collective decision-making under biased persuasion by studying the ex ante expected probability that voters collectively choose the correct alternative (given their preferences). Note that this criterion is equivalent to the maximization of the ex ante expected utility of a representative voter, which is given by:

$$
U(\Omega^k, v) = \int_{\Omega} \left( \sum_{s \in S^n} P[s] u_m(\delta(v(\Omega^k, s)), \omega) \right) f(\omega) d\omega.
$$

**No persuasion benchmark.** As a benchmark, we first characterize the voters’ equilibrium behavior in the absence of expert persuasion. The details of this characterization will be useful below when we study equilibrium in the presence of expert communication. Note that we restrict attention to symmetric voting equilibria. Thus, suppose that all voters $j \in I$, $j \neq i$ follow the same voting strategy $v$. Denote the vote-profile across these $n - 1$ voters by the vector $v(s_{-i})$, where:

$$
v(s_{-i}) \equiv (v(s_1), \ldots, v(s_{i-1}), v(s_{i+1}), \ldots, v(s_n)).$$

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7 There are other symmetric Nash equilibria (for example, when every voter always votes for Y irrespective of the private signal received) of the voting continuation game, but these are ignored as they correspond to lower expected utility of the voters.
Then voter $i$’s interim expected utility from submitting a vote $v_i \in A$ conditional on his private signal $s_i$ is given by:

$$
\int_{\Omega} \left( \sum_{s_{-i} \in S^{n-1}} P[s_{-i} \mid \omega] u(\delta(v_i, v(s_{-i})), \omega) \right) f(\omega|s_i) d\omega.
$$

As the collective decision is made according to the simple majority rule, voter $i$’s vote affects the outcome of the election only if the remaining $n - 1$ votes are split equally across the two alternatives. To ease notation, we define as $n_X(v(s_{-i}))$ the number of votes for alternative $X$ cast by the $n - 1$ voters other than $i$. Voter $i$ is pivotal in the sense that his vote affects the outcome if and only if $n_X(v(s_{-i})) = (n - 1)/2$. We introduce the following notation to describe this event:

$$
\Pi_i(v) \equiv \{ s_{-i} \in S^{n-1} : n_X(v(s_{-i})) = (n - 1)/2 \}.
$$

With this notation, we can write as follows the difference in voter $i$’s ex post utility from voting for $X$ rather than $Y$ in any state of the world $\omega \in \Omega$:

$$
|u(\delta(X, v(s_{-i})), \omega) - u(\delta(Y, v(s_{-i})), \omega)| = \begin{cases} 
\bar{u} - \bar{y} & \text{if } n_X(v(s_{-i})) = \frac{n-1}{2} \\
0 & \text{otherwise}.
\end{cases}
$$

This, together with the fact that voters’ signals are conditionally independent, allows us to express the difference in interim expected utility as follows:

$$
\sum_{s_{-i} \in \Pi_i(v)} (\bar{u} - \bar{y}) P[s_{-i}] (2F(\omega|s_i, s_{-i}) - 1).
$$

Since we restrict attention to symmetric pure voting strategies $v$, any signal-profile $s_{-i} \in \Pi_i(v)$ must consist of $(n - 1)/2$ $X$-signals and $(n - 1)/2$ $Y$-signals. Therefore, the set $\Pi_i(v)$ contains $(\frac{n-1}{2})$ signal-profiles. Conditional signal-independence means that every profile $s_{-i} \in \Pi_i(v)$ arises with the following probability:

$$
P[s_{-i}] = P[s_{-i} \mid \omega \leq \omega_i] F(\omega_i) + P[s_{-i} \mid \omega > \omega_i] (1 - F(\omega_i)),
$$

where:

$$
P[s_{-i} \mid \omega \leq \omega_i] = P[s_{-i} \mid \omega > \omega_i] = p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2}}.
$$

This implies that being pivotal reveals no information to voters, so that $F(\omega_i|s_i, s_{-i}) = F(\omega_i|s_i)$ for every $s_{-i} \in \Pi_i(v)$. It is now easy to characterize the optimal voting-behavior of any voter $i$. As $\bar{u} > \bar{y}$:

- vote for alternative $X$ if the interim utility difference is positive, which is the case iff $F(\omega_i|s_i) > 1/2$;
- vote for alternative $Y$ if the interim utility difference is negative, which is the case iff $F(\omega_i|s_i) < 1/2$.

Note that if $F(\omega_i|s_i) = 1/2$, voter $i$ is indifferent between voting for $X$ or $Y$. For simplicity, we assume in this case that any voter $i$ acts in line with the expert’s preference, which is to vote for alternative $X$.

**Proposition 1.** In the absence of public persuasion by an expert, the most efficient voting equilibrium has the following features. For every voter $i \in I$ and all $s_i \in S$:
1. if \( F(\omega_v) > 1/2 \): \( v(s_i) = s_i \) for \( p > F(\omega_v) \), and \( v(s_i) = X \) otherwise;

2. if \( F(\omega_v) < 1/2 \): \( v(s_i) = s_i \) for \( p > 1 - F(\omega_v) \), and \( v(s_i) = Y \) otherwise.

The proof is available from the authors upon request. It is worth noting that, according to Prop. 1, there are only two scenarios in which the unique equilibrium of the game without expert persuasion involves committee members voting in line with their private signals (namely if either \( p > F(\omega_v) > 1/2 \), or \( 1/2 > F(\omega_v) > 1 - p \)). Such a strategy is called informative voting by Austen-Smith and Banks (1996). It is only in these two scenarios that the Condorcet Jury Theorem applies in our model, according to which the probability of a correct committee decision approaches 1 as the number \( n \) of voters becomes arbitrarily large.

3 Main results: information aggregation

In this section, we present our main insights into how the likelihood of making a correct collective decision is affected by the expert’s communication and how this in turn affects the optimal committee size where optimality refers to the minimum committee-size that can achieve the highest possible ex ante probability that the voters collectively choose the correct alternative. We denote this ‘number’ by \( n_{\text{min}} \). We then state results that show that the expert’s communication can adversely affect the likelihood of a correct decision in both large and small committees.

3.1 Information aggregation under low likelihood of preference alignment (High Bias)

We begin by presenting our main results for the case of a low likelihood of preference alignment between the expert and the voters (\( F(\omega_v) < 1/2 \)). In this case, Fig. 1 provides a graphical illustration of the parameter-pairs \( (p, F(\omega_v)) \) for which the minimum committee size needed to achieve the maximum probability of a correct collective decision is \( n_{\text{min}} = 1 \) (the white unshaded area of Fig. 1), \( n_{\text{min}} = 3 \) (the gray shaded area), or \( n_{\text{min}} = 5 \) (the black shaded area).

![Figure 1: Optimal committee size with persuasion](image-url)
Our first main result states formally what is illustrated graphically in Fig. 1:

**Theorem 1.** Under biased expert persuasion in settings with low likelihood of preference alignment (i.e. \( F(\omega_v) < 1/2 \)):

1. if \( F(\omega_v) > 1 - p \):
   
   a. and \( F(\omega_v) \in \left[ \frac{2p-1}{2p}, 1/2 \right) \), then \( n_{\text{min}} = 1 \);
   
   b. and \( F(\omega_v) \in \left( \frac{(1-p)(2p-1)(2p+1)}{p(4p(1-p)+1)}, \frac{2p-1}{2p} \right) \), then \( n_{\text{min}} = 3 \);
   
   c. and \( F(\omega_v) < \frac{(1-p)(2p-1)(2p+1)}{p(4p(1-p)+1)} \), then \( n_{\text{min}} = 5 \).

2. if \( F(\omega_v) < 1 - p \):
   
   a. for \( p < \bar{p}_1 = \sqrt{2}/2 \approx 0.70711 \), then \( n_{\text{min}} = 1 \);
   
   b. for \( \bar{p}_1 < p < \bar{p}_3 \approx 0.76069 \), then \( n_{\text{min}} = 3 \);
   
   c. if \( p > \bar{p}_3 \), then \( n_{\text{min}} = 5 \).

This result shows that in settings with a low chance of preference alignment between the expert and the voters, the smallest committee size needed to maximize the probability of a correct decision increases with the precision \( p \) of voters’ private signals if \( F(\omega_v) > 1 - \bar{p}_3 \approx 0.24 \). On the other hand, if \( F(\omega_v) < 1 - \bar{p}_3 \) then the optimal size is non-monotonic. The non-monotonicity is a quantitative outcome that comes from the interplay of optimal persuasion strategies adopted by the expert that elicit signal-invariant or signal-revealing voting behavior from the voters for specific intervals of the states of the world, the length of which depends on the restrictions of the parameters. This is characterized in Proposition 2 later.

The next figure illustrates our second main result regarding the desirability of expert persuasion. In particular, Fig. 2 shows (in the gray shaded areas) for which parameter-pairs \((p, F(\omega_v))\) the presence of the expert generates a higher probability of a correct decision than a committee of three (left-hand panel of Fig. 2) or a committee of five voters (right-hand panel of Fig. 2) can achieve without the expert.

In order to state the general result, we need to consider the conditional probability

\[
P[\delta(s) = X|\omega > \omega_v] = 1 - J_n(p),
\]

where

\[
J_n(p) \equiv \sum_{j=n+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}
\]

is the probability that more than half of the voters receive the correct private signal vis-a-vis the true state of the world. The condition stated in the following definition provides us with a threshold that determines whether the likelihood \( P[\delta(s) = X|\omega > \omega_v] \) is deemed high or low by the expert. This, in turn, determines his optimal persuasion strategy. The persuasion strategy then determines the voting behavior of the voters, which in turn determines the probability of correct decision-making of the committee.

**Definition 1.** In the event of preference misalignment between the expert and the voters, the likelihood that the expert’s favorite outcome \( X \) is chosen when committee members vote in line with their private signals is said to be **high** if:
Figure 2: Desirability of expert persuasion in committees of three and five voters

\[ 1 - J_n(p) > \frac{F(\omega_v)/(1 - F(\omega_v))}{(p/(1 - p)) - 1}. \]  

(1)

The threshold on the right-hand side of (1) is an odds ratio, where the numerator contains the odds in favor of preference alignment, and the denominator contains the scaled conditional odds in favor of a correct signal. \(^8\) Note that this threshold is always positive, so that a necessary condition for (1) is that the odds ratio be strictly below 1. This, as the reader can easily verify, is the case if and only if the probability of preference misalignment exceeds the odds of an incorrect signal: \(1 - F(\omega_v) > (1 - p)/p\). From this inequality it follows immediately that condition (1) fails when the number of voters is large because \(J_n(p) \to 1\) as \(n \to \infty\). The following result may now be stated regarding the desirability of receiving biased expert persuasion by the committee.

**Theorem 2.**  
1. Let \(F(\omega_v) > 1 - p\).

   a. When condition (1) fails, the ex ante probability of a correct collective decision is higher with biased expert persuasion than without if \(F(\omega_v) < G_n(p)\), where:

   \[ G_n(p) \equiv \frac{p}{1 - p} \left(1 - J_n(p)\right). \]  

   (2)

   If instead \(F(\omega_v) > G_n(p)\), then expert persuasion adversely affects the probability of a correct decision.

   b. When condition (1) holds, the ex ante probability of a correct collective decision is higher with biased expert persuasion than without it.

\(^8\)Note that the numerator odds take values in \(\mathbb{R}^{++}\), while the odds \(p/(1 - p)\) take values in \((1, \infty)\) due to our assumption that private signals are informative (i.e. \(p > 1/2\)). In order to be able to form a meaningful odds ratio, the denominator odds have to be scaled so as to also take values in \(\mathbb{R}^{++}\).
2. Let $F(\omega) < 1 - p$. The ex ante probability of a correct collective decision is higher with biased expert persuasion than without it.

We now provide some intuition for the results in Theorem 2. Whether expert persuasion is desirable to the committee members or not depends on the specific nature of persuasion resorted to by the expert in equilibrium (this is characterized in Sec. 4). Consider the parameter values such that the optimal nature of persuasion in part (1.a) (that is described in Prop. 2.1.a) results in voters following a signal-invariant voting strategy, thereby yielding a unanimous decision. In this case, the probability of a correct decision is independent of the number of voters. In the expert’s absence, voters vote according to their private signals, which implies that in large committees a large number of private signals is being aggregated into a collective decision. The desirability of expert persuasion is therefore to be judged by comparing the merits of these two regimes, and this is captured in the inequality $F(\omega) < G_n(p)$.

Suppose instead that the model parameters $p$, $F(\omega)$, and $n$ are such that in the event of preference misalignment there is a high chance that alternative $X$ is chosen when voters vote according to their respective private signals. This situation corresponds to part (1.b) of Theorem 2, where our characterization of the equilibrium in Prop. 2.1.b shows that expert persuasion makes voters vote for their favorite alternative $X$ regardless of their respective private signals in all states $\omega \in [0, \omega^*]$, where $\omega^* < \omega_v$. In contrast, without expert persuasion voters would cast their vote in line with their private signals in these states. In all the remaining states, they would vote according to their private signals both in the cases where the expert is present and when he is not. It is therefore intuitive that expert persuasion enhances the chances that the electorate makes the correct decision. At this stage we note that for each pair $(n, p)$, the graph of $G_n(p)$ is a strict subset of the graph of $G_{n+1}(p)$. This means that the zone of benefit shrinks with committee size.

Finally, the scenario described in part 2 of the above theorem corresponds to the case where the signal precision is sufficiently low, so that voters in the absence of expert communication vote for $Y$ irrespective of their private signals. Given that the likelihood of preference misalignment is high, the expert’s power to manipulate voters into voting for the wrong alternative is limited and is outweighed by his help in guiding voters to the correct alternative when the true state is in $[0, \omega_v]$. Hence expert persuasion increases the likelihood of a correct collective decision in this case.

3.2 Information aggregation under high likelihood of preference alignment (Low Bias)

Here we present our results for the case of a high likelihood of preference alignment between the expert and the voters: $F(\omega_v) > 1/2$.

**Theorem 3.** Under biased expert persuasion in settings with high likelihood of agreement (i.e. $F(\omega_v) > 1/2$), the probability of a correct collective decision is constant for all $n$, and so $n_{\text{min}} = 1$.

Thus, when the chance of preference misalignment between expert and voters is low, committee size is irrelevant and a small committee with the minimum number of just one voter generates the same probability of correct decision-making as any larger electorate. This is because the strategic persuasion of the expert is directed towards the voters in such a manner that each of them vote irrespective of their private signals for all possible states of the world, and therefore
the probability of correctness of the committee decision becomes invariant to the number of members.

Our final key result shows that even when the expert and the voters are highly likely to agree on what is the correct decision, the expert’s presence can still adversely affect the probability of a correct decision. In general, whether expert persuasion harms information aggregation or not depends on the size $n$ of the electorate. We will see that with seven or more voters, persuasion unambiguously reduces the probability of a correct collective decision relative to a scenario without expert persuasion. However, with three and with five voters, the situation is more nuanced in that the result will depend on the interplay of signal precision $p$ and the preference bias of the expert. In particular, for an intermediate level of signal precision, the probability of a correct decision will be higher with persuasion, while for low and high levels of signal precision it will be higher without persuasion.

**Theorem 4.** The presence of a biased expert has the following impact on the committee’s ability to make the correct decision:

1. If $p < F(\omega_v)$, the ex ante probability that voters collectively choose the correct alternative is unaffected by the expert’s presence.

2. If $p > F(\omega_v)$ and furthermore:
   a. if $n \geq 7$, the ex ante probability of choosing the correct alternative is higher without expert persuasion;
   b. if $n = 3, 5$, the ex ante probability of a correct collective decision is higher with biased expert persuasion if $F(\omega_v) < G_n(p)$, where $G_n(p)$ is defined in (2). If instead $F(\omega_v) > G_n(p)$, then expert persuasion adversely affects the probability of a correct decision.
   c. if $n = 1$, the ex ante probability of choosing the correct alternative is always higher with persuasion.

In order to gain some intuition for the results in Theorem 4, note that when signal precision is low (i.e. $p < F(\omega_v)$) each member votes for $X$ in all states of the world irrespective of his private signal and irrespective of the number of voters (see Prop. below). This rationalizes the expert’s decision not to transmit any information to the voters. As a result, the probability of making the correct decision is the same whether or not an expert is present.

But when the signal strength is high (i.e. $p > F(\omega_v)$), the probability of making the correct decision is higher without persuasion for a sufficiently large electorate (i.e. $n \geq 7$). The reason is as follows: in the presence of an expert, the information provided is such that voters opt for the wrong alternative $X$ when $\omega \in (\omega_v, \omega^*)$, regardless of how many voters there are. If, instead, there is no expert and voters vote in line with their private signals, the chance of a wrong decision in states $\omega \in (\omega_v, \omega^*)$ diminishes as the size of the electorate grows. A noteworthy feature of our result is that the critical size of the electorate is $n = 7$.

Now consider the case of $n = 3$ or $n = 5$ voters. In these cases, our results in Prop. below imply that the probability that voters make the right choice when voting in line with their private signals is low. It is here that the analysis gets interesting due to the non-monotonicity in the ranking of the probabilities of a correct decision with and without the expert: for $p > F(\omega_v)$, the length of the interval $(\omega_v, \omega^*)$ over which the expert can manipulate voters into choosing the wrong decision decreases with $p$. I.e. for low $p$ there is a large range of states for which voters
are being manipulated, meaning that the probability of a correct decision is higher without 
the expert. If, instead, the signal precision $p$ is very high, the probability that voters will 
collectively choose the correct alternative without persuasion is anyway high in all states -
even in such small electorates. Only in an intermediate range of $p$ is it worthwhile to suffer 
the expert’s manipulation in exchange for his help in choosing the correct alternative all states 
except $\omega \in (\omega_0, \omega^*)$.

These results have interesting implications on voting rights and usefulness of expert 
commentary in a democracy as alluded to in Section [1]. In explaining these results, we found that 
the nature of equilibrium persuasion (and the consequent voting behavior) is crucial. In the 
following two sections, we present our results regarding the expert’s equilibrium persuasion 
strategy and the voting behavior that it induces under the various combinations of the model 
parameters $n, p,$ and $F(\omega_0)$.

4 Nature of persuasion under low likelihood of preference alignment

We begin with the scenario where the likelihood of preference alignment between the expert 
and the voters is low: $F(\omega_0) < 1/2$. In this case, the characterization of the expert’s equilibrium 
persuasion strategy depends not only on the likelihood of preference misalignment, but also on 
the interplay of $F(\omega_0)$ with the other model parameters, namely the number of voters $n$ and 
the signal precision $p$. The following result characterizes the expert’s equilibrium persuasion 
strategy and the resulting equilibrium voting behavior of the committee members:

**Proposition 2.** Let $F(\omega_0) < 1/2$. Then the unique coarsest equilibrium features a binary persuasion strategy with threshold $\omega^* \in (0, 1)$ s.t. $\Omega_1^* = [0, \omega^*]$ and $\Omega_2^* = (\omega^*, 1]$. In particular:

1. if $F(\omega_0) > 1 - p$ and:

   a. **condition [1]** fails, then $\omega^*$ solves $F(\omega^*) = F(\omega_0)/p$. This implies $\omega^* > \omega_0$ and induces the following voting behavior: $v_i(\Omega_1^*, s_i) = X$ and $v_i(\Omega_2^*, s_i) = Y$ for all $s_i \in S$. This equilibrium arises under the following parameter constellations:
      i. $n \geq 5$;
      ii. $n = 3$ and $F(\omega_0) \geq (1 - p)(2p - 1)(2p + 1)/p(4p(1 - p) + 1)$;
      iii. $n = 1$ and $F(\omega_0) \geq (2p - 1)/2p$;

   b. **condition [1]** holds, then $\omega^*$ solves $F(\omega^*) = (F(\omega_0) - (1 - p))/p$. This implies $\omega^* < \omega_0$ and induces the following voting behavior: $v_i(\Omega_1^*, s_i) = X$ and $v_i(\Omega_2^*, s_i) = s_i$ for all $s_i \in S$. This equilibrium arises for $n = 3$ if the inequality in 1.a.ii fails, and for $n = 1$ if the inequality in 1.a.iii fails.

2. if $F(\omega_0) < 1 - p$ and:

   a. $n \geq 5$, then the equilibrium is as in 1.a;

   b. $n = 3$ and $p < \bar{p}_3 \approx 0.76069$, then the equilibrium is as in 1.a. If, instead, $p > \bar{p}_3$, then $\omega^*$ solves $F(\omega^*) = F(\omega_0)/(1 - p)$. This implies $\omega^* > \omega_0$ and induces the following voting behavior: $v(\Omega_1^*, s_i) = s_i$ and $v(\Omega_2^*, s_i) = Y$ for all $s_i \in S$;

   c. $n = 1$ and $p < \bar{p}_1 = \sqrt{2}/2 \approx 0.70711$, then the equilibrium is as in 1.a. If, instead, $p > \bar{p}_1$, then the equilibrium is the same as in case 2.b with $p > \bar{p}_3$. 

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The proof is in Section 7.1 in the appendix. To gain some intuition for the results in Prop. 2, recall first that the expert wants the voters to collectively choose X (his favorite alternative). In particular, he would like voters to choose X in the event of disagreement. If condition (1) in Definition 1 holds, the expert is willing to let voters follow their respective private signals for a large stretch of the state space (i.e. for all \( \omega > \omega^* \), where \( \omega^* < \omega_{\Omega} \)) because the chance that voters collectively choose X in the event of disagreement (i.e. \( 1 - J_n(p) \)) is sufficiently high.

5 Nature of persuasion under high likelihood of preference alignment

We now mention briefly the case of high likelihood of preference alignment between the expert and the voters: \( F(\omega_{\Omega}) > 1/2 \). The following proposition provides a full characterization of the equilibrium in this case:

**Proposition 3.** Let \( F(\omega_{\Omega}) > 1/2 \). Then the unique coarsest equilibrium features a binary persuasion strategy with threshold \( \omega^* > \omega_{\Omega} \) s.t. \( \Omega_1^3 = [0, \omega^*] \) and \( \Omega_2^3 = (\omega^*, 1] \). In particular:

1. if \( p < F(\omega_{\Omega}) \), then \( \omega^* = 1 \) and \( v_i(\Omega_1^3, s_i) = X \) for all \( s_i \in S \) (i.e. persuasion yields no information);

2. if \( p > F(\omega_{\Omega}) \), then \( \omega^* \) solves \( F(\omega^*) = F(\omega_{\Omega})/p \). This implies \( \omega^* > \omega_{\Omega} \) and induces the following voting behavior: \( v_i(\Omega_1^3, s_i) = X \) and \( v_i(\Omega_2^3, s_i) = Y \) for all \( s_i \in S \).

The proof is in Section 7.3 in the appendix. To gain some intuition for the result in Prop. 3, note that there are only two candidates for the coarsest equilibrium persuasion strategy: First, the binary strategies described in Prop. 3 (where, regardless of signal, voters vote for X if \( \Omega_1^2 \) is announced, and vote for Y if \( \Omega_2^2 \) is announced), and second, the class of ternary persuasion strategies \( \Omega_3^2 \) under which voters vote in line with their private signals when the expert submits an intermediate report \( \Omega_3^3 = [\alpha, \beta] \), where \( 0 \leq \alpha < \omega_{\Omega} \leq \beta \leq 1 \) (otherwise, they all vote for the same alternative regardless of signal - namely X if \( \Omega_3^3 = [0, \alpha] \) is reported, and Y if \( \Omega_3^3 = [\beta, 1] \) is reported). The proof shows that the coarsest equilibrium persuasion strategy is unique and is binary in nature. When \( p < F(\omega_{\Omega}) \) it follows from Prop. 3 that whenever no information is conveyed by the expert, every member votes for X irrespective of his private signal its strength is sufficiently low (i.e. \( p < F(\omega_{\Omega}) \)). Since the expert prefers alternative X in all states of the world, this is the ideal scenario for him and therefore he chooses the persuasion strategy that does not transmit any information. This explains part (a) of Prop. 3. However, if voters’ signals are sufficiently informative (i.e. \( p > F(\omega_{\Omega}) \)), then Prop. 3 implies that in the absence of any information from the expert, voters vote in line with their private signals. This scenario is suboptimal for the expert who, in equilibrium, provides information \( \Omega_3^2 \) about the state of the world so that voters choose X irrespective of their private signals when they hear this report. When, instead, \( \Omega_2^2 \) is declared by the expert, the voters always choose the expert’s least preferred alternative Y. In equilibrium, the expert’s persuasion strategy maximizes the length of the interval \( \Omega_2^2 \) for which voters are willing to vote for X regardless of their private signals. To see this, observe that for voters to choose X regardless of their private signal, the expert’s report \( \Omega_2^2 \) must provide information that is sufficiently strongly in favor of X (i.e. the likelihood that a state greater than \( \omega_{\Omega} \) has generated the report \( \Omega_2^2 \) must be sufficiently low) so that the voters choose X even when they receive a private signal of Y.\(^9\)

\(^9\)Note that the expert could, in principle, achieve the same result with an alternative communication strategy:
Remark 1. The threshold $\omega^*$ that describes the unique equilibrium persuasion strategy is a decreasing function of $p$. This implies that more informed voters receive more accurate public information from the expert.

6 Conclusion

In this paper we have studied the effect of expert persuasion on the general conclusion of the Condorcet Jury Theorem that larger committees take better collective actions. Our main finding is that in a common-interest non-deliberating voting model where collective decisions are reached via the simple-majority rule, a small committee (of size $\leq 5$) is ‘good enough’. We also find that persuasion never limits information aggregation if the precision of voters’ private signals is low. Otherwise, persuasion will hurt information aggregation in large committees. This is because the information conveyed through the equilibrium persuasion strategy overpowers voters’ private information and invariably makes them vote for a particular alternative. In contrast, without persuasion voters will vote according to their private signals so that the probability of the correct decision increases with the size of the electorate. Thus, absence of expert advice can actually improve information aggregation in large constituencies. Another key insight of this paper is that a similar issue arises even in small constituencies, even though not for all constellations of the model parameters. We also find that in general the amount of information transmitted through expert persuasion increases when each member gets more precise private information.

We have used these results to draw conclusions regarding voting rights and the desirability of expert commentary in a democracy. Our results can also enhance one’s understanding on how firms may take their business decisions. Yermack (1996) studies the Fortune 500 firms and finds a negative correlation between firm value and the size of a firm’s board of directors who take decisions on behalf of the firm. This finding is not only confined to large firms. By studying small and mid-sized Finnish firms, Eisenberg et al (1998) find a negative relation between board size and profitability. The same negative relation is confirmed in other contexts by Bhagat and Black (2002), Mak and Kusnadi (2005) and Conyon and Peck (1998). On the other hand, there are also studies which support the opposite idea that group size and group performance are positively linked. Guo and Schick (2003) surveyed 294 chairpersons and 223 members from 334 ethics committees in the USA in 2000, and found that larger committees are perceived to be more successful. Hence the empirical evidence regarding group size and group performance goes in both directions, and a theoretical framework is warranted to understand the relationship better. Our results suggest the following in this regard. If decisions are majority, very large firms with many shareholders should take decisions by allowing all shareholders to vote and by disallowing independent expert recommendations while smaller firms should form small committees and invite outside experts for advice.

In our analysis we have assumed that the committee does not deliberate and this assumption is common to most of the literature around CJT. If it did deliberate, then, given there is no conflict of interest amongst voters, the problem would be akin to the one with a partially informed single decision maker (or receiver) who receives $n$ independent private signals in addition to any information transmitted by the expert. A single decision-maker with many binary signals can send a message which indicates that his least preferred alternative is the correct choice some (but not all) of the time. Whenever the expert remains silent voters will infer that with sufficiently high probability, the state is s.t. the expert’s favorite alternative is the correct choice. A similar persuasion strategy is used in Kamenica and Gentzkow (2011) for a single decision maker.
may be harder to influence than one with a single binary signal because with more signals there might be a chance that the expert’s information is drowned out by the private information of the decision-maker. In any event, as private information does not hurt voters in our model, and as expert persuasion cannot be detrimental to the social objective under a single-member committee, we expect that larger deliberating committees will make (weakly) better judgements and expert commentary will also make a (weakly) positive impact.

Possible extensions to our model include alternative voting rules (such as approval voting or cumulative voting), and three or more alternatives. It would also be quite natural to introduce multiple experts with either similar or conflicting biases, and examine the effect of their communication on the electorate’s chances of making the correct decision. We reserve these for future research. Another aspect of our model that one could think of relaxing is the assumption that voters receive their private signals without having to pay for them. In relation to this possible extension, it is worth highlighting that Gershkov and Szentes (2009) study the design of an optimal collective decision mechanism in a scenario without conflict of interest among voters (like in the present paper) and without expert persuasion (unlike the present paper). In their model, voters need to pay in order to obtain partially informative signals, which gives rise to free-rider problems - particularly when voters are homogeneous. In order to mitigate this source of inefficiency, Gershkov and Szentes (2009) show that the optimal mechanism must involve an interesting protocol: voters are selected one-by-one at random and asked to acquire a costly signal whose realization they must subsequently report to the mechanism designer. Voters are neither informed about their position in this sequence nor of the other voters’ reports. As this is a very general property of an optimal mechanism (beyond the environments studied by Gershkov and Szentes (2009)), we conjecture that it would remain optimal even if voters were subject to persuasion by a biased expert.

7 Appendix

7.1 Proof of Proposition 2

Step 1: From Prop. 1 we know that when the expert’s persuasion strategy is $\Omega^1$ (which means no information is provided to the voters), then $v(\Omega^1, s_i) = Y$ for all $i \in I$ and all $s_i \in S$. This cannot be an equilibrium as this voting behavior results in the expert’s least preferred alternative being chosen.

Step 2: Now suppose the expert uses a binary persuasion strategy $\Omega^2$ with $\Omega^2_1 = [0, \omega']$ and $\Omega^2_2 = (\omega', 1]$ with $\omega < \omega' \leq 1$. Furthermore, the threshold $\omega'$ that defines this binary strategy is chosen so as to induce the following voting behavior: $v(\Omega^2_1, s_i) = X$ and $v(\Omega^2_2, s_i) = Y$ for all $s_i \in S$. To see how the equilibrium threshold $\omega'$ is determined, consider the voting behavior of a representative voter after receiving the public message $\Omega^2_1$ from the expert. Given that all voters other than $i$ vote for $X$ regardless of their private signals, voter $i$ is not pivotal. In this case, we assume voter $i$ votes on the basis of his private signal $s_i$ and the expert’s message $\Omega^2_1$ as if his vote alone determined the outcome. Computing voter $i$’s interim utility difference from voting for $X$ rather than $Y$ given his private signal $s_i$ yields:

$$\int_{\Omega^2_1} (u(X, \omega) - u(Y, \omega)) f(\omega|s_i, \Omega^2_1)d\omega = (\bar{u} - y)(2F(\omega|s_i, \Omega^2_1) - 1).$$
Therefore, voter $i$ votes for $X$ if $F(\omega_i|s_i, \Omega_1^2) \geq 1/2$, and otherwise he votes for $Y$. In particular, if $s_i = X$, we obtain by Bayes’ Rule:

$$F(\omega_i|X, \Omega_1^2) = \frac{pF(\omega_i)}{pF(\omega_i) + (1-p)(F(\omega') - F(\omega_i))}.$$ 

Thus, voter $i$ with private signal $s_i = X$ votes for $X$ if $F(\omega') \leq F(\omega_i)/(1-p)$, and otherwise he votes for $Y$.

If, instead, voter $i$’s private signal is $s_i = Y$, we obtain by Bayes’ Rule:

$$F(\omega_i|Y, \Omega_1^2) = \frac{(1-p)F(\omega_i)}{(1-p)F(\omega_i) + p(F(\omega') - F(\omega_i))}.$$ 

Thus, if $s_i = Y$ voter $i$ votes for $X$ if $F(\omega') \leq F(\omega_i)/p$, and otherwise he votes for $Y$.

Now consider voter $i$’s voting behavior following the public message $\Omega_2^3$ from the expert. Given that all other voters vote for $Y$ regardless of their private signals, voter $i$ is not pivotal. Casting his vote as if it alone determined the outcome, voter $i$ will vote for $Y$ as $F(\omega_i|s_i, \Omega_2^3) = 0$.

To summarize: As $p > 1/2$, it follows that given a message of $\Omega_2^3 = [0, \omega']$ by the expert, with $\omega' \in (\omega_i, 1]$ s.t. $F(\omega') \leq F(\omega_i)/p$, voter $i$ votes for $X$ regardless of his signal. Given a message of $\Omega_2^3 = (\omega', 1]$ with any $\omega' \in (\omega_i, 1]$, voter $i$ votes for $Y$ regardless of his signal.

We now look for the expert’s optimal threshold value $\omega' \in (\omega_i, 1]$ s.t. $F(\omega') \leq F(\omega_i)/p$. Note that the expert’s expected payoff from such a binary persuasion strategy is:

$$\int \sum_{s \in S^n} \mathbb{P}[s|\omega]u_m(\delta(v(\Omega_2^3, s)), \omega)f(\omega)d\omega$$

$$= \int_0^{\omega'} \tilde{u}_m f(\omega)d\omega + \int_{\omega'}^1 u_m f(\omega)d\omega$$

$$= u_m + (\tilde{u}_m - u_m)F(\omega'). \quad (3)$$

It is therefore optimal for the expert to choose the highest possible threshold value for $\omega'$, which is the one defined implicitly by the equation $F(\omega') = F(\omega_i)/p$.

**Step 3:** Now suppose the expert uses a ternary persuasion strategy $\Omega_3^3$ with $\Omega_3^3 = [0, \alpha]$ and $\Omega_3^3 = [\beta, 1]$, where $\alpha \in (0, \omega_i)$ and $\beta \in (\omega_i, 1]$. Furthermore, the thresholds $\alpha$ and $\beta$ that characterize this strategy are chosen so as to induce the following voting behavior:

$v(\Omega_3^3, s_i) = X$ for all $s_i \in S, v(\Omega_2^3, s_i) = s_i$, and $v(\Omega_3^3, s_i) = Y$ for all $s_i \in S$.

To see how the equilibrium thresholds $\alpha$ and $\beta$ are determined, we start by considering the voting behavior of a representative voter. Recall that for any $t = 1, 2, 3$ voter $i$ will vote for $X$ if $F(\omega_i|s_i, \Omega_t^3) \geq 1/2$. Otherwise, he votes for $Y$. It is straightforward to see that after a public message of $\Omega_1^3$ from the expert, we have $F(\omega_i|s_i, \Omega_1^3) = 1$ regardless of the realization of the private signal. So while voter $i$ is not pivotal given that all other voters vote for $X$ regardless of their private signals, it is optimal for $i$ to follow the same strategy due to our assumption that he will vote as if his ballot alone determines the outcome.

Similarly, following a public message of $\Omega_2^3$ from the expert, we have $F(\omega_i|s_i, \Omega_2^3) = 0$ regardless of the realization of the private signal. So while voter $i$ is not pivotal given that all other voters vote for $Y$ regardless of their private signals, it is optimal for $i$ to follow the same strategy due to our assumption that he will vote as if his ballot alone determines the outcome.
Finally, following the public message $\Omega_3^2$ from the expert, all voters other than $i$ vote according to their respective signals. Therefore, voter $i$ is pivotal for any signal-profile $s_{-i} \in \Pi_i(v)$. In particular, if $s_i = X$, we obtain by Bayes’ Rule:

$$F(\omega_i | X, \Omega_3^2) = \frac{p(F(\beta) - F(\omega_i))}{p(F(\beta) - F(\omega_i)) + (1-p)(F(\omega_i) - F(\alpha))}.$$ 

Thus, voter $i$ with private signal $s_i = X$ votes for $X$ if $F(\beta) \geq F(\omega_i) + \frac{1-p}{p}(F(\omega_i) - F(\alpha))$, and otherwise he votes for $Y$.

If, instead, voter $i$’s private signal is $s_i = Y$, we obtain by Bayes’ Rule:

$$F(\omega_i | Y, \Omega_3^2) = \frac{(1-p)(F(\beta) - F(\omega_i))}{(1-p)(F(\beta) - F(\omega_i)) + p(F(\omega_i) - F(\alpha))}.$$ 

Thus, if $s_i = Y$ voter $i$ votes for $Y$ if $F(\beta) < F(\omega_i) + \frac{p}{1-p}(F(\omega_i) - F(\alpha))$. Otherwise he votes for $X$.

**To summarize:** As $p > 1/2$ it is a best response for voter $i$ to vote in line with his signal when message $\Omega_3^2$ is conveyed by the expert and the other voters vote in line with their private signals.

We now look for the expert’s optimal threshold values $\alpha$ and $\beta$. Note that the expert’s expected payoff from the above ternary persuasion strategy is:

$$\int_{\Omega} \sum_{s \in \Sigma}^{\mathbb{P}[s]} \omega_{um}(\delta(s(\Omega_3^3), \omega)) f(\omega) d\omega$$

$$= \int_{\alpha}^{\omega} \tilde{u}_m f(\omega) d\omega + \int_{\alpha}^{\omega} (J_n(p)(\tilde{u}_m - y_m) + y_m) f(\omega) d\omega$$

$$+ \int_{\alpha}^{\beta} (\tilde{u}_m - J_n(p)(\tilde{u}_m - y_m)) f(\omega) d\omega + \int_{\beta}^{1} y_m f(\omega) d\omega$$

$$= \tilde{u}_m + (\tilde{u}_m - y_m) \left[ (F(\alpha) + F(\beta))(1 - J_n(p)) - F(\omega_i)(1 - 2J_n(p)) \right],$$

(4)

where $J_n(p) = \sum_{j=0}^{n-1} \binom{n}{j} p^j (1-p)^{n-j}$. As $\tilde{u}_m > y_m$ and $J_n(p) < 1$, the expert maximizes his expected payoff from the ternary persuasion strategy by choosing the thresholds $\alpha$ and $\beta$ so as to maximize $F(\alpha) + F(\beta)$ subject to the constraints: (i) $F(\beta) \geq F(\omega_i) + \frac{1-p}{p}(F(\omega_i) - F(\alpha))$, and (ii) $F(\beta) < F(\omega_i) + \frac{p}{1-p}(F(\omega_i) - F(\alpha))$.

In order to solve the optimization problem that determines the expert’s ternary persuasion strategy, and to compare its performance to that of the optimal binary persuasion strategy, we have to distinguish two cases:

**Step 3.1: $F(\omega_i) > 1 - p$:** Note that the threshold $\alpha$ determines the interval boundaries constraining the choice of threshold $\beta$. In particular, if $\alpha$ approaches its maximum admissible value $\omega_i$, then $\beta = \omega_i$ and the ternary persuasion strategy collapses to a binary one with suboptimal threshold $\omega' = \omega_i$. Instead, we shall set the threshold $\beta$ equal to its maximum admissible value: $\beta^* = 1$. With this value, constraint (ii) yields the following upper bound on $\alpha$: $F(\alpha) < (p - (1 - F(\omega_i)))/p$. As $p > 1 - F(\omega_i)$ by assumption, this upper bound is positive and the optimal threshold $\alpha^*$ is determined implicitly by the following equation: $F(\alpha^*) = 1 - ((1 - F(\omega_i))/p)$.

Having characterized the optimal thresholds $\alpha^*$ and $\beta^*$, we can now compute the difference
in the expert’s expected payoffs from the optimal binary and ternary persuasion strategies:

\[
\int_{\Omega} \sum_{s \in S} \mathbb{P}[s|\omega] \left( u_m(\delta(\Omega^3, s), \omega) - u_m(\delta(\Omega^2, s), \omega) \right) f(\omega) d\omega
= \frac{(\bar{u}_m - u_m)(2p - 1)(1 - F(\omega_1))}{p} \left( 1 - J_n(p) - \frac{F(\omega_1)/(1 - F(\omega_1))}{(p/(1 - p)) - 1} \right).
\]

If the payoff difference in (5) is negative, then the binary persuasion strategy is optimal for the expert, and if it is positive, then the ternary persuasion strategy is optimal. The sign of the payoff difference in (5) is determined by whether condition (1) holds or fails. The following result shows for which parameter values this is the case:

**Lemma 1.** Let \( p > 1 - F(\omega_1) \):

1. if \( F(\omega_1) \in ((\sqrt{2} - 1)/\sqrt{2}, 1/2) \) and:
   
   (a) \( n = 1 \), then condition (1) in Definition 1 fails for all \( F(\omega_1) \geq (2p - 1)/2p \) and holds otherwise;
   
   (b) \( n \geq 3 \), then condition (1) fails for all \( p > 1 - F(\omega_1) \);

2. if \( F(\omega_1) \in ((7\sqrt{7} - 10)/(7\sqrt{7} + 17), (\sqrt{2} - 1)/\sqrt{2}) \) and:
   
   (a) \( n = 1 \), then condition (1) holds for all \( p > 1 - F(\omega_1) \);
   
   (b) \( n \geq 3 \), then condition (1) fails for all \( p > 1 - F(\omega_1) \);

3. if \( F(\omega_1) < (7\sqrt{7} - 10)/(7\sqrt{7} + 17) \) and:
   
   (a) \( n = 1 \), condition (1) holds for all \( p > 1 - F(\omega_1) \);
   
   (b) \( n = 3 \), condition (1) holds for \( p < \hat{p}^F_2 \) and fails otherwise, where \( \hat{p}^F_2 \) is the biggest real root in \((1/2, 1)\) of \( 4p^3 - 4p^2 - p + 1 + F(\omega_1)/(1 - F(\omega_1)) \). Note that this can be expressed equivalently by saying that for given \( p \) s.t. \( F(\omega_1) > 1 - p \), condition (1) holds for \( F(\omega_1) < (1 - p)(2p - 1)(2p + 1)/p(4p(1 - p) + 1) \);
   
   (c) \( n \geq 5 \), condition (1) fails for all \( p > 1 - F(\omega_1) \).

The proof of Lemma 1 is given below in Sec. 7.2. With the results from Lemma 1, item 1. of Prop. 2 follows immediately.

**Step 3.2: \( F(\omega_1) < 1 - p \):** In this case, the expert’s problem of maximizing his expected payoff off from the ternary persuasion strategy subject to the constraints \( F(\beta) \geq F(\omega_1) + \frac{1 - p}{p} (F(\omega_1) - F(\alpha)) \) and \( F(\beta) < F(\omega_1) + \frac{1 - p}{p} (F(\omega_1) - F(\alpha)) \) yields optimal threshold values \( \alpha^* = 0 \) and \( \beta^* \) defined implicitly by \( F(\beta^*) = F(\omega_1)/(1 - p) \). By substituting these optimal threshold values into the equation for the expert’s expected payoff from the ternary persuasion in (4), we can obtain the following expected payoff difference:

\[
\int_{\Omega} \sum_{s \in S} \mathbb{P}[s|\omega] \left( u_m(\delta(\Omega^3, s), \omega) - u_m(\delta(\Omega^2, s), \omega) \right) f(\omega) d\omega
= -F(\omega_1)(\bar{u}_m - u_m) \left[ J_n(p)(2p - 1) - p^2 - p + 1 \right] / p(1 - p).
\]
Label as \( \eta(n, p) \) the term in square brackets: \( \eta(n, p) \equiv J_n(p)p(2p - 1) - p^2 - p + 1. \) Note that \( \eta(5, 1/2) = 1/4, \eta(5, 1) = 0, \) and that \( \eta(5, p) = 0 \) has no solution in \( p \in (1/2, 1). \) Thus, \( \eta(5, p) > 0 \) for all \( p \in (1/2, 1). \) As \( p > 1/2 \) it follows that \( \eta(n, p) \) is increasing in \( J_n(p). \) We now make recourse to the following result that is derived in the proof of Lemma 2 in Karotkin and Paroush (2003):

**Lemma (Karotkin and Paroush 2003).** \( J_n(p) \) is increasing in \( n. \) In particular: \( J_{n+1}(p) - J_n(p) = p(2p - 1)\left( \frac{n}{n+1} \right) p^{\frac{n+1}{2}} (1 - p)^{\frac{n-1}{2}}. \)

By Karotkin and Paroush’s lemma it follows that \( \eta(n, p) \) is increasing in \( n. \) This implies that \( \eta(n, p) > 0 \) for all \( n \geq 5, \) which establishes the result in part 2.(a) of Prop. 2.

To prove part 2.(b), set \( n = 3. \) Note that \( \eta(3, 1/2) = 1/4, \eta(3, 1) = 0, \) and that \( \eta(3, p) = 0 \) has a unique solution in \( p \in (1/2, 1) \) given by \( \hat{p}_3 = ((27 - 3\sqrt{78})^{1/3} + (3\sqrt{78} + 27)^{1/3})/6 \approx 0.76. \) This shows that for all \( p \in (1/2, \hat{p}_3), \) we have \( \eta(3, p) > 0, \) while for all \( p \in (\hat{p}_3, 1), \) we have \( \eta(3, p) < 0. \) This proves part 2.(b).

Finally, to prove part 2.(c), set \( n = 1. \) Note that \( \eta(1, 1/2) = 1/4, \eta(3, 1) = 0, \) and that \( \eta(3, p) = 0 \) has a unique solution in \( p \in (1/2, 1) \) given by \( \hat{p}_1 = \sqrt{2}/6 \approx 0.71. \) This shows that for all \( p \in (1/2, \hat{p}_1), \) we have \( \eta(1, p) > 0, \) while for all \( p \in (\hat{p}_1, 1), \) we have \( \eta(1, p) < 0. \) This proves part 2.(c) and completes the proof.

### 7.2 Proof of Lemma 1

To prove the result, we start by re-arranging condition (1) as follows:

\[
(1 - J_n(p)) \left( \frac{2p - 1}{1 - p} \right) > \frac{F(\omega_k)}{1-F(\omega_k)}.
\]

For ease of notation, we define as \( H_n(p) \) the function of \( p \) on the left-hand side of this inequality. The proof idea is best gleaned from Fig. 3 which illustrates condition (1) for five different committee sizes. The upward-sloping line and the curved single-peaked shapes in the figure represent the respective functions \( H_n(p) \) which capture the components of condition (1) that vary with signal precision \( p. \) The horizontal line in Fig. 3 represents the odds in favor of agreement. It is obvious that the graph of \( H_1(p) = 2p - 1 \) has a unique intersection point \( p_1 = 1/2(1 - F(\omega_k)) \) with the horizontal line at \( F(\omega_k)/(1 - F(\omega_k)) \) for any \( F(\omega_k) \in (0, 1/2). \) Furthermore, we have \( p_1 \geq 1 - F(\omega_k) \) if \( F(\omega_k) \geq (\sqrt{2} - 1)/\sqrt{2}. \) Thus, for \( n = 1 \) condition (1) holds for all \( p > p_1 \) if \( F(\omega_k) \geq (\sqrt{2} - 1)/\sqrt{2}, \) and otherwise it holds for all \( p > 1 - F(\omega_k). \)

In the remainder of this proof, we show first that for any committee size \( n \geq 3, \) the function \( H_n(p) \) features a single-peaked graph which, when intersected by a horizontal line, yields exactly two strictly ordered intersection points \( p_n^1 \) and \( p_n^2. \) We then show for a five member committee that when signal precision \( p \) exceeds the likelihood of disagreement \( 1 - F(\omega_k), \) both intersection points with a horizontal line at any \( F(\omega_k)/(1 - F(\omega_k)) \) below the maximum value \( H_5(p^2_5) \approx 0.217549 \) lie outside the range of admissible values \( p \in (1 - F(\omega_k), 1). \) Therefore, condition (1) fails for \( n = 5 \) and all \( p > 1 - F(\omega_k). \) Finally, by virtue of the fact that \( H_{n+2}(p) < H_n(p) \) for all \( p \in (1/2, 1) \) due to the aforementioned lemma by Karotkin and Paroush (2003), it follows immediately that condition (1) also fails for all \( p > 1 - F(\omega_k) \) and all \( n > 5. \) Thus, only in one-member and in three-member committees can condition (1) be met for some \( p \in (1 - F(\omega_k), 1). \)
As an illustration, it is straightforward to verify that a drop in the value of $\bar{P}$ of more than 1. Thus, for any $\bar{P}$ and therefore condition (1) holds for all $\bar{P}$.

First, note that for all $\bar{P}$, the corresponding lower bound on $\bar{P}$, which is equivalent to $\bar{P} < (7\sqrt{7} - 10)/7\sqrt{7} + 17$, then condition (1) cannot hold for any $\bar{P}$. Observe that for $\bar{P} = (7\sqrt{7} - 10)/7\sqrt{7} + 17$, the corresponding lower bound on $\bar{P}$ is $\bar{P} = (7\sqrt{7} - 17)/2 \approx 7.6013$. Therefore: $p^2_3 > p(F(\omega)) = (7\sqrt{7} - 17)/2$. To see that for any $F(\omega) < (7\sqrt{7} - 10)/7\sqrt{7} + 17$ the corresponding lower bound $p(F(\omega))$ is smaller than the intersection point $p^2_3(F(\omega))$ of the function $H_3(p)$ with the horizontal line at $F(\omega)/(1 - F(\omega))$, note that a change in the value of $F(\omega)$ affects these two benchmarks in the following way: $dp(F(\omega))/dF(\omega) = -1$ and $dp^2_3(F(\omega))/dF(\omega) = 1/H_3(p^2_3(F(\omega)))(1 - F(\omega))^2 < 0$ for all $p^2_3(F(\omega)) > p^2_3$. In particular, it is straightforward to verify that a drop in the value of $F(\omega)$ raises the value of the lower bound $p(F(\omega))$ by 1, while it raises the value of the intersection point $p^2_3(F(\omega))$ by more than 1. Thus, for any $F(\omega) < (7\sqrt{7} - 10)/7\sqrt{7} + 17$ we have $p^2_3(F(\omega)) > p(F(\omega))$, and therefore condition (1) holds for all $p \in (p(F(\omega))$, $p^2_3(F(\omega)))$.

**General shape of $H_n$.** We now characterize the shape of the function $H_n$ for arbitrary $n \geq 3$.

First, note that for all $n$: $H_n(1/2) = 0$ and $H_n(1) = 0$. Next, consider the monotonicity and curvature of $H_n$. To this end, we define for ease of notation $L_{n,j}(p) \equiv \binom{n}{j} p^j (1-p)^{n-j}$. Also let:

$$K_n(p) \equiv \sum_{j=0}^{n-1} \binom{n}{j} p^j (1-p)^{n-j} = \sum_{j=0}^{n-1} L_{n,j}(p).$$

As $n$ is odd, the integer $(n-1)/2$ is even. We can therefore express equivalently the function $H_n$ as: $H_n(p) = ((2p - 1)/(1-p))K_n(p)$. In order to obtain the derivatives of this function, it is useful to note that:

$$L'_{n,j}(p) = n (L_{n-1,j-1}(p) - L_{n-1,j}(p)).$$

Thus, $K'_n(p) = \sum_{j=0}^{n-1} n (L_{n-1,j-1}(p) - L_{n-1,j}(p)) = -n L_{n-1,\frac{n-1}{2}}(p)$. We therefore obtain our
desired derivative:

\[
H_n'(p) = \frac{1}{(1-p)^2} K_n(p) - \frac{n(2p-1)}{1-p} L_{n-1, \frac{v-1}{2}}(p)
\]

\[
= \sum_{j=0}^{n-1} \binom{n}{j} p^j (1-p)^{n-j-2} - n(2p-1) \left( \frac{n-1}{n-1} \right) p^{\frac{n-1}{2}} (1-p)^{\frac{n-3}{2}}.
\]

We first evaluate the derivative at \( p = 1/2 \):

\[
H_n'(1/2) = 4K_n(p) = \frac{4}{2n} \sum_{j=0}^{n-1} \binom{n}{j}.
\]

Note that \( \sum_{j=0}^{n} \binom{n}{j} = 2^n \), and since \( n \) is odd, we also have \( \sum_{j=0}^{n-1} \binom{n}{j} = (1/2) \sum_{j=0}^{n} \binom{n}{j} \). Thus, \( H_n'(1/2) = 2 \), which shows that \( H_n \) is strictly increasing at \( p = 1/2 \).

Next, we evaluate the derivative \( H_n' \) at \( p = 1 \):

\[
H_n'(1) = \sum_{j=0}^{n-1} \binom{n}{j} 1 \cdot 0^{n-j-2} - n \left( \frac{n-1}{n-1} \right) 1 \cdot 0^{\frac{n-3}{2}}
\]

First consider the case \( n = 3 \):

\[
H_3'(1) = \left( \frac{3}{0} \right) 1 \cdot 0^1 + \left( \frac{3}{1} \right) 1 \cdot 0^0 - 3 \left( \frac{2}{1} \right) 1 \cdot 0^0.
\]

As \( 0^0 = 1 \), this expression reduces to: \( H_3'(1) = 3 - 6 = -3 \). Next, consider \( n \geq 5 \). In these cases, the expression for \( H_n'(1) \) no longer features any expressions involving \( 0^0 \), but only expressions where \( 0 \) is raised to a strictly positive power, which are all equal to zero. Thus, \( H_n'(1) = 0 \) for all \( n \geq 5 \). We can therefore infer from the fact that \( H_n \) is strictly increasing at \( p = 1/2 \) and non-increasing at \( p = 1 \), that \( H_n \) must have a global maximum somewhere in \( (1/2, 1) \). In fact, we now argue that \( H_n \) has a unique critical point in \( (1/2, 1) \), which must therefore be the unique global maximum.

**Uniqueness of critical point of \( H_n \).** For \( n \geq 3 \), at any critical point \( p_n^* \), the following condition holds:

\[
\frac{1}{(1-p_n^*)^2} K_n(p_n^*) - \frac{n(2p_n^*-1)}{1-p_n^*} L_{n-1, \frac{v-1}{2}}(p_n^*) = 0.
\]  \( \text{(7)} \)

Multiplying through \( (7) \) by \( (1-p_n^*)^2 \) yields:

\[
K_n(p_n^*) = n(1-p_n^*)(2p_n^*-1)L_{n-1, \frac{v-1}{2}}(p_n^*).
\]

To establish that there is a unique \( p_n^* \in (1/2, 1) \) for which this equality holds, we study separately the functions \( K_n(p) \) and \( n(1-p)(2p-1)L_{n-1, \frac{v-1}{2}}(p) \equiv \Lambda_n(p) \). In fact, it will be more useful for our purposes to study the behavior of the functions \( K_n(1-x) \) and \( \Lambda_n(1-x) \) for values \( x \in [0, 1/2] \), as illustrated in Fig. 4 for the case of \( n = 5 \). In the remainder of this section of the proof, we establish that for any \( n \geq 3 \), the functions \( K_n(1-x) \) and \( \Lambda_n(1-x) \) behave qualitatively in the way depicted in Fig. 4 so that we can conclude there exists a unique intersection.
point in \((0, 1/2)\) of these two functions.

\[K_n(1-x)\]

\[\Lambda_n(1-x)\]

\[\hat{x}_n, \hat{x}_n^*, \hat{x}_n\]

\[x\]

\[\frac{1}{2}\]

\[1/2\]

\[1/2\]

\[1/2\]

Figure 4: Uniqueness of critical point of \(H_n\).

The reader can easily verify that \(K_n(1) = \Lambda_n(1) = \Lambda_n(1/2) = 0\). Furthermore, it is straightforward to obtain the first two derivatives of \(K_n\) and their signs:

\[K_n'(p) = -\frac{n\Gamma(n)}{\Gamma^2 \left( \frac{n+1}{2} \right)} p^{\frac{n-1}{2}} \left( 1 - p \right)^{\frac{n-1}{2}} < 0 \text{ for all } p \in [1/2, 1),\]

\[K_n''(p) = \frac{n\Gamma(n)}{2\Gamma^2 \left( \frac{n+1}{2} \right)} p^{\frac{n-3}{2}} \left( 1 - p \right)^{\frac{n-3}{2}} (n - 1) (2p - 1) > 0 \text{ for all } p \in (1/2, 1).\]

This implies that, as illustrated in Fig. 4, the function \(K_n(1-x)\) is strictly increasing and convex for all \(x \in (0, 1/2)\). We can also easily compute the first derivative of \(\Lambda_n\):

\[\Lambda_n'(p) = -\frac{n\Gamma(n)}{2\Gamma^2 \left( \frac{n+1}{2} \right)} p^{\frac{n-3}{2}} \left( 1 - p \right)^{\frac{n-3}{2}} \left( 4 \left( 1 + n \right) p^2 - 2(1+2n)p + (n-1) \right).\]

Note that:

\[\Lambda_n'(p) > 0 \iff p > \hat{p}_n \equiv \frac{2n + \sqrt{4n^2 + 5} + 1}{4(n + 1)}, \quad (8)\]

and that \(\Lambda_n'(p) < 0 \iff p < \hat{p}_n\). Thus, \(\Lambda_n(p)\) has a critical point at \(\hat{p}_n\). We can therefore state that the function \(\Lambda_n(1-x)\) is strictly monotonically increasing in \(x\) for \(x < \hat{x}_n \equiv 1 - \hat{p}_n\), and that it is strictly monotonically decreasing for \(x > \hat{x}_n\). Next, note that:

\[K_n'(p) > \Lambda_n'(p) \iff 0 < p^2 - p + \frac{n - 1}{4(1+n)} \]

\[\iff p > \bar{p}_n \equiv \frac{1}{2} \left( 1 + \sqrt{1 + \frac{2}{n+1}} \right), \quad (9)\]

\[\text{In the following expressions, we use the Gamma function to express more compactly the familiar factorial function: } \Gamma(n) \equiv (n-1)!.\]
and that \( K'_n(p) < \Lambda'_n(p) \iff p < \bar{p}_n. \) At the point \( \bar{p}_n \) both derivatives are equal. By comparing the binomial expressions in (8) and (9) that define \( \bar{p}_n \) and \( \bar{p}_n, \) resp., it is easy to verify that \( \bar{p}_n < \bar{p}_n. \) We can therefore state that while both functions \( K_n(1 - x) \) and \( \Lambda_n(1 - x) \) start at \( x = 0 \) with a value of 0, for values \( x < \bar{x}_n = 1 - \bar{p}_n, \) the function \( \Lambda_n(1 - x) \) increases faster than the function \( K_n(1 - x). \) For any \( x \in (\bar{x}_n, \bar{x}_n), \) the function \( K_n(1 - x) \) grows at increasing rate, while the function \( \Lambda_n(1 - x) \) now grows at a rate below that of \( K_n(1 - x). \) Finally, for all \( x > \bar{x}_n, \) the function \( K_n(1 - x) \) continues to grow at increasing rate, while the function \( \Lambda_n(1 - x) \) smoothly falls back to a value of 0. In other words, for any \( n \geq 3, \) the behavior of the functions \( K_n(1 - x) \) and \( \Lambda_n(1 - x) \) is as depicted in Fig. 4, which proves that there exists a unique intersection point \( \bar{x}_n = 1 - p_n^* \) of these two functions in \((0, 1/2). \) This point of intersection constitutes the unique critical point of the function \( H_n, \) which therefore is the unique global maximum of \( H_n \) in \((1/2, 1). \)

Having argued that for any \( n \geq 3 \) the function \( H_n \) starts and ends with a value of zero, and has a unique interior maximum, we now turn to the question of how this function varies with committee size \( n. \) By the above lemma due to Karotkin and Paroush [2003] (see proof of Prop. 2), we know that \( j_{n+1} > j_n(p) \) for all \( p \in (1/2, 1). \) It therefore follows immediately that \( H_n(p) > H_{n+2}(p) \) for all \( p \in (1/2, 1). \)

**Evaluating condition (1) for \( n = 5. \)** For committee size \( n = 5 \) we can explicitly write \( H_n(p) = (1 - p)^2 (2p - 1) (3p + 6p^2 + 1). \) Now fix a value \( F(\omega) = (0, 1/2). \) This implies the following lower bound on the admissible values of \( p: p \geq p = 1 - F(\omega). \) We now ask if the function \( H_n, \) when evaluated at the lower bound \( p, \) satisfies or violates condition (1). I.e. we check if:

\[
H_n(p) \leq \frac{F(\omega)}{1 - F(\omega)} = \frac{1 - p}{p} \iff p(1 - p) (2p - 1) \leq \frac{1}{1 + 3p(1 + 2p)},
\]

where \( p \in [1/2, 1]. \)

First consider the function \( p(1 - p)(2p - 1), \) and note that both its limits as \( p \to 1/2 \) and \( p \to 1, \) resp., are equal to zero. Furthermore, the derivative of this function w.r.t. \( p \) is \( 6p(1 - p) - 1. \) Thus, for \( p \in [1/2, 1/2 + \sqrt{3}/6, \) the function \( p(1 - p)(2p - 1) \) is strictly increasing, while for \( p \in (1/2 + \sqrt{3}/6, 1] \) it is strictly decreasing. At \( p = 1/2 + \sqrt{3}/6, \) the function has a critical point, which constitutes its global maximum as the second derivative, when evaluated at \( p = 1/2 + \sqrt{3}/6, \) takes the value \(-2\sqrt{3}. \) We can therefore conclude that the function \( p(1 - p)(2p - 1) \) reaches its maximum value of \( \sqrt{3}/18 \) at \( p = 1/2 + \sqrt{3}/6, \) while taking strictly lower values for all other \( p. \)

Next, consider the denominator of the fraction on the right-hand side of (10). The quadratic polynomial \( 1 + 3p(1 + 2p) \) is strictly increasing (its derivative is \( 12p + 3 \)) and takes values in \([4, 10]. \) This implies that the right-hand side fraction in (10) is a strictly decreasing function that reaches its global minimum at \( p = 1, \) where it takes the value 1/10. Now observe that the maximum value of \( \sqrt{3}/18 \) of the function \( p(1 - p)(2p - 1) \) is strictly lower than the minimum value of the fraction \( 1/\left(1 + 3p(1 + 2p)\right) \). We can therefore conclude that condition (1) is violated when \( p \) takes its lowest possible value of \( p, \) regardless of what this value \( p \) is.

Now suppose that \( F(\omega) \leq H_n(p^*_n)/(1 + H_n(p^*_n)), \) where \( p^*_n \) is the unique global maximum point of the function \( H_n, \) at which it takes the value \( H_n(p^*_n). \) For any such \( F(\omega), \) the horizontal line at \( F(\omega))/(1 - F(\omega)) \) will intersect the graph of the function \( H_n \) at two points: \( p^*_n(F(\omega)) \) and \( p^*_n(F(\omega)) \) (with \( p^*_n(F(\omega)) < p^*_n(F(\omega))^2), \) where \( H_n(p^*_n(q)) = F(\omega))/(1 - F(\omega)) \) for
all $i = 1, 2$. Fig. 5 provides an illustration. Also indicated in Fig. 5 is the lower bound $p = 0.705$.

Figure 5: Condition (1) for $n = 5$

$1 - F(\omega_i)$ of admissible values $p$, which illustrates our finding above that $H_5(1 - F(\omega_i)) < F(\omega_i)/(1 - F(\omega_i))$. In the remainder of this section of the proof of Lemma 1, we focus on showing that for all $F(\omega_i) < (0, 1/2)$, it holds that $p = 1 - F(\omega_i) > p_5^*$. To see this, note that:

$$H_5(1 - F(\omega_i)) < \frac{F(\omega_i)}{1 - F(\omega_i)} = H_5(p_5^*(F(\omega_i))),$$

so that we have either $1 - F(\omega_i) < p_5^*(q)$ (due to the fact that $H_5$ is strictly increasing for all $p < p_5^*(F(\omega_i))$, or $1 - F(\omega_i) > p_5^*(F(\omega_i))$ (due to the fact that $H_5$ is strictly decreasing for all $p > p_5^*(F(\omega_i))$).

As the function $H_5$ is a polynomial of order 5, we can neither compute analytically its global maximum $p_5^*$, nor the two points $p_5^1(F(\omega_i))$ and $p_5^2(F(\omega_i))$ for given $F(\omega_i) \leq H_5(p_5^*)/(1 + H_5(p_5^*))$. Therefore, we take an indirect approach to establishing that $1 - F(\omega_i) > p_5^*(F(\omega_i))$ by verifying that $H_5'(1 - F(\omega_i)) < 0$ for all $F(\omega_i) \leq H_5(p_5^*)/(1 + H_5(p_5^*))$. To do this, we recall that the function $H_3$ has a global maximum at $p_3^* = (\sqrt{7} + 2)/6$, with associated maximum value $H_3(p_3^*) = (7\sqrt{7} - 10)/27$. Thus, if $F(\omega_i) = F_3^* \equiv (7\sqrt{7} - 10)/(7\sqrt{7} + 17)$, we have $H_3(p_3^*) = F_3^*/(1 - F_3^*)$. Note that $p_3^*$ is in the admissible range because $1 - F_3^* = (7\sqrt{7} - 17)/2 < p_5^* = (\sqrt{7} + 2)/6$.

We now evaluate the derivative $H_5'$ at the point $1 - F_3^*$, which yields $H_5'(1 - F_3^*) = (6517 630 - 2463 433\sqrt{7})/2$. It is easy to verify analytically that this number is strictly negative (and is approx. equal to $-0.54473$). This shows that the maximum of $H_5$ must occur at some point $p_3^* < 1 - F_3^* < p_5^*$. Thus, $H_3(p_3^*) > H_3(p_5^*) > H_5(p_5^*)$, where the latter inequality follows from the aforementioned Lemma by Karotkin and Paroush (2003). Therefore, we know that:

$$F_3^* = \frac{H_3(p_3^*)}{1 + H_3(p_3^*)} > \frac{H_5(p_5^*)}{1 + H_5(p_5^*)}.$$
which implies that for any \( F(\omega_{k}) \leq H_{5}(p_{5}^{*})/(1 + H_{5}(p_{5}^{*})) \) we have:

\[
p_{5}^{*} < 1 - F_{3}^{*} < 1 - \frac{H_{5}(p_{5}^{*})}{1 + H_{5}(p_{5}^{*})} \leq 1 - F(\omega_{k}).
\]

Due to the fact that \( H_{5}^{'}(p) < 0 \) for all \( p > p_{5}^{*} \), we can finally state that \( H_{5}^{'}(1 - F(\omega_{k})) < 0 \). This is what we needed in order to prove that for any \( F(\omega_{k}) \leq H_{5}(p_{5}^{*})/(1 + H_{5}(p_{5}^{*})) \), condition (1) is violated for all \( p \in (1 - F(\omega_{k}), 1) \). This, finally, establishes that condition (1) is violated for \( n = 5 \) and all \( F(\omega_{k}) \in (0, 1/2) \) and \( p \in (1 - F(\omega_{k}), 1) \) as claimed in Lemma 1. \( \square \)

### 7.3 Proof of Proposition 3

First consider the case \( 1/2 < p < F(\omega_{k}) \). From Prop. 1, it follows that when the expert’s persuasion strategy is \( \Omega_{1} \) so that no information is provided, then \( v(\Omega_{1}, s_{i}) = X \) for all \( i \in I \) and all \( s_{i} \in S \). As every voter votes for the expert’s preferred alternative regardless of his private signal, the expert has no incentive to deviate any other persuasion strategy, making \( (\Omega_{1}, v) \) an equilibrium of the game.

Next consider the case \( 1/2 < F(\omega_{k}) < p \). Suppose the expert uses a binary persuasion strategy as defined above in the proof of Prop. 2. Just as in the case of a low likelihood of preference alignment, the optimal binary persuasion strategy in the case of a high likelihood of preference alignment features a threshold \( \omega^{*} \) defined implicitly by the equation \( F(\omega^{*}) = F(\omega_{k})/p \), which yields the expert an expected payoff of \( y_{m} + (\bar{u}_{m} - y_{m})F(\omega_{k})/p \).

Next, consider the performance of the ternary persuasion strategies defined in the proof of Prop. 2. In the present case of a high likelihood of preference alignment, the expert’s problem of maximizing his expected payoff from the ternary persuasion strategy is identical to the one in the proof of Prop. 2 (see Step 3.1) since \( p > 1/2 > 1 - F(\omega_{k}) \). Therefore, if the payoff difference in (5) above is negative, then the binary persuasion strategy is optimal for the expert, and if it is positive, then the ternary persuasion strategy is optimal. The sign of the payoff difference in (5) is determined by the sign of the expression in the large brackets. Note that this expression is decreasing in the value of \( J_{1}(p) \). By Karotkin and Paroush’s lemma given above in the proof of Prop. 2, it follows that if the expression in large brackets in (5) is negative for \( n = 1 \), then it will also be negative for all odd \( n > 1 \).

It is easy to see that this expression is negative for \( n = 1 \). Note that \( J_{1}(p) = p \). Suppose now that the expression in large brackets in (5) is non-negative for \( n = 1 \):

\[
1 - J_{1}(p) \geq \frac{F(\omega_{k})/(1 - F(\omega_{k}))}{(p/(1 - p)) - 1} \iff 2p \geq 1 + \frac{F(\omega_{k})}{1 - F(\omega_{k})}
\]

It is easy to verify that this latter inequality cannot hold: as \( F(\omega_{k}) > 1/2 \), the expression on the right-hand side of the inequality is at least 2, while the linear function on the left-hand side is always below 2. We can therefore conclude that the difference in the expert’s expected payoffs from the optimal ternary and binary persuasion strategies is negative for all \( n \geq 1 \).

Finally, note that in an equilibrium with a binary persuasion strategy, the expert’s expected payoff in (5) is increasing in \( \omega' \), and the maximum value of \( \omega' \) under this type of persuasion strategy is attained when the condition \( F(\omega') = F(\omega_{k})/p \) holds. What would happen if the expert attempted to use a binary persuasion strategy with \( \omega' \) s.t. \( F(\omega') > F(\omega_{k})/p \)? In this case, we would obtain \( F(\omega_{k}|Y, \Omega_{1}^{2}) < 1/2 \), meaning that voters vote in line with their respective
private signals whenever the message $\Omega_1^2$ is received. Note that such a persuasion strategy corresponds to a ternary strategy with thresholds $\alpha = 0$ and $\beta = \omega'$. It is easy to see that such a ternary persuasion strategy involves a lower expected payoff for the expert than the equilibrium under optimal binary persuasion with $\omega'$ s.t. $F(\omega') = F(\omega_0)/p^{11}$ This proves part (b) of the proposition and concludes the proof.

### 7.4 Proof of Theorem [1]

First consider the case $F(\omega_0) > 1 - p$. If condition [1] fails, then by Prop. [2] 1, the probability of making the correct collective choice is constant and will therefore not rise if additional voters are added: $1 - F(\omega_0)(1 - p)/p$. Now contrast this with the case where condition [1] holds. In this latter case, the probability of making the correct collective choice is $1 - (1 - J_n(p))(1 - F(\omega_0))/p$. Note that the former probability of correct decision-making is higher than the latter iff:

$$1 - J_n(p) > \frac{F(\omega_0)(1 - p)}{1 - F(\omega_0)}.$$  \hspace{1cm} (11)

To see that this inequality holds whenever condition [1] holds, note that we can rewrite condition [1] in Definition 1 as follows: $(1 - J_n(p))(2p - 1) > F(\omega_0)(1 - p)/(1 - F(\omega_0))$. So whenever this inequality holds, it follows immediately that the inequality in (11) is satisfied, which implies that the probability of correct decision-making is higher in situations where condition [1] fails.

Now consider the case of $F(\omega_0) < 1 - p$. By item 2. of Prop. [2] the probability of correct decision-making is $1 - F(\omega_0)(1 - p)/p$ when either $n \geq 5$, or $n = 3$ and $p < \bar{p}_3$, or $n = 1$ and $p < \bar{p}_1$. Thus, for $p < \bar{p}_1 < \bar{p}_3$ committee size is irrelevant, and a single voter generates the same probability of correct decision-making as any larger committee. Next consider $p \in (\bar{p}_1, \bar{p}_3)$ and compare a committee with three or more voters to one with just a single decision-maker, for whom the probability of a correct decision is $1 - F(\omega_0)$. Comparing these two probabilities of correct decision-making, we find that a ‘large’ committee (with $n \geq 3$ voters) is superior to a single decision-maker. Finally, consider $p > \bar{p}_3$ and compare a committee with five or more voters to one with just three voters, for which the probability of a correct decision is $1 - F(\omega_0)(1 - J_3(p))/(1 - p)$. Comparing the probabilities of correct decision-making across these two committee sizes, we find that a ‘large’ committee (with $n \geq 5$ voters) is superior to the three-member one iff:

$$1 - J_3(p) > \frac{(1 - p)^2}{p} \iff \frac{(2p - 1)(1 + p)(1 - p)^2}{p} > 0.$$  \hspace{1cm} (12)

As we assume that $p \in (1/2, 1)$, the inequality on the right-hand side of (12) always holds. Thus, any ‘large’ committee with five or more voters maximizes the probability of correct decision-making when $p \in (\bar{p}_3, 1 - F(\omega_0))$. \hfill \Box

### 7.5 Proof of Theorem [2]

First consider the case where $F(\omega_0) > 1 - p$, and suppose that condition [1] fails. Rewriting condition [1], it is easy to verify that this is the case iff $F(\omega_0) \geq (1 - J_n(p))(2p - 1)/(p -

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11 There is only one other possible class of binary persuasion strategies $\Omega^2$, namely the one with $\Omega_1^2 = [0, \omega')$ and $\Omega_2^2 = [\omega', 1]$, where $\omega' < \omega_0$ s.t. $v_i(\Omega_1^2, s_i) = X$ and $v_i(\Omega_2^2, s_i) = Y$ for every $s_i \in S$. This class clearly yields a lower expected payoff for the expert than the optimal binary persuasion strategy, and therefore can never be optimal for the expert.
Now recall that by Prop. 1 the probability of a correct decision in the absence of persuasion is $J_n(p)$. With persuasion, Prop. 2.1.a implies that the correct collective decision is made with probability $1 - F(\omega) (1 - p)/p$. Thus, expert persuasion aides information aggregation iff $F(\omega) < G_n(p)$. Observe that these two boundaries on the value of $F(\omega)$ are mutually compatible iff $J_n(p) < (1 - 3p(1 - p))/p(2p - 1)$, which is true for all $p \in (1/2, 1)$.

Now note that $G_1(p) = p$ so that $1 - p < F(\omega) < 1/2 < G_1(p)$ for all $p \in (1/2, 1)$. This implies that persuasion always enhances the likelihood that a single decision-maker takes the correct decision. Next, note that for all $n \geq 3$, the function $G_n$ is continuously differentiable, that $G_n(0) = G_n(1) = 0$, and that $G_n(1/2) = 1/2$. The behavior of $G_n$ is further characterized by the following lemma:

**Lemma 2.** For all $n \geq 7$, the function $G_n(p)$ is strictly decreasing for all $p \in [1/2, 1)$.

The proof of this lemma is in Section 7.6 below, and Fig. 6 provides a graphical illustration of $G_n$ for select $n \geq 7$. Furthermore, Fig. 7 shows that for committee sizes $n = 3, 5$ the functions $G_3$ and $G_5$ have a unique maximum in the interior of the range $(1/2, 1)$ at $p_3$ and $p_5$, resp. Thus, as $F(\omega) < 1/2$, it is obvious that for every $n$ there exists an interval whose boundaries arise from the intersection of the horizontal line at $F(\omega)$ with the graph of $G_n$. For signal precision $p$ within this interval, expert persuasion - despite its bias - enhances the probability of a correct collective decision relative to the benchmark of no persuasion.

![Figure 6: Three illustrations of the function $G_n$ defined in (2)](image)

Now let $F(\omega) > 1 - p$ and suppose that condition 1 holds. In this case, Prop. 2.1.b implies that the probability of a correct collective decision is $1 - (1 - J_n(p))(1 - F(\omega))/p$. It is straightforward to verify that this probability exceeds the probability $J_n(p)$ that voters will make a correct decision in the absence of persuasion.

Finally, consider the case where $p < 1 - F(\omega)$. By Prop. 1 the probability of a correct decision in the absence of persuasion is $1 - F(\omega)$, regardless of the number of voters. In the presence of the expert, Prop. 2.2.a implies that for $n \geq 5$ voters, the probability of a correct collective decision is $1 - F(\omega)/(1 - p)/p$. Thus, expert persuasion always harms information.
aggregation. The same results follows by Prop. 2.2.b for \( n = 3 \) voters and signal precision \( p < \bar{p}_3 \), and for \( n = 1 \) voters and signal precision \( p < \bar{p}_1 \). If, instead, \( n = 3 \) and \( p > \bar{p}_3 \), or \( n = 1 \) and \( p > \bar{p}_1 \), then Prop. 2.2.b implies that the probability of a correct collective decision is \( 1 - F(\omega_v)(1 - J_n(p))/ (1 - p) \). It is straightforward to verify that this probability exceeds the probability \( J_n(p) \) with which voters make a correct decision in the absence of persuasion. \( \Box \)

### 7.6 Proof of Lemma 2

Recall the function \( G_n \) which was introduced in (2) in the main text. Using the notation introduced above in the proof of Lemma 1, we can write: \( G_n(p) = (p/(1 - p))K_n(p) \). Furthermore, recalling the expression for \( \hat{\Lambda}_n(p) \) given there, we obtain:

\[
G_n'(p) = \frac{1}{(1 - p)^2}K_n(p) + \frac{p}{1 - p}K_n'(p)
\]

\[
= \frac{1}{(1 - p)^2}K_n(p) - \frac{np}{1 - p}L_{n-1,(n-1)/2}(p).
\]

Observe that we have \( G_n'(p) < 0 \iff K_n(p) < np(1 - p)L_{n-1,(n-1)/2}(p) = \hat{\Lambda}_n(p) \). To see that this inequality holds for all \( p \in [1/2, 1] \) and any \( n \geq 7 \), we study the behavior of the two functions \( K_n \) and \( \hat{\Lambda}_n \). In the proof of Lemma 1, we have already obtained the first two derivatives of \( K_n \), and so we know that it is a strictly decreasing and convex function that starts at \( p = 1/2 \) with a value of \( K_n(1/2) = 1/2 \), and ends at \( p = 1 \) with a value of \( K_n(1) = 0 \). Next consider the function \( \hat{\Lambda}_n \). Its first derivative is:

\[
\hat{\Lambda}_n'(p) = -\frac{n(n + 1)\Gamma(n)}{2\Gamma^2\left(\frac{n+1}{2}\right)}(2p - 1)p^{\frac{n-1}{2}}(1 - p)^{\frac{n+1}{2}},
\]

which is negative for all \( p \in (1/2, 1) \), with \( \hat{\Lambda}_n'(1/2) = 0 \). The second derivative is:

\[
\hat{\Lambda}_n''(p) = \frac{n^2(n + 1)\Gamma(n)}{\Gamma^2\left(\frac{n+1}{2}\right)}(p(1 - p))^\frac{n-3}{2}\left(p^2 - p + \frac{n-1}{4n}\right),
\]
which is negative if \( p < (\sqrt{n} + 1)/2\sqrt{n} \). Thus, \( \Lambda_n \) is decreasing and concave for \( p < (\sqrt{n} + 1)/2\sqrt{n} \), and decreasing and convex for \( p > (\sqrt{n} + 1)/2\sqrt{n} \). Furthermore:

\[
\hat{\Lambda}_n(1/2) = \frac{n\Gamma(n)}{2^{n+1}1^{\frac{n+1}{2}}} \quad \text{and} \quad \hat{\Lambda}_n(1) = 0.
\]

Note that for \( n = 7 \), we have \( \hat{\Lambda}_7(1/2) = 35/64 \approx 0.54688 \). Also, an increase in committee size raises the value of \( \Lambda_n(1/2) \) as \( \hat{\Lambda}_{n+2}(1/2) - \hat{\Lambda}_n(1/2) = \hat{\Lambda}_n(1/2)/(n + 1) \). We can therefore conclude that for any odd \( n \geq 7 \), the value \( \Lambda_n(1/2) \) strictly exceeds \( 1/2 \). This means that at \( p = 1/2 \), the function \( K_n \) starts at a lower value than the function \( \hat{\Lambda}_n \).

To complete the proof, we now show by contradiction that \( K_n(p) < \hat{\Lambda}_n(p) \) for all \( p \in [1/2, 1) \). For this purpose, we assume that the two functions intersect at some point \( p_n^* \) (given that both functions are strictly decreasing, there can be at most one intersection point), as illustrated in Fig. 8. As the figure illustrates, this implies that there exist two points \( \tilde{p}_n \) and \( \hat{p}_n \) (with \( \tilde{p}_n < p_n^* < \hat{p}_n \)) at which the first derivatives of the two functions are identical, so that for any \( p \in (\tilde{p}_n, \hat{p}_n) \) the function \( \Lambda_n \) falls more steeply than \( K_n \). However, a comparison of the two derivatives shows:

\[
\hat{\Lambda}_n'(p) < K_n'(p) \iff p > \frac{1}{2} + \frac{1}{n+1}.
\]

Thus, there is only the single point \( 1/2 + 1/(n + 1) \) at which the two derivatives are identical. Note that there cannot be an intersection point to the left of this value, as \( K_n \) starts below \( \Lambda_n \) and initially declines more sharply than \( \hat{\Lambda}_n \). Furthermore, there cannot be an intersection to the right of \( 1/2 + 1/(n + 1) \) as \( \hat{\Lambda}_n \) falls more steeply than \( K_n \) and would have to end up taking a negative value at \( p = 1 \), in contradiction to the fact that \( \hat{\Lambda}_n(1) = 0 \). We can therefore conclude that there exists no intersection point of \( K_n \) and \( \hat{\Lambda}_n \).

\[
\text{Figure 8: Monotonicity of } G_n \text{ for } n \geq 7.
\]

7.7 Proof of Theorem 3

By Prop. 3 we know that all voters use the same signal-independent voting strategy. This results in a constant probability of making the correct choice (namely \( F(\omega_v) \)) in the scenario
where \( p < F(\omega) \), and \( 1 - F(\omega)(1 - p) / p \) if \( p > F(\omega) \) regardless of the size of the electorate.

\[ \blacksquare \]

### 7.8 Proof of Theorem 4

First consider the case \( p < F(\omega) \). Item 1 follows immediately from Prop. 1 and Prop. 3. Next, consider the case \( p > F(\omega) \). From Prop. it follows that, in the absence of an expert, voters cast their votes in line with their respective private signals. As a result, they choose the correct alternative with probability \( J_n(p) \), which converges to 1 as the size of the electorate gets large. By Prop. 2 we know that, in the presence of an expert, the equilibrium \((\Omega^2, v)\) induces the following probability of making the correct decision: \( 1 - F(\omega)(1 - p) / p \). Thus, expert persuasion harms information aggregation iff \( F(\omega) > G_n(p) \), where the function \( G_n(p) \) is defined in (2). As we have \( F(\omega) > 1/2 \), it is immediate that the condition \( F(\omega) > G_n(p) \) holds for all \( p \in (1/2, 1) \) if \( n \geq 7 \). This establishes that expert persuasion impedes information aggregation in electorates with seven or more voters (item 2.a). In the case of a single decision-maker (item 2.c), the result is obvious as \( G_1(p) = p \) and therefore \( F(\omega) < G_1(p) = p \) for all \( p \) under consideration.

We now turn to the proof of item 2.b of Theorem 4. The idea behind the proof can be readily understood by looking at Fig. 7, which shows that the functions \( G_3 \) and \( G_5 \) each have a unique maximum in the interior of the range \((1/2, 1)\). Thus, if \( F(\omega) \) exceeds this maximum, then expert persuasion harms information aggregation. If, instead, \( F(\omega) \) is below this maximum, then there exists an interval whose boundaries arise from the intersection of the horizontal line at \( F(\omega) \) with the graph of \( G_n \) \((n = 3, 5)\). For signal precision \( p \) within this interval, expert persuasion - despite its bias - enhances the probability of a correct collective decision relative to the benchmark of no persuasion.

To show this formally, we start by considering the case of \( n = 3 \) voters. The left-hand panel of Fig. 7 shows the graph of the function \( G_3(p) = p + p^2 - 2p^3 \). Note that \( G'_3(1/2) = 1/2 \), and that the first-order condition \( G'_3(p) = -6p^2 + 2p + 1 = 0 \) yields a unique critical point \( p_3 = (1 + \sqrt{7})/6 \approx 0.60763 \). As \( G''_3(p) < 0 \) for all \( p \in [1/2, 1] \), the function \( G_3 \) is strictly concave everywhere in this range, and so the point \( p_3 \) is the unique global maximum of \( G_3 \). The corresponding functional value is \( G_3(p_3) = (7\sqrt{7} + 10)/54 \), which we label as \( q_3 \). Note that since \( q_3 \in (1/2, 1) \), it follows immediately that in settings where \( F(\omega) > q_3 \), expert persuasion is detrimental to information aggregation. Now suppose instead that \( 1/2 < F(\omega) < q_3 \). As \( G_3 \) is strictly concave and \( G'_3(p) < 1 \) for all \( p \in [1/2, 1] \), the equation \( p + p^2 - 2p^3 = F(\omega) \) has two real-valued solutions \( \tilde{p}_3^F \) and \( \tilde{p}_3^F \) with \( F(\omega) < \tilde{p}_3^F < \tilde{p}_3^F < 1 \). For all \( p \in (\tilde{p}_3^F, \tilde{p}_3^F) \) we have \( G_3(p) > F(\omega) \), and for all \( p \notin [\tilde{p}_3^F, \tilde{p}_3^F] \) we have \( G_3(p) < F(\omega) \).

Next, consider the case of \( n = 5 \) voters. The right-hand panel of Fig. 7 shows the graph of the function \( G_5(p) = p(1 - p)^2(6p^2 + 3p + 1) \). Note that \( G'_5(1/2) = 1/8 \), and that the first-order condition \( G'_5(p) = (1 - p)(1 + 3p + 6p^2 - 30p^3) = 0 \) yields a unique critical point \( p_5 = (2 + (548 - 30\sqrt{290})^{1/3} + (548 + 30\sqrt{290})^{1/3})/30 \approx 0.51761 \). Note that whilst it is tedious to compute the roots of the cubic function \( G'_5(p) = 2((1 + 3p) - (2p^2(27 - 30p))) \) in order to ascertain the curvature of \( G_5 \), it is easy to verify that \( G''_5(p) < 0 \) for all \( p \in [1/2, 3/5] \). To see this, note that both the functions \( 1 + 3p \) and \( 2p^2(27 - 30p) \) are strictly increasing for \( p \in [1/2, 3/5] \), and that the maximum value of the former function (i.e. 2.8 when \( p = 3/5 \)) is strictly lower than the minimum value of the latter function (i.e. 6 when \( p = 1/2 \)). Therefore, \( G_5 \) is strictly concave for all \( p \in [1/2, 3/5] \). From this, we can conclude that the point \( p_5 \) is the unique global maximum of \( G_5 \).

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Rather than incur the tedium of verifying directly that the corresponding functional value \( q_5 \equiv G_5(p_5) \) lies strictly in the range \( (1/2, 1) \), we simply argue that for any \( n \) it holds that \( G_n(p) < 1 \) for all \( p \in [1/2, 2/3] \). The condition \( G_n(p) < 1 \) is equivalent to \( (1 - J_n(p)) < (1 - p)/p \). Recall that \( J_n(1/2) = 1/2 \) and \( J_n(1) = 1 \). Thus, \( (1 - J_n(p)) \in [0, 1/2] \) for all \( p \in [1/2, 1] \).

Now note that \( (1 - p)/p \) strictly exceeds \( 1/2 \) for all \( p \in [1/2, 2/3] \), which immediately implies that \( G_5(p_5) < 1 \). We can therefore state that in settings where \( F(\omega_v) > q_5 \), expert persuasion is detrimental to information aggregation.

Finally, the fact that \( G_5 \) is strictly concave for all \( p \in [1/2, 3/5] \), that \( G_5(p) < 1 \) for all \( p \in [1/2, 1] \), and that \( G_5(3/5) = 1488/3125 \approx 0.47616 < 1/2 \) implies immediately that in settings where \( 1/2 < F(\omega_v) < q_5 \), the equation \( G_5(p) = F(\omega_v) \) has two real-valued solutions \( \hat{p}_F^F \) and \( \tilde{p}_F^F \) with \( F(\omega_v) < \hat{p}_F^F < \tilde{p}_F^F < 1 \). For all \( p \in (\hat{p}_F^F, \tilde{p}_F^F) \) we have \( G_5(p) > F(\omega_v) \), and for all \( p \notin (\hat{p}_F^F, \tilde{p}_F^F) \) we have \( G_5(p) < F(\omega_v) \).

### References


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