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Abstract

We investigate the effects of higher education on the evolution of income inequality. In so doing we propose a novel overlapping generations model with three social classes: the rich, the middle class, and the poor. We show that there is an initial phase in which no social class invests in higher education of their children such that income inequality is driven by wealth accumulation and bequests. Once a certain income threshold is surpassed, the rich start to invest in higher education of their children, which partially crowds out bequests and thereby reduces inheritance flows and income inequality in the short run. The better educated children of the rich, however, enjoy higher wage incomes later on in their lives such that income inequality starts to rise again, this time mainly driven by inequality in educational attainment. Over time, the middle class and potentially also the poor start to invest in higher education. As the economy proceeds toward a balanced growth path, educational differences between social groups and, thus, income inequality decline again. We argue that (1) the proposed mechanism provides a candidate explanation for the observed U-shaped evolution of income inequality and inheritance flows, as well as the differential investments in higher education by richer and poorer households, (2) the currently observed increase in income inequality is likely to level off in the future.

\textbf{JEL classification:} I23, I24, I25, O11, O41.

\textbf{Keywords:} Higher education, inequality, bequests, growth regime switch, middle income trap.
The main force pushing toward reduction in inequality has always been the diffusion of knowledge and the diffusion of education.
(Thomas Piketty)

1 Introduction

The most salient features of the evolution of income inequality and inheritance flows in industrialized countries throughout the last century are i) relatively high income inequality and high inheritance flows as a share of total income at the beginning of the 20th Century; ii) a substantial drop of both variables after World War II; iii) a relatively constant level throughout the 1950s, 1960s, and 1970s, i.e., for around one generation; and vi) strongly rising income inequality and inheritance flows thereafter (Atkinson et al., 2011; Piketty and Saez, 2003; Piketty, 2014; Piketty and Zucman, 2015; Alvaredo et al., 2017a). This pattern is displayed for the United States in Figure 1 by means of the evolution of the top 10% income share (Alvaredo et al., 2017b) and in Figure 2 as the flow of inheritances in terms of GDP in each year (Alvaredo et al., 2017a).1 Many explanations for the U-shaped evolution of income inequality and inheritances have been proposed: the disruptions of World War II had a negative impact on wealth, while substantial inheritance taxes and high marginal income tax rates in the period 1950-1970 exacerbated the drop in inheritance flows and income inequality after World War II. As far as the increase in inheritance flows and income inequality from the late 1980s onwards is concerned, potential explanations include i) decreases in marginal income tax rates and inheritance taxes (particularly in the United States and the United Kingdom), ii) skill-biased technological change, which disproportionately benefited the well-educated, iii) decreases in population growth, which increased the concentration of bequests, iv) globalization, which put additional pressure on low incomes because low-skilled labor-intensive production has often been outsourced to low-wage countries, v) that wealthier agents invest a larger fraction of their wealth in assets with a higher rate of return, and vi) that contractionary monetary policy has the potential to raise income inequality if it raises unemployment predominantly among low-income workers (see Acemoglu, 2002; Goldin and Katz, 2009; Acemoglu and Autor, 2012; Elsby et al., 2013; Piketty, 2014; Coibion et al., 2017; Kasa and Lei, 2018, for different arguments).

We propose a mechanism that is complementary to the above-mentioned forces and which provides an additional explanation for the joint U-shaped evolution of income inequality and inheritance flows throughout the 20th Century. The central driving force is the increase in costly higher education after World War II, where the most wealthy groups were the first to invest massively in the college education of their children. This slows down intergenerational wealth accumulation among the rich because household’s resources are

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1The U-shaped evolution of the top 10% income share and of inheritances is also documented for most other rich countries in Piketty (2014), Piketty and Zucman (2015), and Alvaredo et al. (2017a).
re-allocated away from bequests (wealth accumulation) toward higher education (human capital accumulation). The fact that a certain amount of time elapses between education and labor market entry implies that inheritance flows and income inequality are reduced in the short run. However, the increase in the skills of the children of the rich raises their wage income once that they enter the labor market and leads to a steeper age-income profile later on in their lives. Consequently, after around one generation, income inequality rises again. While income inequality is mainly driven by differential wealth accumulation
before the rich start to invest massively in college education of their children, income inequality afterwards originates in differences in wage incomes due to differences in educational attainment. This pattern is consistent with the data on income inequality and inheritances and with the established view on which factors have been the drivers of the evolution of income inequality at different points in time (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014).

The increase in the importance of higher education is also highly visible in the data and the timing coincides with the timing that our model implies: While in 1940 only 4.6% of the population above the age of 25 had a college degree, 32% did so in the year 2015 (United States Census Bureau, 2015). As far as the joint evolution of income and higher educational attainment between the different income groups is concerned, the predicted pattern of our model is consistent with the data as reported by The Pell Institute (2015). They show that in 1970, 40% of the dependent members of families in the top income quartile had a bachelor’s degree by the age of 24. This number almost doubled to 77% in 2013. In the second highest income quartile, 15% of dependent family members had a bachelor’s degree in 1970, which more than doubled to 34% in 2013. In the lowest income quartile, however, only 6% of the dependent family members had a bachelor’s degree in 1970 and the number barely rose over time to 9% in 2013. These numbers are summarized in Table 1 and the increase of educational attainment in terms of a bachelor’s degree depending on family income is illustrated in Figure 3.

Table 1: Bachelor’s degree attainment by age 24 depending on the family’s income quartile (Source: The Pell Institute, 2015)

<table>
<thead>
<tr>
<th></th>
<th>1970</th>
<th>2013</th>
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</thead>
<tbody>
<tr>
<td>Top</td>
<td>40%</td>
<td>77%</td>
</tr>
<tr>
<td>Third</td>
<td>15%</td>
<td>34%</td>
</tr>
<tr>
<td>Second</td>
<td>11%</td>
<td>17%</td>
</tr>
<tr>
<td>Bottom</td>
<td>6%</td>
<td>9%</td>
</tr>
</tbody>
</table>

As far as the literature on the connection between family income and educational attainment of children is concerned, empirical evidence supports a positive and independent impact of family income on children’s educational attainment and the effect even became stronger over the past decades. Blanden and Gregg (2004) document that the intergenerational transmission of incomes has increased for children born in 1970 as compared to those born in 1958. Based on evidence from the United Kingdom, Blanden and Machin (2004) argue that low family income has an independent effect on children’s outcomes after controlling for several dimensions of family background and child ability. Evidence for the United States also suggests that higher family income has a positive effect on educational attainment (Clark-Kaufman et al., 2003). Blanden et al. (2007) are able to account for over 80% of the rise in the intergenerational correlation coefficient by the stronger relationship between family income and education. Galindo-Rueda and Vignoles (2005)
Figure 3: Illustration of the evolution of the bachelor’s degree attainment by age 24 depending on the family’s income quartile (Source: The Pell Institute, 2015).

demonstrate that the role of cognitive test scores in determining educational attainment has declined between the 1958 and 1970 cohorts. This suggests that the role of parental income in children’s educational outcomes has increased. The growing role of family background for the access to higher education has also been documented in a number of other studies, for example, in Glennester (2002). Furthermore, Acemoglu and Pischke (2001) exploit changes in the distribution of family income that have taken place over the past 30 years to estimate the effect of parental resources on college education. Their estimates suggest large effects of family income on enrollment with a 10 percent increase in family income being associated with a 1.4 percentage point increase in the probability of children attending a four-year college. Finally, given the observed increases in education inequality, our model predicts that lifetime income inequality should have been on the rise over the recent decades, which is consistent with recent empirical evidence for Germany and the United States (Bönke et al., 2015; Guvenen et al., 2017).

We conceptualize our mechanism by assuming that there are three social classes, the rich, the middle class, and the poor. Initially, income inequality is driven by the accumu-
mulation of wealth because the rich are able to save more than the other social classes. The reason is that there are subsistence consumption needs that have to be fulfilled, which implies that poorer households spend a larger fraction of their income on consumption and a smaller fraction on saving. During this stage of economic development, which we call the neoclassical regime, investments in college education are not yet widespread because higher education for the children is seen as a luxury good from the perspective of households. However, at some point in time an income threshold is surpassed, above which the rich start to invest in college education of their children. In the aftermath of the regime switch to the high-skill regime, income inequality and inheritance flows decline. The reason is that high-skilled human capital accumulation of the children is costly for the parents not only in terms of tuition fees and living expenses but also in terms of the opportunity cost of foregone household income. Consequently, the rich reduce their savings rate and therefore the intergenerational accumulation of wealth. This phase lasts for around one generation, i.e., inheritance flows and therefore income inequality are reduced for around 30 years. Afterwards, income inequality rises again because the better educated children of the rich earn higher labor incomes, are thus able to bequeath more wealth to their own offspring, and are also able to invest more in their offspring’s education than the less well educated children of the middle class and the poor. Subsequently, incomes of the middle class and potentially also incomes of the poor surpass the threshold levels above which investments in their children’s higher education becomes a utility-maximizing strategy.

While our mechanism provides one additional complementary explanation for the observed U-shaped evolution of income inequality and inheritance flows, our framework does not imply that top income shares approach 1 in the long-run (cf. Piketty, 2014). If all social classes manage to transit to the high-skill regime, the growth rates of human capital converge among the different classes in the long run, such that income inequality declines again and the top income share settles at a level substantially below 1. Crucial for the emergence of declining income inequality, however, is that all social classes switch to the high-skill regime. As we show, this is by no means guaranteed because the poor face the risk of being disconnected from the growth process. In this case, targeted policy interventions such as stipends and tuition fee waivers for the poor would be an effective instrument to reduce inequality.

The following articles are closely related to our research conceptually and/or content-wise. Galor and Zeira (1993) show that initial wealth disparities matter for the long-run distribution of incomes if capital markets are imperfect and there are indivisibilities in education investments. In so doing they explain the emergence of a Kuznets curve, i.e., increasing inequality first and then decreasing inequality over the course of economic development. Galor and Moav (2004) propose a model of development in which physical capital as the main driving force of economic growth in earlier stages of the Industrial Revolution...
is replaced by human capital accumulation as the engine of growth in later phases. They show that inequality has a positive effect on growth in the physical capital based growth regime, while inequality has a negative effect on economic growth in the human capital based growth regime. Ray (2006) analyzes the endogenous emergence of inequality in a model of human capital accumulation with credit constraints. He shows that inequality is a necessary endogenous outcome in the sense that even for an initially equal distribution, some individuals will end up with a low level of education in low-paying professions and some will end up with a high level of education in high-paying professions. Galor and Moav (2006) analyze the historical demise of the capitalist-worker class structure in an overlapping generations model with intergenerational transfers and public education. They show that the increasing importance of human capital in production after the Industrial Revolution led to a cooperation between capitalists and workers to the extent that capitalists started to support public education for the masses. The central reason for doing so is the complementarity between skills and physical capital in the sense that a better educated workforce raises the rate of return on physical capital.

In a related paper, Galor et al. (2009) show how inequality in landownership adversely affected the emergence of institutions that promoted basic education during the Industrial Revolution. The reason is that, due to a lower degree of complementarity between human capital and land (as compared to human capital and physical capital), landowners are interested in a reduction of the mobility of rural workers who might otherwise move to cities and work in factories. Consequently, they oppose mass education. In countries, where landownership is very unequal, the theory of Galor et al. (2009) implies that the adverse effect of education on landowners is very large such that they have a stronger incentive to oppose mass education. Altogether, and consistent with the empirical evidence, countries with a higher inequality in landownership got surpassed by countries with a lower degree of inequality in landownership during the process of industrialization. These frameworks successfully explain the emergence of publicly funded basic education for the masses and the connection between education and inequality from a historical perspective.

In contrast to these papers, we are concerned with the connection between higher education in terms of a college degree, which gained importance in the second half of the 20th Century, and the patterns of income inequality after World War II. We therefore focus on the next phase of increasing inequality throughout the period 1980-2010 and show that, under certain circumstances, inequality might decline again in the future. Related to the recent increase in inequality, Böhm et al. (2015) develop a very interesting directed technical change model to analyze the extent to which low-skilled workers benefit from trickle-down effects of public education policies that are targeted toward the high-skilled workers. They show that such policies reduce the wages of low-skilled workers and raise inequality in the short run, while they are beneficial to low-skilled workers in the

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4Galor and Tsiddon (1997) focus on a different question and investigate the interactions between technological inventions and inequality. They show that periods with a lot of inventions go hand in hand with high inequality but also with high intergenerational mobility, while periods with a lot of innovations are associated with lower inequality but also with lower intergenerational mobility.
long run. While Böhm et al. (2015) consider an exogenous distribution of skilled and unskilled households, in our setting the switch from the neoclassical regime to investments in higher education and, thus, the distribution of skills is endogenous. Moreover, this regime switch is the central mechanism explaining the U-shaped evolution of inequality over time. Finally, by construction, Böhm et al. (2015) focus on the episode of increasing inequality, while our paper addresses the whole U-shaped pattern in the joint evolution of income inequality and inheritances. For the sake of clarity, however, we switch off the channel of skill-biased technical change because it would merely reinforce the increase in income inequality after the regime switch that we find and therefore lead to a more pronounced U-shaped pattern. At the same time, introducing skill-biased technical change into our framework with endogenous education would complicate our model beyond the point of analytical tractability. As such, the mechanism that we describe below should be seen as complementary to the explanations advanced by Goldin and Katz (2009); Acemoglu and Autor (2012); Böhm et al. (2015).

The paper is organized as follows: Section 2 describes the model, Section 3 analyzes the dynamics, Section 4 illustrates the model solution by means of a numerical example, and Section 5 concludes and describes potential policy measures to reduce income inequality and its negative economic effects.

## 2 The model

We now turn to the discussion of our theoretical framework with the aim to isolate the economic channels that have been described above and which may provide an additional candidate explanation for the observed empirical regularities. In so doing, we are able to present a clear analytical and graphical analysis of the channels that are operative in this setting. A comprehensive assessment of the entire inequality dynamics that includes other potential explanations such as World War II, changes in taxation, globalization, and skill-biased technological change would require a large-scale computable overlapping generations model. This is beyond the scope of the present paper in which we aim to isolate some of the driving factors of income inequality dynamics analytically.

As far as the terminology is concerned, we follow the standard practice and refer to wage inequality as the part of inequality that is driven by wage income differentials as determined by the economy-wide wage rate per unit of effective labor on the one hand and by the differential human capital levels of individuals on the other hand. By contrast, income inequality also includes inequality due to different amounts of interest income as derived from differential wealth holdings, i.e., by wealth inequality. Note in this context, that the only savings vehicle is physical capital. Since human capital is embodied and used to earn wage income, we do not lump physical capital and human capital together but keep the natural distinction that human capital differences drive inequality in wage

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5For the conceptual complications arising from a directed technical change setting with endogenous human capital formation, see Schaefer (2014).
income, while differences in the holdings of physical capital drive inequality in terms of interest income. Finally, we refer to wealth that is not consumed at old age but passed over to the next generation as bequests.

Consider an economy populated by individuals whose life cycle consists of three distinct phases: childhood, adulthood, and retirement. Time is discrete, indexed by \( t \), and ranges from 0 to \( \infty \). A continuum of small firms \( i \in (0, 1) \) produce aggregate output \( Y_t \) with the representative firm having access to a constant returns to scale production function of the Cobb-Douglas type

\[
Y_t = AK_t^\alpha H_{Y,t}^{1-\alpha},
\]

where \( K_t \) is aggregate physical capital, \( H_{Y,t} \) is aggregate human capital, \( A > 0 \) refers to the total factor productivity (TFP), and \( \alpha \in (0, 1) \) denotes the elasticity of output with respect to physical capital. Perfect competition implies that the wage rate per unit of effective labor, \( w_t \), and the capital rental rate, \( R_t \), are given by

\[
w_t = (1 - \alpha)Ak_t^\alpha, \quad \text{and} \quad R_t = \alpha Ak_t^{\alpha - 1},
\]

where \( k_t = K_t/H_{Y,t} \) denotes physical capital per unit of effective labor. We follow the standard practice in overlapping generations models by assuming that physical capital fully depreciates over the course of one generation, i.e., the interest rate is given by \( r_t = R_t - 1 \). This is a reasonable assumption given that a generation covers a time span of 30 years. Moreover, the interest rate is determined at the integrated world capital market such that \( R_t = R \) is constant and the equilibrium capital stock per unit of effective labor is given by

\[
k_t \equiv k = \left( \frac{\alpha A}{R} \right)^{\frac{1}{1-\alpha}}.
\]

This in turn implies that the wage rate per unit of effective labor is constant and given by

\[
w_t \equiv w = (1 - \alpha)A \left( \frac{\alpha A}{R} \right)^{\frac{\alpha}{1-\alpha}}.
\]

Each member of generation \( t \) belongs to one out of three social classes that are indexed by \( j = r, m, p \): the rich are referred to by \( r \), the middle class by \( m \), and the poor by \( p \). Adults are endowed with one unit of time and \( h_{j,t} \) units of human capital. They work, consume the amount \( c_{j,t} \), give birth to \( n > 0 \) children, and potentially provide each child with \( e_{j,t} \geq 0 \) units of higher education. Moreover, adults save the amount \( s_{j,t} \) to cover their own consumption needs in their last period of life, \( c_{j,t+1} \), and to bequeath the amount \( nb_{j,t+1} \) to their offspring.

We describe the lifetime utility of agent \( j \), who was born in \( t - 1 \), by using the utility
function

\[ u_{j,t} = \log(c_{j,t} - \bar{c}) + \gamma \log(h_{j,t+1}) + \beta \left[ \log(c_{j,t+1}) + \theta \log(b_{j,t+1}) \right], \quad (5) \]

where \( \bar{c} > 0 \) is the subsistence level of consumption, \( \beta \in (0,1) \) represents the discount factor, \( h_{j,t+1} \) refers to the level of human capital per child, \( \gamma > 0 \) denotes the utility weight that parents attach to the human capital level of their children, and \( \theta \in (0,1) \) represents the utility weight of the bequests to each child. The fact that bequests and the human capital of children show up in the utility function is motivated by the “warm glow” motive of giving (Andreoni, 1989). This is a short-cut formulation that leads to similar results as in case of a dynastic utility function in which the consumption level of children is an argument of the parental utility function. The reason for the similarity is that the consumption level of the children is determined by their income, which is in turn determined by their education and their inheritance. As usual, the logarithmic specification allows obtaining analytical results. Note that our utility function is less restrictive than the standard specification because, due to the presence of \( \bar{c} > 0 \), it allows the savings rate to positively depend on the income level. This is consistent with the available empirical evidence (see Steger, 2000; Dynan et al., 2004; Strulik, 2010, for the generality of growth models with subsistence consumption and for empirical evidence on the relation between income and the savings rate that is in line with the assumption of subsistence needs).

Population growth is treated as exogenous because we do not aim to analyze the quality-quantity trade-off, the demographic transition, and the take-off to long-run economic growth.\(^6\) Instead, we assume that the economy already escaped the Malthusian stagnation in the past and is now industrialized. This means that the transition from the Malthusian regime to a Solovian regime has already occurred (Hansen and Prescott, 2002; Galor and Moav, 2004). What we are concerned with is the next transition from a physical capital based economy to an economy in which economic growth is primarily driven by human capital accumulation. We are thus interested in the transition from neoclassical to endogenous growth (Lucas, 1988; Romer, 1990).

Higher education is financed by parents and provided by the education sector that employs lecturers who are members of the middle class. The alternative assumption that lecturers belong to the rich is difficult to motivate from an empirical point of view. Furthermore, it has the drawback that the regime switch becomes more difficult for the middle class such that they could be disconnected from the growth process. Since we observe a takeoff of education for the middle class, our assumption implies a better fit between the theoretical implications and the observed educational trajectories. Altogether,

human capital evolves according to

\begin{align}
  h_{j,t+1}^n &= \bar{e}_j, & \text{if } e_{j,t} = 0, \\
  h_{j,t+1}^h &= (\bar{e}_j + e_{j,t})\kappa h_{m,t}^l & \text{if } e_{j,t} > 0,
\end{align}

where \( \kappa \) denotes the productivity of the education sector and \( \bar{e}_j \) represents the baseline level of education that each child of each group \( j \) obtains by observing her parents and peers (see, for example, Strulik et al., 2013) and/or because it is provided costlessly by the community in the form of public schooling.\(^7\) We treat \( \bar{e}_j \) as exogenous for the sake of analytical clarity. Public schooling could, however, easily be endogenized via income taxes, such that the tax-financed level of \( \bar{e}_j \) would depend on the structural parameters of our model (see de la Croix and Doepke, 2004, for more details).\(^8\)

If \( e_{j,t} > 0 \), the corresponding social class is in the high-skill regime as indicated by the superscript \( l = h \) and otherwise it is in the neoclassical regime as indicated by the superscript \( l = n \). To reduce notational clutter, we omit the superscript whenever this is possible. It is straightforward to assume that \( \bar{e}_r \geq \bar{e}_m \geq \bar{e}_p \), i.e., that the rich acquire no less baseline education than the middle class, which in turn acquire no less baseline education than the poor. While the strict inequality would follow in a straightforward manner from the fact that the peer group that children observe tends to consist of members of the same social class, it is important to note that our parameter restriction explicitly allows for the equality of baseline education levels among the different social classes, i.e., \( \bar{e}_r = \bar{e}_m = \bar{e}_p \). Furthermore, our main qualitative result regarding the U-shaped evolution of income inequality due to a differential regime switch of the different social classes holds irrespective of whether or not the assumption is fulfilled with strict equalities or strict inequalities.

\(^7\)Note that our formulation is a special case of de la Croix and Doepke (2003, 2004) and Glomm and Ravikumar (1992): \( h_{j,t+1} = (\bar{e}_j + e_{j,t})\eta h_{m,t}^l \), where \( \bar{h}_{j,t}^l \) denotes average human capital, \( \nu \) the intergenerational transmission of human capital, and \( \eta \) the impact of education on human capital. From a conceptual point of view, the presence of average human capital can also be interpreted as a spillover effect which is sizable according to recent findings (see Choi, 2011). We set \( \eta = 1 \) and \( \nu = 0 \) for notational convenience without affecting the generality of our results. Moreover, we allow the productivity of the education sector, \( \kappa \), to differ from 1.

\(^8\)Note, at this stage, that the outcome of the parental optimization problem does not depend on whether parents derive utility directly from investing in education or from the human capital level that the child attains. The reason is that the human capital level of the children \( h_{j,t+1} \) is the product of the parental choice of education and the productivity of the higher education sector. Since the latter is from the perspective of optimizing parents taken an given, the result of the optimization problem is the same, irrespective of whether the education choice of parents or the human capital level of their children enters the utility function. Analogously, for the optimal bequest of parents, it is irrelevant whether bequests enter the utility function or the capital stock of children. The reason is that the interest rate is treated as exogenous in the optimization problem of parents.
The budget constraints for adults and retirees are given by

\[
I^l_{j,t} = \begin{cases} 
  c^h_{j,t} + s^h_{j,t} + wh^{l}_{m,t}ne^h_{j,t}, & \text{if } e_{j,t} > 0, \\
  c^n_{j,t} + s^n_{j,t}, & \text{if } e_{j,t} = 0,
\end{cases}
\]

(8)

\[
s^l_{j,t} = \frac{c^l_{j,t+1} + nb^l_{j,t+1}}{R},
\]

(9)

with income being denoted by \(I^l_{j,t} = wh^l_{j,t}(1 - zn) + Rb^l_{j,t}\), where \(z \in (0, 1)\) is the time share necessary to raise one child to adulthood.\(^9\) Note that the term \(Rb^l_{j,t}\) refers to the bequests (plus interest payments) that a member of the cohort born in \(t - 1\) gets from her parents. The following lemma summarizes households' optimal decisions.\(^10\)

**Lemma 1.**

(i) If \(l = n\), agents maximize (5) subject to (8) and (9), such that

\[
c^n_{j,t} = \frac{I^n_{j,t} + [(1 + \theta)\beta]\bar{c}}{1 + (1 + \theta)\beta},
\]

(10)

\[
c^n_{j,t+1} = \frac{\beta}{1 + (1 + \theta)\beta}[I^n_{j,t} - \bar{c}]R,
\]

(11)

\[
b^n_{j,t+1} = \frac{\theta\beta}{n[1 + (1 + \theta)\beta][I^n_{j,t} - \bar{c}]R},
\]

(12)

with \(e_{j,t} = 0\) implying that \(h^n_{j,t+1}\) is constant according to (6).

(ii) If \(l = h\), agents maximize (5) subject to (7), (8), and (9), such that

\[
c^h_{j,t} = \frac{I^h_{j,t} + (\gamma + (1 + \theta)\beta)\bar{c} + wh^h_{m,t}n\bar{e}_j}{1 + \gamma + (1 + \theta)\beta},
\]

(13)

\[
c^h_{j,t+1} = \frac{\beta}{1 + \gamma + (1 + \theta)\beta}[I^h_{j,t} - \bar{c} + wh^h_{m,t}n\bar{e}_j]R,
\]

(14)

\[
b^h_{j,t+1} = \frac{\theta\beta}{n[1 + \gamma + (1 + \theta)\beta][I^h_{j,t} - \bar{c} + wh^h_{m,t}n\bar{e}_j]R},
\]

(15)

\[
e^h_{j,t} = \frac{\gamma}{wh^h_{m,t}n[1 + \gamma + (1 + \theta)\beta]}(I^h_{j,t} - \bar{c}) - \frac{[1 + (1 + \theta)\beta]\bar{c}_j}{1 + \gamma + (1 + \theta)\beta},
\]

(16)

with \(h^h_{j,t+1}\) evolving according to (7).

**Proof.** See Appendix A.1.

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\(^9\)Recall that lecturers are recruited from the middle class, such that the costs of higher education depend on \(h^h_{m,t}\).

\(^10\)See also Solow (1956), Ramsey (1928), Cass (1965), Koopmans (1965), and Diamond (1965) for the analysis of capital accumulation in a neoclassical setting without human capital accumulation. The last reference is closest to our framework because of its discrete time overlapping generations formulation.
whereas they decrease with the subsistence consumption level ($\bar{c}$). The existence of $\bar{c} > 0$ implies a hierarchy of needs for households: expenditure shares of first period consumption are declining with income, while expenditure shares of second period consumption and of bequests are increasing with income. Hence, richer households save more and bequeath more wealth to their children, which is a well-known fact and which is the driver of income inequality in the neoclassical regime.

In the high-skill regime, educational investments are positive and decrease with the preference for bequests ($\theta$), population growth ($n$), the discount factor ($\beta$), the subsistence consumption level ($\bar{c}$), and the baseline education level ($\bar{e}$), whereas they increase with the preference for education ($\gamma$). Moreover, in light of (16), we observe that the regime switch from the neoclassical to the high-skill regime occurs if and only if the level of income, $I_{j,l,t}$, is sufficiently high. This is expressed formally in the following proposition.

**Proposition 1.**

A member of social class $j = r, m, p$ invests in education, i.e., $e_{j,t} > 0$, if her income exceeds the critical threshold $I'_{j,t}$ defined as

$$I'_{j,t} = \frac{wh_{m,t}^l e_j}{\gamma} [1 + (1 + \theta)\beta] + \bar{c}, \quad l = n, h.$$  

(17)

Hence, $e_{j,t} = 0$ if $I_{j,t} \leq I'_{j,t}$.

**Proof.** See Appendix A.2.

Regarding the critical level of income that induces the regime switch to the high-skill regime, several remarks are in order. First, the threshold level of income necessary to induce the regime switch depends positively on the level of subsistence consumption, $\bar{c}$, on the sum of the weights of first and second period consumption, $1 + \beta$, on the weight of bequests, $\theta \beta$, on the level of baseline education, $\bar{e}_j$, and on education cost, $wh_{m,t}^h n$. By contrast, a higher preference for education, $\gamma$, reduces the critical income level.

Second, in light of Proposition 1, the high-skill regime applies to households of class $j$ if their income level is sufficiently high such that investments in education above the basic level (that can be acquired costlessly) deliver a higher amount of additional utility than using the same amount of income for consumption or bequests. In this case we have $I_{j,t} > I'_{j,t}$, such that $e_{j,t} > 0$. The optimal solutions are then described by item (ii) of Lemma 1. If the income level of households in social class $j$ falls short of the critical threshold $I'_{j,t}$, these households find it optimal not to invest in education of their children because the children acquire a certain amount of human capital costlessly and the additional investments in human capital would deliver less additional utility than if the same amount of income was instead spent on consumption or was bequeathed. These households find themselves in the neoclassical regime, i.e., $l = n$ and $e_{j,t} = 0$, such that their optimal decisions are described by item (i) of Lemma 1.

Third, as long as the middle class does not switch to the high-skill regime, $I'_{j,t}$ is constant. If the middle class starts to invest in higher education of their children, $I'_{j,t}$ is
growing with the growth rate of $h_{m,t}$. If, furthermore, the middle class switches before the poor and after the rich, the described mechanism becomes crucial for the poor because they are at risk that $I'_{p,t}$ is growing faster than their incomes such that a regime switch of the poor would not occur without policy interventions in terms of publicly financed universities, college education subsidies such as stipends, or tuition fee waivers (Rauh, 2017). In this context it is also important to notice that an alternative modeling of the neoclassical regime would not affect our results qualitatively. If we allowed for perpetual growth in the neoclassical regime by assuming an $Ak$-production technology (see, for example, Mierau and Turnovsky, 2014), the critical threshold level of income ($I'_{j,t}$) would rise with the productivity growth rate of the middle class. If all classes benefit equally from productivity gains, both the critical threshold and incomes would grow at the same constant rate. The omission of other engines of growth is therefore a convenient shortcut. Similarly, in a closed economy, capital accumulation increases wages and, thus, incomes, while $I'_{j,t}$ is increasing by the same amount. Hence, the small open economy assumption is innocuous with respect to the regime switch, unless incomes of the poor grew faster than wages in the education sector, which, however, does not seem to be an empirically relevant case.

Fourth, in case of $\bar{e}_r = \bar{e}_m = \bar{e}_p$, it follows that all social classes exhibit the same threshold of income for the regime switch but that, nevertheless, social classes switch at different dates depending on the distribution of wealth. Thus, even without any differences in the level of basic education among the social classes, there is a risk that the poor will be disconnected from the growth process in the sense that their incomes are lagging behind their critical income level $I'_{p,t}$. In case of $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $I'_{r,t} > I'_{m,t} > I'_{p,t}$. However, a social group with a higher baseline level of education reaches its higher threshold income level earlier if its accumulation rate of wealth is sufficiently high.\(^{11}\) This implies that differences in basic education or in innate abilities (Eckwert and Zilcha, 2007) with an initial advantage for the rich would only reinforce our mechanism.

Taking into account the time required for child-care ($z \cdot n$), each social class supplies

$$L_{j,t} = (1 - zn)N_{j,t}$$  \hspace{1cm} (18)

units of raw labor, where $N_{j,t}$ is the number of individuals who belong to social class $j$. Consequently, the aggregate supply of human capital can be obtained as the sum of human capital within each social class, which is in turn given by the product of class-specific labor supply ($L_{j,t}$) and average class-specific human capital ($h_{j,t}$):

$$H_t = h_{r,t}L_{r,t} + h_{m,t}L_{m,t} + h_{p,t}L_{p,t}.$$  \hspace{1cm} (19)

Taking into account that human capital used for production ($H_{Y,t}$) is equal to aggregate

\(^{11}\)The critical level of wealth compatible with $I'_{j,t}$ reads $b'_{j,t} = (1/R)[wh_{m,t}\bar{e}_j[1 + (1 + \theta)\beta]/\gamma + \bar{c} - w(1 - zn)\bar{e}_j]$. The distance of social group $j$ to $b'_{j,t}$ shrinks at rate $b'_{j,t} - b_{j,t+1}$. An increase in $\bar{e}_j$ reduces the latter if $\partial(b_{j,t} - b_{j,t+1})/\partial\bar{e}_j < 0$, which is the case if $h_{m,t} < \gamma(1 - zn) \left[1/[1 + (1 + \theta)\beta] + \thetaR^2/(n[1 + (1 + \theta)\beta]^2)\right]$.\)
human capital net of the human capital that is employed in the education sector, which is recruited from the middle class, we have

$$H_{Y,t} = \begin{cases} H_t - (e_{r,t}N_{r,t} + e_{m,t}N_{m,t} + e_{p,t}N_{p,t})h_{m,t}L_{m,t}, & \text{if } l = h, \\ H_t, & \text{if } l = n. \end{cases}$$  \hfill (20)

Before we turn to the discussion of the dynamics, we define the market equilibrium of the economy as follows.

**Definition 1.** The market equilibrium of the economy is defined by the solution of the system of equations (10)-(12) for \( l = n \) and by the solution of the system of equations (13)-(16) for \( l = h \), where \( I_{n,j,t} \) and \( I_{h,j,t} \) are given by (9), the wage rate per unit of effective labor and the capital rental rate are determined by (4), the capital stock per unit of effective labor is given by (3), and the market for human capital is cleared according to (20). Since the markets for physical capital and human capital clear according to this definition, Walras’ law implies that also the goods market is cleared.

3 Dynamics

3.1 Neoclassical regime

In light of Proposition 1, the neoclassical regime is characterized by \( I_{j,t} \leq I'_{j,t} \), such that \( e_{j,t} = 0 \). Consequently, the level of human capital is constant and equal to the level of baseline education, i.e., \( h_{n,j,t+1} = \bar{e}_j \). Thus, income of a member of social class \( j \) is given by

$$I_{n,j,t} = w\bar{e}_j(1 - zn) + Rb^n_{j,t}. \hfill (21)$$

In the following, we denote stationary variables with an asterisk in the subscript. Given that labor incomes are stationary in the neoclassical regime, the accumulation of wealth via bequests is the only source of income growth. Income, \( I^n_{j,t} = \bar{w}_j(1 - zn) + Rb^n_{j,t} \), is thus constant when bequests approach their steady-state level, \( b^n_{j,*} \). This implies that a regime switch from the neoclassical regime to the high-skill regime can only occur if the threshold levels of income that are compatible with the regime switch are below their steady-state levels. In mathematical terms, a necessary condition for the regime switch is \( I^n_{j,t} > I'_{j,t} \), where \( e_{j,t} > 0 \) if and only if the income of a member of social class \( j \) in period \( t \) (\( I^n_{j,t} \)) exceeds the threshold level \( I'_{j,t} \), which requires that \( I^n_{j,t} > I'_{j,t} \). Note that this would be the case even in the presence of productivity growth fueled by other sources than human capital accumulation (e.g., by technological progress) because disposable incomes and the threshold level \( I'_{j,t} \) were growing at the same rate. The following proposition specifies this aspect in more detail. A graphical illustration is presented in Figure 4.

**Proposition 2.**
(i) According to (12) and (21), the accumulation of wealth within social class $j$ is governed by

$$b^n_{j,t+1} = \frac{\theta \beta}{n[1 + (1 + \theta)\beta]}[w\bar{e}_j(1 - zn) + Rb^n_{j,t} - \bar{c}]R$$

(22)

with a unique and stable steady state at

$$b^n_{j,*} = \frac{\theta \beta R}{n[1 + (1 + \theta)\beta] - \theta \beta R^2}[(1 - zn)w\bar{e}_j - \bar{c}],$$

(23)

given that $\theta \beta R^2 < n[1 + (1 + \theta)\beta]$, which implies that the slope of the $b^n_{j,t+1}$-locus is smaller than 1.

(ii) The regime switch requires $I_{j,*} = (1 - zn)w\bar{e}_j + Rb^n_{j,*} > I'_{j,t}$, which implies in light of (21) and (23) that

$$b^n_{j,*} > \frac{wh_{m,t}n\bar{e}_j}{\gamma R}[1 + (1 + \theta)\beta] + \frac{\bar{c}}{R} - \frac{(1 - zn)w\bar{e}_j}{R}$$

(24)

and

$$\bar{e}_j > \bar{e}^n \equiv \frac{\gamma \bar{c}}{\gamma (1 - zn)w - wh_{m,t}[n(1 + (1 + \theta)\beta) - \theta \beta R^2]}$$

(25)

if $l = h$, and

$$\bar{e}_j > \bar{e}^n \equiv \frac{\gamma \bar{c}}{\gamma (1 - zn)w - w\bar{e}_m[n(1 + (1 + \theta)\beta) - \theta \beta R^2]}$$

(26)

if $l = n$.

Proof. See Appendix A.3. □

With regard to item (i) in Proposition 2, labor income must exceed the level of subsistence consumption, i.e., $(1 - zn)w\bar{e}_j > \bar{c}$, otherwise the steady state is economically meaningless. Moreover, global stability of $b^n_{j,*}$ requires that $\theta \beta R^2 < n[1 + (1 + \theta)\beta]$. In addition, note that the location of the $b^n_j$-locus as defined by (22) depends positively on $\bar{e}_j$. If $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $b^n_{r,*} > b^n_{m,*} > b^n_{p,*}$.

A transition into the high-skill regime [item (ii)] requires that the long-run value of bequests in social class $j$ as given by $b^n_{j,*}$ is larger than the threshold level of wealth $b^n_{j,t}$ that is associated with the threshold level of income $I'_{j,t}$. This is only possible if $\bar{e}_j > \bar{e}$. From item (i), we know that a feasible and globally stable steady state, $b^n_{j,*} > 0$, requires that $n[1 + (1 + \theta)\beta] - \theta \beta R^2 > 0$. As $\gamma \bar{c} > 0$ and $\bar{e}_j > 0$, (25) and (26) hold only if

---

12For $(1 - zn)w\bar{e}_j < \bar{c}$, a steady state exists only if $\theta \beta R^2 > n[1 + (1 + \theta)\beta]$, which implies, in turn, that $b^n_{j,*}$ is globally unstable.

13This is a direct implication of Proposition 1 and the requirement that $I'_{j,t} < I^n_{j,*}$, or – equivalent to the latter – that $b^n_{j,t} < b^n_{j,*}$, where $b^n_{j,t} = (1/R)\{wh_{m,t}\bar{e}_j[1 + (1 + \theta)\beta]\}/\gamma + \bar{c} - w(1 - zn)\bar{e}_j$. Otherwise, the critical threshold level would be larger than the long-run value and could not be reached if $b^n_{j,0} < b^n_{j,*}$.
\( \gamma (1 - zn)w > \{ n[1 + (1 + \theta )\beta ] - \theta \beta R^2 \} \). If the middle class is in the neoclassical regime, the poor need a minimum \( \bar{\epsilon}_p > \hat{\epsilon}_n \), otherwise their steady-state income falls short of their threshold income. In case that the middle class already experienced a regime switch, costs of higher education are increasing such that the threshold level of income necessary for the regime switch is also increasing. As long as the baseline level of education of the poor fulfills (25), the regime switch is still possible.

If \( \bar{\epsilon}_j \leq \hat{\epsilon}_l \), accumulated assets of social class \( j \) sustained by the neoclassical regime fall short of \( I'_{j,t} \), such that a switch to the high-skill regime cannot occur for social class \( j \). If \( \hat{\epsilon}_n > \bar{\epsilon}_r \), no social class would ever invest in higher education of their children. In this case the economy is trapped in the neoclassical regime as described by the Solow (1956) model and growth ceases at a certain point. This could be a relevant description of the “middle income trap”, i.e., that countries, which successfully escaped the phase of stagnation at the subsistence level, cannot manage to switch to an innovation-based high-skill economy (cf. Eichengreen et al., 2012, 2013, who observe that middle income traps are less likely to be an obstacle for countries with a well educated population). By contrast, a switch of all social classes to the growth regime is guaranteed for \( \bar{\epsilon}_p > \hat{\epsilon}_l \).

### 3.2 High-skill regime

In the high-skill regime, income of social class \( j \) exceeds \( I'_{j,t} \), such that the parents belonging to class \( j \) invest in higher education of their children. Their optimal decisions are represented by item (ii) of Lemma 1 and the evolution of human capital in social class \( j \)
is then governed by (7). We summarize the dynamic behavior in the high-skill regime in the subsequent proposition.

**Proposition 3.**

(i) In the high-skill regime, the evolution of human capital and bequests is governed by the following system of difference equations

\[
\begin{align*}
    h_{j,t}^{h+1} &= (\bar{e}_j + e_{j,t}) \kappa h_{m,t}^h, \\
    b_{j,t}^{h+1} &= \frac{\theta \beta}{n(1 + \gamma + (1 + \theta)\beta)} [wh_{j,t}^h (1 - zn) + Rh_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j] R.
\end{align*}
\]

(ii) The ratio between bequests and human capital is constant and the same for all social classes that switched to the high-skill regime, i.e.,

\[
\frac{b_{j,t}^{h+1}}{h_{j,t}^{h+1}} = \frac{w\theta \beta R}{\gamma \kappa} = \text{const.}
\]

(iii) Let \(x_{j,t}^h\) denote the ratio of human capital between social class \(j\) and the middle class, i.e., \(x_{j,t}^h = h_{j,t}^h/h_{m,t}^h\), such that

\[
\begin{align*}
    x_{j,t}^{h+1} = \frac{e_{j,t} + \bar{e}_j}{e_{m,t} + \bar{e}_m} = \left[\left(1 - zn\right) + \frac{\theta \beta R^2}{\gamma}\right] x_{j,t}^h - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_m n.
\end{align*}
\]

The stationary solution is then given by

\[
x_{j,s}^h = \lim_{t \to \infty} x_{j,t+1} = \bar{e}_j/\bar{e}_m,
\]

with \(x_{j,s}^h \geq 1\), if \(\bar{e}_j \geq \bar{e}_m\).

(iv) The gross growth rate of human capital is given by

\[
\begin{align*}
    \frac{h_{j,t+1}^h}{h_{j,t}^h} &= (\bar{e}_j + e_{j,t}) h_{m,t}^h = \frac{\gamma \kappa \left[\left(1 - zn\right) + \frac{\theta \beta R^2}{\gamma} - \frac{\bar{c}}{wh_{m,t}^h} + \frac{ne_j}{x_{j,t}^h}\right]}{n(1 + \gamma + (1 + \theta)\beta)}
\end{align*}
\]

\[
\begin{align*}
    &= \frac{\gamma \kappa \left[\left(1 - zn\right) + \frac{\theta \beta R^2}{\gamma} - \frac{\bar{c}}{wh_{m,t}^h} + \frac{ne_j}{x_{j,t}^h}\right]}{n[1 + \gamma + (1 + \theta)\beta]}. \quad (33)
\end{align*}
\]

In the long-run, it converges to

\[
\lim_{t \to \infty} \frac{h_{j,t+1}^h}{h_{j,t}^h} = \frac{\gamma \kappa (1 - zn + \bar{e}_m n) + \theta \beta R^2}{n[1 + \gamma + \beta(1 + \theta)]}. \quad (34)
\]

(v) The long-run ratio of bequests between social class \(j\) and the middle class converges
\[ 
\lim_{t \to \infty} \frac{b^{h}_{j,t+1}}{b^{h}_{m,t+1}} = \frac{b^{h}_{j,*}}{b^{h}_{m,*}} = \frac{w^{\bar{e}_j}}{w^{\bar{e}_m}} \left( 1 - zn \right) + \frac{w^{\theta \beta R^2}}{\gamma \kappa} - \bar{c} + w n \bar{e}_j, 
\]

which implies that
\[ \frac{b^{h}_{j,*}}{b^{h}_{m,*}} \ll 1 \quad \text{for} \quad \bar{e}_j \ll \bar{e}_m. \]

The proof follows from item (i)-(iii) of this proposition.

**Proof.** See Appendix A.3.

The switch to the high-skill regime induces growing wage incomes, \( wh^{h}_{j,t} \), and increasing levels of bequests [item (i) of Proposition 3], such that total incomes, \( I^{h}_{j,t} \), grow as well. After the regime switch, the ratio between bequests and human capital is constant and the same for all social classes [item (ii)], such that incomes grow at the same rate. The ratio of bequests to human capital, \( b^{h}_{j,t+1}/h^{h}_{j,t+1} \), declines with the weight of children’s education in the parental utility function, \( \gamma \), and the productivity of the education sector, \( \kappa \). By contrast, the ratio of bequests to human capital increases with those factors that are responsible for an increase in bequests, i.e., wages, \( w \), which also increase education costs, the weight of bequests in the parental utility function, \( \theta \beta \), and the capital rental rate, \( R \).

Income inequality is determined by the initial distribution of wealth, the differential evolution of bequests, and the differential investment in human capital between the social classes. In item (iii), we capture the evolution of inequality in terms of education by the dynamics of the ratio between human capital of social class \( j \) and the level of human capital of the middle class, i.e., \( x_{j,t} = h^{h}_{j,t}/h^{h}_{m,t} \). As regards the evolution of relative human capital, two cases have to be distinguished. (1) If social classes do not differ with respect to their baseline levels of education such that \( \bar{e}_r = \bar{e}_m = \bar{e}_p \), it follows in light of (30) that the influence of subsistence consumption on the evolution of \( x_{j,t} \) is approaching zero in the long run (\( \lim_{t \to \infty} \bar{c}/wh^{h}_{j,t} = 0 \)). Thus, \( x_{j,t} \) converges to 1 as \( t \) approaches infinity. Item (ii) implies that all social classes bequeath the same amount of assets to their children, such that incomes are equal among the social classes in the long run. Thus, there is only scope for long-run income inequality in this case if not all social classes switch to the high-skill regime. Transitory income inequality is determined by the initial distribution of wealth for a given constellation of \( I^{h}_{j,t} < I^{h}_{j,s} \) determining the timing of the regime switch in social class \( j \). It is important to note in this context that the transition phase of these types of models typically last for a very long time period (several generations) such that income inequality could be observed for centuries if such a model represented the underlying data generating process. (2) If, in turn, \( \bar{e}_j \gtrless \bar{e}_m \), it follows that \( x_{j} \) approaches \( \bar{e}_j/\bar{e}_m \gtrless 1 \) if \( \bar{e}_j \gtrless \bar{e}_m \). Thus, differences in the level of baseline education translate into differences in relative human capital endowments, differences in the levels of bequests, and differences
in the levels of income. A constant $b/h$ ratio implies then that households with lower human capital endowments also exhibit lower bequests, even in the long run.

The growth rate of human capital [item (iv)] depends positively on the $b/h$ ratio, positively on time devoted to work, and positively on $\gamma/\{n[1 + (1 + \theta)\beta]\}$, which drives the expenditure share of higher education. Moreover, the growth rate of human capital is positively affected by the baseline level of education, $\bar{e}_j$, and the productivity of the education sector, $\kappa$. During the transition, the growth rate of human capital is adversely affected by subsistence needs, $\bar{c}$, and by $x_{jt}^h$. The latter reflects a neoclassical convergence mechanism. In the long run, due to increasing wage incomes, the impact of subsistence needs on the evolution of human capital approaches zero. Altogether, $x_{jt}$ converges to $\bar{e}_j/\bar{c}_m$, such that the growth factor of human capital converges to expression (34). Since differences in the baseline level of education drive long-run differences in human capital, it follows that differences in bequests across social classes are also driven by differences in $\bar{e}_j$ [item (v)].

Crucial for our theory is the emergence of first declining and then increasing income inequality after the regime switch from the neoclassical to the high-skill regime. The main argument is summarized in the following proposition. The possible decline in income inequality in a later phase of the high-skill regime will be discussed further below.

**Proposition 4.** A social class that reaches the critical income level $I'_{jt}$ reduces the accumulation of wealth.

**Proof.** At the moment of the regime switch, a member of social class $j$ is equipped with a
wealth level of at least $b'_{j,t}$, which assures the necessary level of income that generates the regime switch ($I'_{j,t}$, as defined by Proposition 1). In the neoclassical regime, according to Lemma 1, an income level of $I^0_{j,t}$ induces a level of bequests of

$$b^0_{j,t+1} = \frac{\theta \beta R}{n[1 + (\theta + (1 + \theta)\beta)]} [I^0_{j,t} - \bar{c}].$$

(36)

In the high-skill regime, the level of bequests is given by

$$b^h_{j,t+1} = \frac{\theta \beta R}{n[1 + \gamma + (1 + \theta)\beta]} [I^0_{j,t} - \bar{c} + w_t \tilde{h}^h_{n,t} n \bar{e}_j].$$

(37)

From the last two expressions, we obtain $b^0_{j,t+1} > b^h_{j,t+1}$ if

$$I^0_{j,t} > [1 + (\theta + (1 + \theta)\beta)] \frac{w^h_{n,t} n \bar{e}_j}{\gamma} + \bar{c},$$

(38)

which, in light of Proposition 1, implies that $I^0_{j,t} > I'_{j,t}$.

According to Proposition 4, a household switching to the high-skill regime starts to invest in higher education of the children but at the expense of bequests per child. To put it differently, households shift resources from the accumulation of wealth to the accumulation of human capital. The rich are the first social class that experiences the regime switch and reduces bequests, followed by the middle class and possibly also the poor. Given that the expenditure share of bequests is, due to the existence of subsistence needs, increasing with income, the decline in bequeathed assets in the highest income percentiles happens at a time when there are increasing levels of bequests in the other social classes, such that income inequality falls. As labor incomes of the children of the rich are increasing because of human capital accumulation (which, in turn, stimulates increasing levels of bequests and education), income inequality starts to rise again at a later stage. The reason that inequality unambiguously falls in the short- to medium run is that the higher human capital level of the children only leads to higher wages later on in their lives, whereas the effect of lower bequests on inequality is immediately. Altogether, income inequality and the flow of bequests follow the U-shaped pattern as we observe it in the data described in the introduction.

A graphical illustration of our arguments is presented in Figure 5. The evolution of bequests follows the arrows $A$, $B$, and $C$. In period $t$, the members of social class $j$ exhibit a level of assets indicated by $b^0_{j,t}$, which exceeds $b'_{j,t}$ associated with the threshold income $I'_{j,t}$. Thus, social class $j$ switches to the high-skill regime and leaves the $b^0_{j,t+1}$-locus. From Proposition 4 it follows that the $b^h_{j,t+1}$-locus compatible with $b^0_{j,t}$ must be located below the $b^0_{j,t+1}$-locus, such that the level of bequests shrinks ($B$). Since the evolution of social class $j$ is now described by Proposition 3, item (i), the $b^h_{j,t+1}$-locus moves upwards such that bequests increase again ($C$). Obviously, the speed of the upward shift depends (ceteris paribus) on the quality of the education sector reflected by the education productivity.
parameter $\kappa$. Thus, $\kappa$ also affects the evolution of the flow of bequests and the evolution of income inequality.

Regarding the evolution of income inequality after the regime switch, it is important to note that the discussed fall and rise of inequality is a transitory phenomenon. Relative human capital stocks will converge to their initial values and income inequality will decline toward its long-run value. Nevertheless, and this will be clarified further below, the emergence of declining income inequality and its potential amount depends crucially on whether or not all social groups experience a regime switch to the high-skill regime. In light of Lemma 1, expenditures for higher education are zero in the neoclassical regime, such that we obtain from Proposition 3, item (iii), initial relative human capital endowments in the neoclassical regime as

$$x_{r,t}^n = \frac{\bar{e}_r}{\bar{e}_m} > 1, \quad x_{m,t}^n = 1, \quad x_{p,t}^n = \frac{\bar{e}_p}{\bar{e}_m} < 1,$$

which are equal to the corresponding long-run values in the high-skill regime, $x_{j^*,h}^h$ [see (31)]. Due to a higher level of wealth and a higher convergence speed, the rich surpass their critical threshold income first, followed by the middle class and possibly also by the poor. Higher education of the rich can be expressed as

$$\epsilon_{r,t} = \gamma \left[ (1 - zn) + \frac{R^2 \theta \beta}{\gamma} \right] x_{r,t}^h - \frac{\bar{c}}{wh_{m,t}} - \frac{[1 + (1 + \theta) \beta]}{1 + \gamma(1 + \theta)\beta},$$

(40)

With $h_{m,t}$ being constant, $x_{r,t}^h$ increases after the regime switch of the rich above $x_{r,t}^n$ and induces increasing expenditures on education, which amplifies the income gap to the other social classes later on. After the middle class experiences a regime switch, the resulting increase in $h_{m,t}$ affects the relative endowments of the other social groups because the price of education changes, while $x_{m,t}$ remains at 1. If the poor are still not investing in higher education, their relative human capital stock shrinks and falls short of $x_{p,t}^n$. Moreover, their threshold income ($I'_{p,t}$) is now increasing with the growth rate of human capital in the middle class. If, under these circumstances, $I'_{p,t}$ is increasing above $I_{p,t}^*$, the poor will never switch (or they may just temporarily switch) to the high-skill regime given that the growth rate of their incomes is below the growth rate of their threshold income $I'_{p,t}$.

Initially, $\epsilon_{r,t}$ increases because of the increase in $x_{r,t}^h$, indicating that education is comparatively cheap for the rich. When the middle class starts to invest in education, there is a dampening effect on $x_{r,t}$ because of the increase in education costs due to the increase in $h_{m,t}$, but a second reinforcing effect on education setting in through the diminishing role of subsistence needs, $\bar{c}/(wh_{m,t})$. The latter, however, is only a transitory effect that becomes smaller and smaller as the middle class accumulates human capital. Thus, the rich reduce the growth rate of expenditures on education below the level of the middle class such that $x_{r,t}^h$ converges from above to its long-run value $x_{r,t}^{h,*} = x_{r,t}^n$. A symmetric argument holds for the poor. The regime switch of the middle class adversely affects the poor’s relative human capital stock, $x_{p,t}$. Thus, their expenditures fall short of
the level achieved in the middle class and \( x_{p,t}^h \) shrinks below \( x_{p,t}^n \). However, the shrinking importance of subsistence needs dampens the decline over time, which implies that the growth in expenditures on education in the middle class ceases to the extent that \( x_{p,t}^h \) adjusts from below to its long-run value \( x_{p,t}^h = x_{p,t}^n \). The convergence of relative human capital stocks to their initial values is precisely the mechanism responsible for the decline in income inequality. But again, it is important to stress the feasibility of the regime switch for the poor. Since the poor may start to invest in higher education after the regime switch of the middle class has occurred, their threshold income may grow faster than their actual incomes. Thus, the poor might never (or only temporarily) switch to the high-skill regime. In this case, their relative human capital shrinks toward zero and the initial distribution of wealth will affect long-run income inequality. In this context, the overall amount of income inequality is a poor predictor for the feasibility of the regime switch for all social classes. What matters is the income gap between the poor and the middle class. A comparatively large distance between the poor and the middle class induces a relatively early switch of the middle class to the high-skill regime, which is responsible for a relatively fast growing threshold income of the poor.

The role of the quality of the education sector, as reflected by \( \kappa \), is twofold. An increase in \( \kappa \) increases labor incomes and bequests, such that the decline in bequests after the regime switch will be reduced or even non-existent. Moreover, an increase in the productivity of the education sector may disconnect the poor from the growth process because of an increase in the growth rate of their threshold income. However, a reduction in the quality of the education sector reduces income inequality during the transition because of a slow increase in labor incomes and bequests. But precisely the slower increase in incomes may again reduce the growth rate of incomes of the poor below the growth rate of their threshold incomes. Thus, for a given amount of income inequality, there exists a certain range in which the productivity of the education sector ensures that a regime switch occurs for all social classes. We discuss and illustrate this argument in the numerical section in more detail.

Altogether, we can summarize our analytical results on the evolution of income inequality over time as follows: (1) At early stages of economic development, the accumulation of wealth and bequests is the engine of income growth. Subsistence consumption constraints imply that savings rates increase with income such that the richer parts of the population accumulate (and bequeath) wealth at a higher rate than the other social classes. This leads to rising income inequality. (2) If the economy manages to avoid the middle income trap, the rich start to invest in higher education of their children, which is costly and crowds out bequests. At that stage, wealth accumulation of the rich is reduced and income inequality starts to shrink. This phase lasts for one generation, i.e., for around 30 years. (3) Over time, the better educated children of the rich start to earn higher wage incomes. The engine of growth becomes human capital accumulation and income inequality starts to rise again. During that phase, the driving force behind the evolution of income inequality is the skill differential between those with higher education and those without. (4) Even-

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tually, all social classes might be able to invest in higher education and the growth rates of human capital converge. As a consequence, the level of income inequality decreases. This, however, crucially hinges on whether or not the poor become disconnected from the process of human capital accumulation.

The described development of the model economy is consistent with the data on wealth concentration, income inequality, the flow of bequests, and the fraction of individuals that receive higher education depending on parental income as described in Figures 1, 2, and 3 and the literature cited in the introduction. Furthermore, the evolution of income inequality that our model predicts is consistent with the notion that the main driver of income inequality before WW I was the differential accumulation of wealth, while, in modern times, the wage gap between highly educated individuals and poorly educated ones becomes another crucial driver of income inequality (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014).

4 Numerical experiments

In this section, we conduct numerical experiments to illustrate the theoretical results form the previous sections and to get an impression of the transitional dynamics that our framework implies. In so doing, we also analyze the effects of different amounts of initial wealth inequality on the evolution of income inequality over time.

We choose the parameters of the model such that the balanced growth path fits to empirical observations of developed economies considering that, in our model, one period has a length of thirty years. We fix the capital income share in the production of output, $\alpha$, at 0.3. The real interest rate is set to 4% per year, i.e., $R = 1.04^{0.3}$. The literature on business cycles suggests a discount factor of future consumption of around 0.99 per quarter, such that $\beta = 0.99^{1/4}$. The projections of the United Nations suggest a stationary world population in the long run such that we set $n = 1$. As far as the child-rearing time is concerned, we fix the time share necessary to raise one child to adulthood, $z$, at 0.027, which implies an opportunity cost of around 15% of parents’ time endowment per child. Moreover, the weight of human capital in the parental utility function, $\gamma$, is set to 0.27.\(^\text{14}\) The remaining values are calibrated in an iterative way assuring long-run human capital growth (per year) between one and two percent and expenditures on education not exceeding 6%. Altogether, this procedure implies $A = 10$, $\kappa = 3.3$, $\theta = 0.4$, $\bar{c} = 1.8$, $\bar{e}_r = 0.28$, $\bar{e}_m = 0.27$, and $\bar{e}_p = 0.26$.

In Figure 6, we depict the evolution of bequests [(a), (c), and (e)] and education [(b), (d), and (f)] in social class $j$ for different amounts of initial inequality in the distribution of wealth (solid lines). In Figure 7 (solid lines), we depict the corresponding evolution of income inequality as expressed by the income share of the rich and the evolution of relative human capital stocks governed by Equation (30). In both figures, the dashed lines show the hypothetical transition if the corresponding social class had not switched

\(^{14}\)For further details on the evaluation of these values see de la Croix and Doepke (2003).
Figure 6: Evolution of bequests ($b_j$) and education ($e_j$): (a), (b) low initial inequality; (c), (d) high initial inequality; (e), (f) high inequality between middle class and poor.
low initial inequality in wealth

Figure 7: Incomes of the rich relative to total incomes \( \frac{I_{r,t}}{I_t} \) and relative human capital endowments \( x_{j,t} \).
to the high-skill regime. Scenario (a), (b) is characterized by a comparatively equal initial distribution of wealth, while the Scenarios (c), (d) and (e), (f) exhibit not only a higher amount of initial wealth inequality, but also a different distance between the middle class and the poor. The latter is crucial for the poor to conduct a regime switch to the high-skill regime. Indeed, it may be misleading to assess the evolution of the economy just by means of initial income inequality.

Since $\bar{e}_r > \bar{e}_m > \bar{e}_p$ for this exercise, social classes transit along different trajectories toward different steady states in the neoclassical regime (see Proposition 2 and Figure 4) in the sense that $b_{r,s}^n > b_{m,s}^n > b_{p,s}^n$. Moreover, Proposition 1 implies that the three social classes exhibit different threshold incomes: $\hat{I}_r > \hat{I}_m > \hat{I}_p$. Due to a higher level of assets, the rich surpass their threshold level of income first and start to invest in higher education of their children. In the previous section, we argued that the regime switch to the high-skill regime induces a reduction in bequests (see Proposition 4 and Figure 5). In Figures 6 (a), (c), and (e) we see indeed that bequests of the rich fall short of the level in the neoclassical regime after the regime switch. Nevertheless, increasing investments in education increase labor incomes in subsequent periods such that the level of bequests increases again. Figures 6 (b), (d), and (f) show the evolution of parental expenditures on education ($e_{j,t}$). The rich are the first to invest in education of their children, while the middle class and the poor exhibit constant levels of human capital until they experience a regime switch as well. Thus, human capital and incomes of the rich grow faster than in case of the other classes. Symmetrically, the poor are the last social group to begin to invest in education. After the middle class switched to the high-skill regime, the education sector is characterized by growing levels of human capital, which in turn induces increasing costs of higher education. Hence, from the perspective of the poor, the regime switch goes hand in hand with shrinking bequests and increasing costs of higher education, such that the education levels of the poor undershoot. This is the mechanism responsible for the increase in income inequality after the regime switch of the poor. Moreover, comparing Figures 6 (a), (b) with (c), (d) shows that initial wealth inequality does not affect differences in education. However, comparing both scenarios with (e), (f) demonstrates that the initial distribution affects the possibility of the regime switch of the poor, i.e., what matters is the distance between the level of wealth of the poor and those of the middle class. In the last scenario, wealth inequality is initially even lower than in Scenario (c), (d), but the poor start with a lower wealth level and accumulate wealth at a lower rate than the middle class due to subsistence consumption constraints. The middle class switches to the high-skill regime, which induces a continuous increase in the poor’s threshold income that exceeds the growth rate of their actual incomes. It follows that the poor never switch to the high-skill regime.\footnote{\textsuperscript{15}For moderate increases in the poor’s initial level of wealth, the poor may initially switch to the high-skill regime but since their income growth cannot keep up with the growth rate of their threshold income, $I_{p,t}$ overtakes their incomes and the poor switch back to the neoclassical regime. In this case bequests converge from below to the neoclassical trajectory.}

In light of Proposition 3, item (iii), relative human capital stocks converge to $\bar{e}_{j} / \bar{e}_{m}$
such that the increase in income inequality caused by rising education levels of the children of the rich after their regime switch is just a transitory phenomenon. Income inequality will fall to a level determined by the differences in the levels of baseline education. We depict the evolution of income inequality expressed by the dynamics of the share of income of the rich in Figure 7. Initially, income inequality is declining due to the forces of a neoclassical convergence mechanism. After the regime switch to the high-skill regime, the income share of the rich drops below the level of the neoclassical regime because of the decline in bequests. The income share of the rich starts to increase again when the other social classes invest in education because their bequests are reduced, while labor incomes and bequests of the rich are increasing again. After investments in education of the rich have peaked, income inequality starts to decline toward its steady-state level. The evolution of income inequality in later phases of the high-skill regime thus follows the evolution of relative human capital endowments (see right-hand panel of Figure 7).

Higher initial inequality due to a wealthier rich class amplifies the time interval during which the regime switches of the different social groups occur such that the rich invest earlier in education and exhibit earlier increasing levels of bequests. The income share of the rich drops during the neoclassical regime from a higher level and increases due to the described differences in educational attainment to a higher level again. Nevertheless, differences in initial wealth inequality do not affect the long-run income share of the rich. Long-run income shares are only affected if at least one group does not switch to the high-skill regime. The long-run amount of income inequality is then dependent on the population shares of the different social classes.

In Figure 8, we illustrate the effect of variations in the quality of the education sector, \( \kappa \), on the evolution of income inequality. Panel (a) presents the evolution of bequests for the rich and in panel (b) we depict the evolution of the income share of the rich. As is obvious, the productivity of the education sector steers the magnitude of the decline in income inequality after the regime switch to the high-skill regime. The lower the productivity of the education sector, the longer the period during which individuals reduce their bequests below the level of the neoclassical regime. Although it is a utility-maximizing strategy to invest in higher education at the expense of bequests, a lower productivity of the education sector implies a slower increase in wage incomes for subsequent generations. This contributes to a stronger transitory decline in income inequality. A relatively low \( \kappa \) (solid red line) may be equally harmful for the poor as a relatively high \( \kappa \). If \( \kappa \) is low, the reduction in bequests and the slow increase in labor incomes may create a situation that is characterized by a faster increase in the threshold income of the poor compared to their actual incomes, such that the poor will cease to invest in higher education. Hence, income inequality rises after the regime switch of the middle class because the poor are disconnected from the high-skill regime and the growth process. As regards the long-run

\[ 16 \text{Inequality shrinks if the rich are – compared to the other social classes – already close to their steady state. Otherwise income inequality would increase due to the subsistence needs that imply that the poorer classes cannot accumulate wealth at such a high rate as the rich.} \]
effects, this scenario is symmetric to the one characterized by a relatively high productivity in the education sector (dotted red line). In the latter scenario, the poor’s threshold income is increasing faster than their actual incomes right from the beginning, such that the poor would not even invest in higher education during the transition. Only for an intermediate range of $\kappa$, the regime switch of all social classes is guaranteed. A moderate increase in the quality of the education sector (black dotted line) reduces the decline in income inequality after the regime switch and increases the peak of income inequality. However, after the peak in inequality is surpassed, it declines faster toward its long-run value.

5 Conclusions

We set up a novel overlapping generations model with three social classes: the rich, the middle class, and the poor. Initially, the economy is in the neoclassical regime in which income growth is entirely driven by the accumulation of wealth. During this stage of economic development, investments in higher education are not widespread because higher education for the children is seen as a luxury good from the perspective of households. After a certain threshold level of income is surpassed, richer households start to invest in higher education of their children. This, however, reduces their savings and therefore their bequests, which reduces income inequality for at least one generation. The corresponding earlier onset of higher education and therefore faster human capital accumulation of the children of the rich leads to an increase of their incomes. Subsequently, also the mid-
middle class and possibly the poor start to invest in higher education. Since different social groups accumulate human capital at different rates, income inequality starts to rise again. This pattern is consistent with the data on bequests, income inequality, and education depending on parental income as described in the introduction. Furthermore, the pattern is also consistent with the view that the main driver of income inequality has been the differential accumulation of wealth in the past but that nowadays a large part of income inequality can be explained by differences in wage incomes based on differences in educational attainment (Atkinson et al., 2011; Meschi and Scervini, 2014; Piketty, 2014). Finally, the implications of our model with respect to a rising lifetime income inequality of generations that are born later is consistent with the empirical evidence for Germany and the United States over the past decades (Bönke et al., 2015; Guvenen et al., 2017).

Our proposed mechanism should be understood as a complementary force to the ones that have already been analyzed in the literature (e.g., the disruptions of World War II, changes in taxation, globalization, changes in monetary policy, and skill-biased technological change) and it gives rise to an additional explanation for the observed U-shaped evolution of income inequality and inheritance flows. In contrast to some more stylized models of rising inequality, however, we argue that the top income share does not approach 1 in the long-run. In the high-skill regime, there is long-run convergence of the growth rates of human capital between the rich, the middle class, and the poor, such that income inequality declines again and the top income share converges toward a level substantially below 1. Crucial for the emergence of declining income inequality, though, is that all social classes switch to the high-skill regime. Essentially, the poor face a risk of being disconnected from the growth process. Regarding the feasibility of the regime switch for the poor, the initial distribution of wealth and thus initial income inequality is not a good predictor. What matters is the distance between the poor and the middle class.

To focus on the higher education channel, we isolated it by deliberately abstracting from other mechanisms that affect inequality such as the disruptions of World War II, changing tax policies, declining population growth, skill-biased technological change, the decline of unionization, and globalization. While we believe that each of these channels have contributed to the pattern of the development of inequality over the last century, the aim of this paper has been to emphasize and formalize the role of higher education. Analyzing the relative importance of all potential mechanisms related to the evolution of income inequality for different countries within a large-scale numerical overlapping generations model is an interesting task in its own right and left for further research.

A central policy implication of our framework to reduce income inequality is to invest in public universities or in education subsidies for the poor, e.g., in the form of stipends and tuition fee waivers (Rauh, 2017). The reason is that such a policy reduces the costs of education for the corresponding social class and thereby reduces the threshold level of income above which it becomes optimal to invest in higher education. On top of that, the mode of financing public education or stipends for the poor could have distributional effects on its own, for example, if the tax system is progressive or if there is cross-subsidization.
of the costs for educating the poor by higher tuition fees for the rich. In these cases there is an additional redistribution from the rich to the poor. In general, a policy that raises education and thereby the human capital level of the population might also be a potential solution for a country to escape the middle income trap as indicated by the results of Eichengreen et al. (2013).

Finally, we want to mention that including an exogenously growing stock of technologies or even endogenous technological progress would leave the substance of our framework unaffected because the threshold levels of income that are necessary for a regime switch would grow with the rate of technological progress.

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Conflict of Interest

The authors declare that they have no conflict of interest.

Mathematical Appendix

A.1 Lemma 1

(1) Item (i):

If \( l = n \), a member of social class \( j \) maximizes (5) subject to (8) and (9). The associated first-order conditions read

\[
\frac{1}{c_{j,t}^n - \bar{c}} = \lambda, \quad \text{(A.1)}
\]

\[
\beta \frac{c_{j,t+1}^n}{R} = \frac{\lambda}{R}, \quad \text{(A.2)}
\]

\[
\theta \beta \frac{b_{j,t+1}^n}{R} = \frac{\lambda n}{R}. \quad \text{(A.3)}
\]

Combining (A.1) with (A.2) and (A.1) with (A.3) yields

\[
c_{j,t+1}^n = \beta R(c_{j,t}^n - \bar{c}), \quad \text{(A.4)}
\]

\[
b_{j,t+1}^n = \frac{\theta}{n} c_{j,t+1}^n. \quad \text{(A.5)}
\]
Combining the last two expressions with the budget constraint gives item (i).

(2) Item (ii):
If \( l = h \), agents maximize (5) subject to (8), (9), and (6). The associated first-order conditions are given by (A.1)-(A.3) for \( l=h \) and

\[
\frac{\gamma}{e_{j,t}^h + \bar{e}_j} = wh_{m,t}^h n \lambda. \tag{A.6}
\]

From the last expression and (A.1), we obtain

\[
e_{j,t}^h = \frac{\gamma}{n} (e_{j,t}^h - \bar{c}) - \bar{e}_j. \tag{A.7}
\]

Combining the last expression with (A.4) and (A.5) verifies item (ii).

A.2 Proposition 1
Noting (16), we obtain \( e_{j,t}^h \leq 0 \), if

\[
\frac{\gamma I_{j,t}}{wh_{m,t}^h n} \leq [1 + (1 + \theta) \beta] \bar{e}_j + \frac{\bar{c}}{wh_{m,t}^h n}, \tag{A.8}
\]

\[
\Rightarrow I_{j,t}' = \frac{wh_{m,t}^h n \bar{e}_j [1 + (1 + \theta) \beta]}{\gamma} + \bar{c}. \tag{A.9}
\]

A.3 Proposition 2
The regime switch requires \( I_{j,\ast} = (1 - zn) w \bar{e}_j + R b_{j,\ast} > I_{j,t}' \), which implies in light of Proposition 1 that

\[
b_{j,\ast}^a > \frac{wh_{m,t}^h n \bar{e}_j [1 + (1 + \theta) \beta] + \bar{c}}{\gamma R} - \frac{(1 - zn) w \bar{e}_j}{R}. \tag{A.10}
\]

Substituting now for \( b_{j,\ast}^a \) by using (23) yields

\[
\frac{\theta R^2}{n [1 + (1 + \theta) \beta] - \theta R^2} [(1 - zn) w \bar{e}_j - \bar{c}] > \frac{wh_{m,t}^h n \bar{e}_j}{\gamma} [1 + (1 + \theta) \beta] + \bar{c} - (1 - zn) w \bar{e}_j. \tag{A.11}
\]

From the last expression, we obtain a minimum level of \( \hat{e}^l \) that assures a regime switch in the future

\[
\hat{e}^l = \frac{\gamma \bar{c}}{\{\gamma (1 - zn) w - wh_{m,t}^h [n(1 + (1 + \theta) \beta) - \theta R^2]\}}, \tag{A.12}
\]
such that

\[
e^h = \frac{\gamma \bar{c}}{\{\gamma(1 - zn)w - w\bar{e}_m n [n(1 + (1 + \theta)\beta) - \theta \beta R^2]\}},
\]

(A.13)

and

\[
e^n = \frac{\gamma \bar{c}}{\{\gamma(1 - zn)w - w\bar{e}_m n [n(1 + (1 + \theta)\beta) - \theta \beta R^2]\}},
\]

(A.14)

Obviously, \(\hat{e}^n\) is constant and a regime switch is possible if \(\hat{e}_j > \hat{e}^h\).

If \(l = h\) it follows that \(\hat{e}^h\) is growing with \(h_{m,t}\), thus moving the critical level of income to the right, such that a regime switch of classes poorer than the middle class becomes infeasible if the threshold level has become greater than their steady-state level of bequests.

Note also that \(\hat{e}^n\) and \(\hat{e}^h\) exhibit a vertical asymptote at

\[
\hat{h}_{m,t} = \frac{(1 - zn)\gamma}{n[1 + (1 + \theta)\beta] - \theta \beta R^2}.
\]

(A.15)

A.4 Proposition 3

(i) The ratio between bequests and human capital is constant along the BGP.

Note that

\[
\bar{e}_j + e_{j,t} = \frac{\gamma(I_{j,t} - \bar{c} + w h_{m,t} \bar{e}_j n)}{w h_{m,t} n [1 + \gamma + (1 + \theta)\beta]}.
\]

(A.16)

Thus

\[
\frac{h_{j,t+1}^h}{h_{j,t}^h} = (\bar{e}_j + e_{j,t}) \kappa \frac{h_{m,t}^h}{h_{j,t}^h} = \frac{\gamma \kappa (I_{j,t} - \bar{c} + w h_{m,t} \bar{e}_j n)}{w h_{m,t} n [1 + \gamma + (1 + \theta)\beta]}.
\]

(A.17)

Note further that

\[
\frac{b_{j,t+1}^h}{b_{j,t}^h} = \frac{\theta \beta R}{n[1 + \gamma + (1 + \theta)\beta]} \left[ \frac{I_{j,t} - \bar{c} + w h_{m,t} \bar{e}_j}{h_{j,t}^h} \right] \]

(A.18)

\[
\Rightarrow \frac{b_{j,t+1}^h}{b_{j,t}^h} = \frac{\theta \beta R}{h_{j,t}^h} \left[ \frac{I_{j,t} - \bar{c} + w h_{m,t} \bar{e}_j}{n[1 + \gamma + (1 + \theta)\beta]} \right].
\]

(A.19)

Combining the last expression with (A.17) yields

\[
\frac{b_{j,t+1}^h}{b_{j,t}^h} = \frac{w \theta \beta R}{\gamma \kappa} = \text{const}.
\]

(A.20)
(ii) The evolution of relative inequality. Noting that \( x_{j,t}^h = \frac{h_{j,t}^h}{h_{m,t}^h} \), we obtain

\[
x_{j,t+1}^h = \frac{\bar{e}_j + e_{j,t}}{\bar{e}_m + e_{m,t}} = \frac{h_{m,t}^h e_i - e_{j,t}}{h_{m,t}^h + e_{j,t}} + \bar{e}_j n
\]

\[
\Rightarrow x_{j,t}^h = \left[ \frac{(1 - zn) + \frac{Rb_{j,t}^h}{w_{j,t}^h}}{(1 - zn) + \frac{Rb_{m,t}^h}{w_{m,t}^h}} \right] x_{j,t}^h - \frac{\bar{e}}{wh_{m,t}^h} + \bar{e}_j n.
\]

Taking into account (A.20), we obtain

\[
x_{j,t+1}^h = \left[ \frac{(1 - zn) + \frac{\theta \beta R^2}{\gamma}}{(1 - zn) + \frac{\theta \beta R^2}{\gamma}} \right] x_{j,t}^h - \frac{\bar{e}}{wh_{j,t}^h} + \bar{e}_j n
\]

As \( \lim_{t \to \infty} \frac{\bar{e}}{wh_{m,t}^h} = 0 \), we obtain from the last expression that

\[
x_{j,t}^* = x_{j,t+1} = 1
\]

if \( \bar{e}_j = \bar{e}_m \) and

\[
x_{j,t}^* = x_{j,t+1} = 1
\]

if \( \bar{e}_j \geq \bar{e}_m \).

(iii) The gross growth rate of human capital.

From (A.17) and the definition of households’ incomes, we obtain

\[
\frac{h_{j,t+1}^h}{h_{j,t}^h} = \frac{\gamma \kappa}{n[1 + \gamma + (1 + \theta)\beta]} \left[ (1 - zn) + \frac{\frac{Rb_{j,t}^h}{w_{j,t}^h}}{1 - zn} + \frac{\bar{e}_j n}{x_{j,t}} \right].
\]

Combining the last expression with (A.20), we obtain

\[
\frac{h_{j,t+1}^h}{h_{j,t}^h} = \frac{\gamma \kappa}{n[1 + \gamma + (1 + \theta)\beta]} \left[ (1 - zn) + \frac{\theta \beta R^2}{\gamma} - \frac{\bar{e}}{wh_{j,t}^h} + \frac{\bar{e}_j n}{x_{j,t}} \right]
\]

such that, in light of (ii), the gross growth rate of human capital reads

\[
\lim_{t \to \infty} \frac{h_{j,t+1}^h}{h_{j,t}^h} = \frac{\gamma \kappa(1 - zn + \bar{e}_m n) + \theta \beta R^2}{n[1 + \gamma + (1 + \theta)\beta]}.
\]

References


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