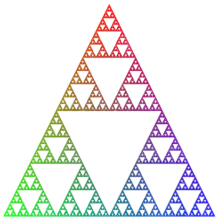
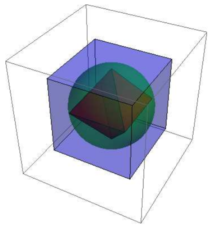
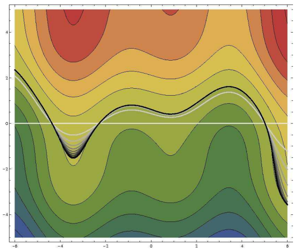






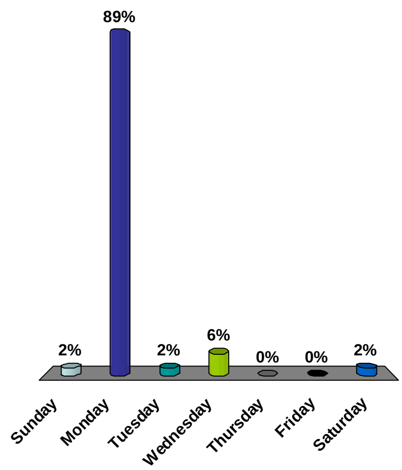
Revision lecture

MA30041: Metric Spaces

Just to become familiar with the clicker:

What day of the week is today?

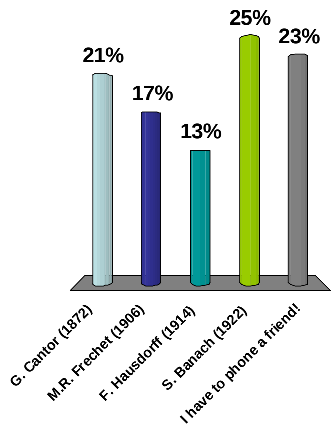
1. Sunday
2. Monday
3. Tuesday
4. Wednesday
5. Thursday
6. Friday
7. Saturday



Just to become familiar with the clicker:

The concept “Metric Space” was introduced by whom?

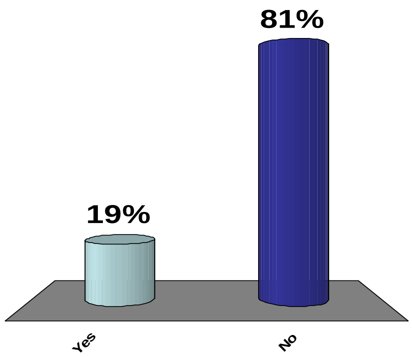
1. G. Cantor (1872)
2. M.R. Fréchet (1906)
3. F. Hausdorff (1914)
4. S. Banach (1922)
5. I have to phone a friend!



Just to become familiar with the clicker:

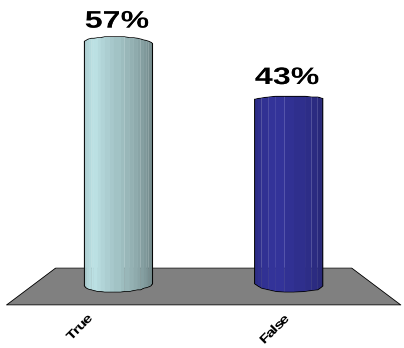
Have you used the Audience Response Systems (ARS) previously?

1. Yes
2. No



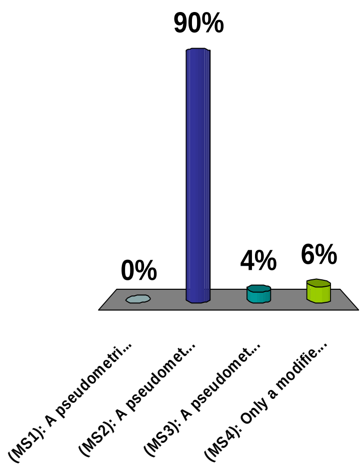
The function dist(Y,Z)=inf{d(y,z)|y in Y, z in Z} for nonempty subsets Y,Z of a metric space (X,d) (where dist(Y,Z)=∞ otherwise), defines a metric.

1. True
2. False



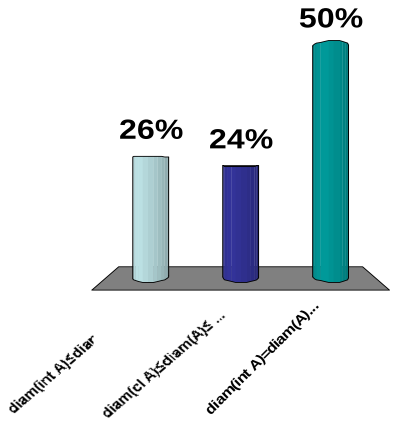
A pseudometric differs from a metric by

1. (MS1): A pseudometric might not be nonnegative
2. (MS2): A pseudometric might not be 0 iff x=y
3. (MS3): A pseudometric might not be symmetric
4. (MS4): Only a modified form weak form of the triangle inequality has to hold.



Let A be a subset of a metric space (X,d). What is the relationship between diam(int A), diam(A) and diam(cl A)?

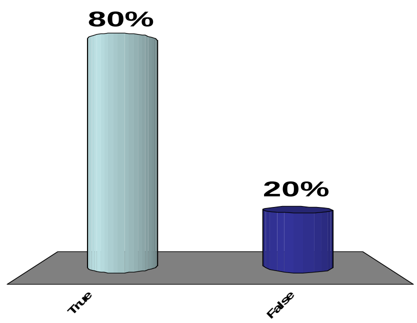
1. diam(int A)≤diam(A)= diam(cl A)
2. diam(cl A)≤diam(A)≤ diam(int A)
3. diam(int A)=diam(A)= diam(cl A)



Let (X,d) be a metric space and (x_n) be a sequence in X. Set A={x_n|n in **N**}.

Is the following statement true or false:
If (x_n) is Cauchy, then the subspace A is totally bounded.

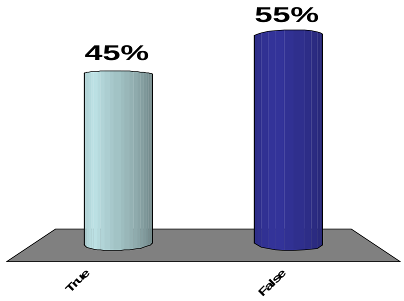
1. True
2. False



Let (X,d) be a metric space and (x_n) be a sequence in X. Set A={x_n|n in **N**}.

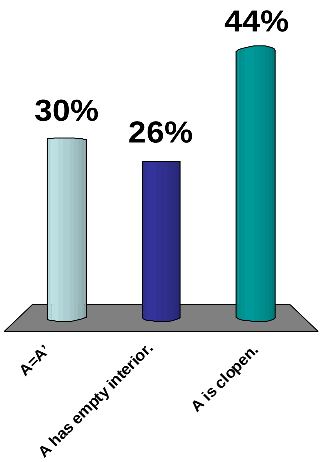
Is the following statement true or false: If (x_n) converges to x, then the union of A and {x} is a compact set.

1. True
2. False



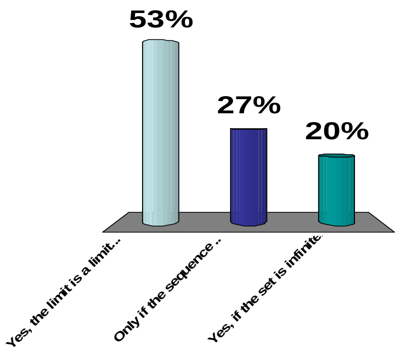
Let A be a subset of a metric space (X,d) s.t. ∂A=Ø. What can you say about A?

1. A=A’
2. A has empty interior.
3. A is clopen.



Is the limit of a converging sequence (x_n) a limit point of the set $\{x_n | n \text{ in } \mathbf{N}\}$?

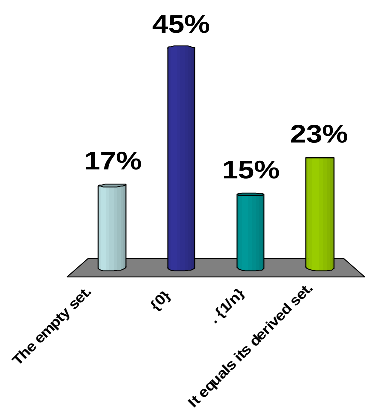
1. Yes, the limit is a limit point.
2. Only if the sequence is not eventually constant.
3. Yes, if the set is infinite.



What is the derived set of

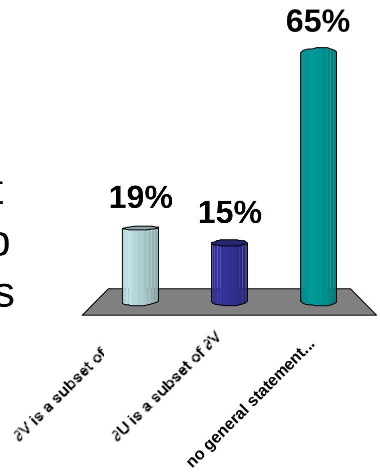
$$\left\{ \frac{1}{m} + \frac{1}{n} \,\middle|\, m, n \in \mathbf{N} \right\}?$$

1. The empty set.
2. {0}
3. $\left\{ \frac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$
4. It equals its derived set.



Let U,V be subsets of a metric space (X,d). If ∂U is a subset of V, and V is a subset of U, then

1. ∂V is a subset of ∂U
2. ∂U is a subset of ∂V
3. no general statement about the relationship between ∂U and ∂V is possible



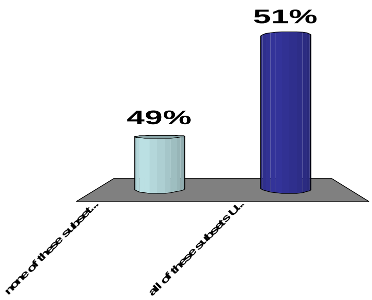
“Quick and dirty” solution:

We have $\partial U = \text{cl } U \cap \text{cl } U^c =$
$\text{cl } U \cap \text{cl } U^c \cap \text{cl } U^c = \partial U \cap \text{cl } U^c$.

But ∂U is a subset of V and thus cl V, U^c is a subset of V^c, thus cl U^c is a subset of cl V^c; therefore ∂U is a subset of $\text{cl } V \cap \text{cl } V^c = \partial V$.

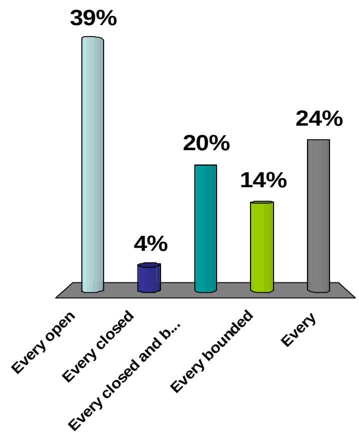
The open subsets of the subspace **R**x{0} of $\mathbf{R}^2$ with the Euclidean metric are precisely those subsets Ux{0} where U is open in **R**. Thus, except the empty set,

1. none of these subsets Ux{0} is also open in $\mathbf{R}^2$
2. all of these subsets Ux{0} is also open in $\mathbf{R}^2$



<u>???</u> interval of **R** is a continuous image of **R** itself.
What can you replace <u>???</u> with?

1. Every open
2. Every closed
3. Every closed and bounded
4. Every bounded
5. Every



Some hints…

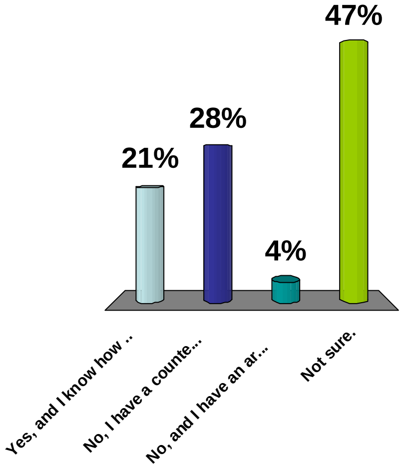
sin(**R**) = [-1,1]

arctan(**R**)=(-π/2,π/2)

f(x) = sin(x) if x≤0 and f(x)=arctan(x) of x>0 is continuous (or f(x)=x if …)!

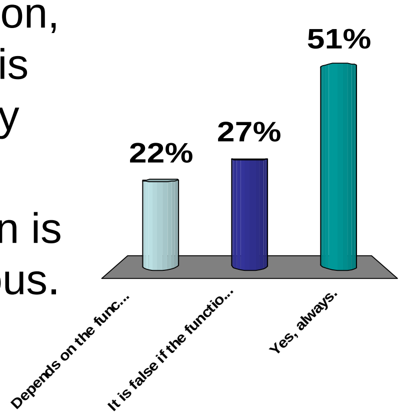
Is a continuous function on a complete metric space in general bounded?

1. Yes, and I know how to prove it.
2. No, I have a counterexample.
3. No, and I have an argument.
4. Not sure.



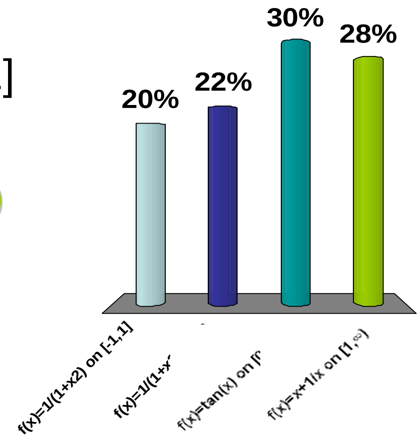
Is a continuous bijection between sequentially compact metric spaces a homeomorphism?

1. Depends on the function, however this function is automatically uniformly continuous.
2. It is false if the function is not uniformly continuous.
3. Yes, always.



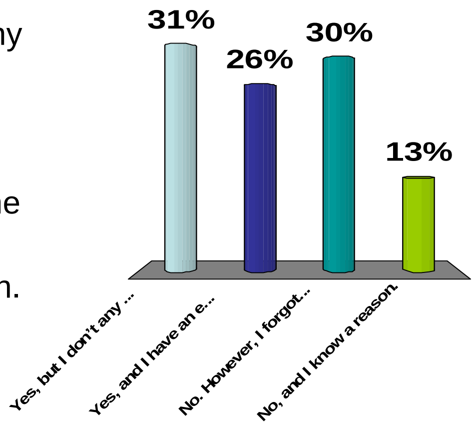
Which of the following functions is ***not*** uniformly continuous on the specified domain?

1. $f(x)=1/(1+x^2)$ on $[-1,1]$
2. $f(x)=1/(1+x^2)$ on **R**
3. $f(x)=\tan(x)$ on $[0,\pi/2)$
4. $f(x)=x+1/x$ on $[1,\infty)$



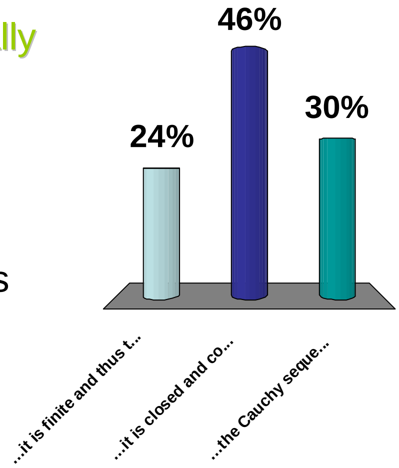
Can a complete metric space be a subspace of an incomplete metric space?

1. Yes, but I don’t know any example.
2. Yes, and I have an example.
3. No. However, I forgot the appropriate theorem.
4. No, and I know a reason.



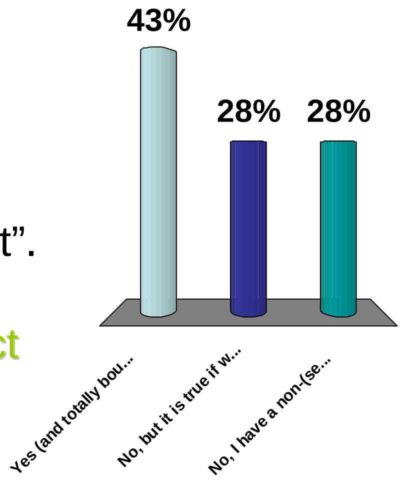
A discrete metric space is sequentially compact iff…

1. …it is finite and thus totally bounded.
2. …it is closed and complete (it is always bounded!).
3. …the Cauchy sequences are eventually constant.



Is every complete metric space sequentially compact?

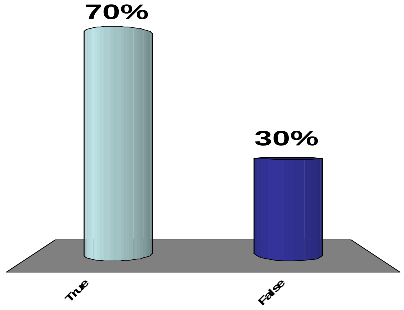
1. Yes (and totally bounded).
2. No, but it is true if we replace "sequentially compact" by "compact".
3. No, I have a non-(sequentially) compact counterexample.



Let (X,d) be a metric space with subsets U,V.

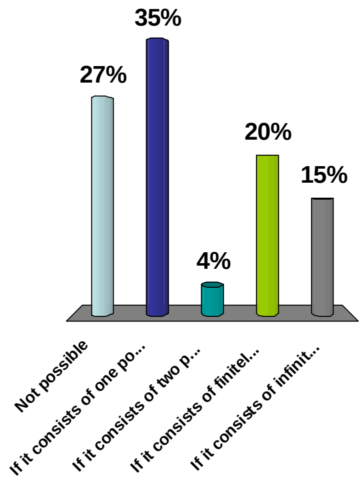
Is the following statement true or false?
If U, V are compact, then their union is also compact.

1. True
2. False



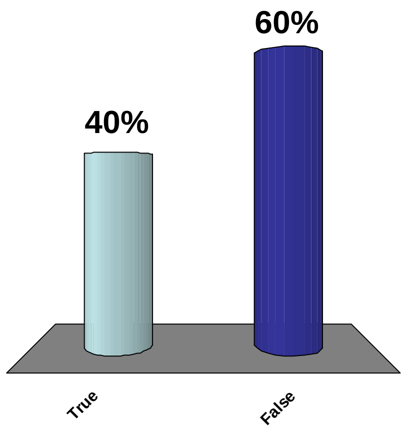
When is a discrete metric space connected?

1. Not possible
2. If it consists of one point only
3. If it consists of two points
4. If it consists of finitely many points
5. If it consists of infinitely but countably many points.



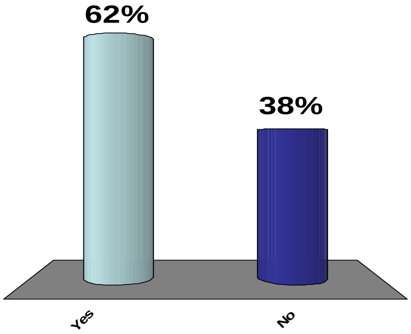
A ball in a connected metric space need not be connected.

1. True
2. False

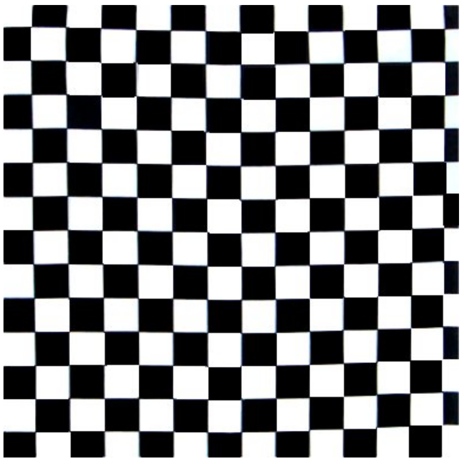


Let A be a subset of a metric space (X,d). If the boundary ∂A is connected, is A itself connected?

1. Yes
2. No



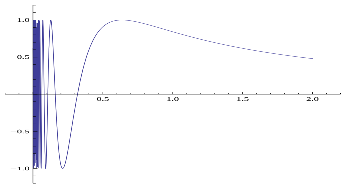
The checkerboard:

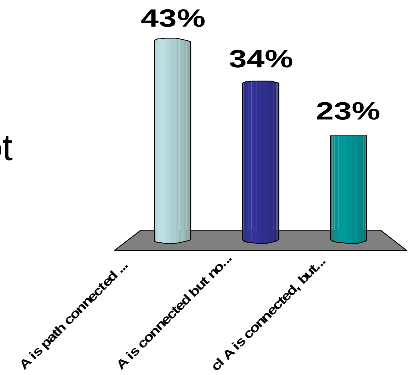


Consider the set $A=\left\{\left(x,\sin\frac{1}{x}\right)\middle|\, x>0\right\}\subset \mathbb{R}^2$.

Which statement is ***false***.

1. A is path connected and connected.
2. A is connected but not path connected.
3. cl A is connected, but not path connected.





Evaluation:

Using the Audience Response Systems (ARS) for this revision session was a good idea.

1. Strongly Agree
2. Agree
3. Neutral
4. Disagree
5. Strongly Disagree