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Natural convection flow from a vertical permeable flat plate with variable surface temperature and species concentration

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Abstract *Concerns the laminar flows from a permeable heated surface which arise in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species. Species concentration levels in air are assumed to be small in many processes in the atmosphere. Under the usual Boussinesque approximations, a set of non-similar equations for combined buoyancy effects and the permeability of the surface are obtained. The resulting equations have been integrated by four distinct methods: perturbation method for small transpiration rate; asymptotic solutions for large transpiration rate; Keller-box methods; and local non-similarity method for any transpiration rate. Effects of various practical values of the Schmidt number, of the multiple buoyancy parameter and that of the transpiration rate of fluid through the surface on the local skin-friction, the local Nusselt number and the local Sherwood number are shown graphically as well as in tabular form.*

Nomenclature

C = concentration in the boundary layer
C_∞ = ambient concentration
C_{fx} = local skin-friction
D = molecular diffusivity of the species concentration
g = gravitational acceleration
Gr_x = modified Grashof number
$Gr_{x,C}$ = Grashof number for concentration diffusion
$Gr_{x,T}$ = Grashof number for thermal diffusion
n = temperature or concentration gradient (equation (5))
N = $N = Gr_{x,C}/Gr_{x,T}$
Nu_x = local Nusselt number
Pr = Prandtl number
Sc = Schmidt number
Sh_x = local Sherwood number
T = temperature of the fluid
T_∞ = free stream temperature
u = axial velocity component
v = velocity component normal to u
w = $w = N/(1 + N)$
V = transpiration velocity
x = axial coordinate
y = coordinate normal to x

Greek letters

α = thermal diffusivity
β_T = volumetric expansion coefficient for temperature
β_C = volumetric expansion coefficient for concentration
ϕ = dimensionless concentration function
ψ = stream function
θ = dimensionless temperature function
ν = kinematic viscosity
μ = dynamic viscosity
ζ = transpiration parameter
η = pseudo similarity variable

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Introduction

The importance of suction and blowing in controlling the boundary layer thickness and the rate of heat transfer has motivated many researchers to investigate its effects on forced and free convection flows. Eichhorn (1960) considered power law variations in the plate temperature and transpiration velocity and gave similarity solutions of the problem. Sparrow and Cess (1961) discussed the case of constant plate temperature and transpiration velocity and obtained series expansions for temperature and velocity distributions in powers of $x^{1/2}$, where x is the distance in the stream-wise direction measured from the leading edge. Later, Merkin (1972; 1975) and Perikh *et al.* (1971) presented numerical solutions for free convection heat transfer with blowing along an isothermal vertical flat plate. Hartnett and Eckert (1975) and Sparrow and Starr (1966) reported the characteristics of heat transfer and skin-friction for pure forced convection with blowing; the former dealt with a non-similar case. Locally non-similar solutions for convection flow with arbitrary transpiration velocity were obtained by Kao (1975; 1976), applying Görtler-Meksin transformations. Free convection flow along a vertical plate with arbitrary blowing and wall temperature has also been investigated by Vedhanayagam *et al.* (1980). Lin and Yu (1988) investigated the free convection flow over a horizontal plate, considering temperature and transpiration rates which both followed power-law variations.

Flows arising from differences in concentration or material constitution, alone or in conjunction with temperature effects, have also received much attention by researchers, as these types of flows are of great practical importance. Clearly atmospheric flows, at all scales, are driven appreciably by both temperature and water concentration differences. Also, flows in bodies of water are driven through equally important effects of temperature, of concentration of dissolved materials, and of suspended particulate matter. Much information on simultaneous heat and mass transfer in laminar free convection boundary layer flows over plates can be found in the monograph by Gebhart *et al.* (1988) and in the papers by Khair and Bejan (1985), Lin and Yu (1995, 1997) and Mongruel *et al.* (1996).

Simultaneous heat and mass transfer in buoyancy-induced laminar boundary layer flow along a vertical plate was studied by Lin and Yu (1995) for any ratio of the solutal buoyancy force to the thermal buoyancy force by using a transformation. Tsuruno and Iguchi (1980) were the first to predict the effect of uniform blowing on combined forced and free convection heat transfer along a vertical isothermal plate, using a method similar to that of Terill (1960) and paying special attention to clarification of the limit between the combined convection and the effectively pure (either forced and free) convection region of flow. Hossain (1992) investigated the effect of uniform transpiration rate on the heat and mass transfer characteristics in mixed convection flow of a viscous incompressible fluid along a vertical permeable plate, under the combined effects of thermal and mass diffusion and subject to uniform wall temperature and species concentration. He considered the transpiration parameter given by

$V_0 = (V/U_0)Re_x^{1/2}/\xi$, where ξ is the mixed convection parameter representing the thermal buoyancy effect, U_0 is the free stream velocity and V is the transpiration at the plate. Recently, Hossain *et al.* (1999) investigated the problem of combined heat and mass transfer above a near-horizontal surface in a porous medium. The conjugate effect of heat and mass transfer in natural convection flow from a vertical wavy surface has been investigated by Hossain and Rees (1999).

The problem considered here is that of a natural convection boundary layer flow, influenced by the combined buoyancy forces from mass and thermal diffusion from a permeable vertical flat surface with non-uniform surface temperature and non-uniform surface species concentration, but with a uniform rate of suction of fluid through the permeable surface. The transformed boundary layer equations are solved numerically near to and far from the leading edge, using extended series solutions and asymptotic series solutions. Solutions for intermediate locations are obtained using the Keller-box technique (Keller, 1978) as well as by the local non-similarity method developed by Minkowycz and Sparrow (1978). In this investigation, consideration is given to the situation where the buoyancy forces have aiding effects, for various possible combinations of the buoyancy ratio parameter w, temperature and concentration gradient n, and Schmidt number, Sc, for fixed Prandtl number $Pr = 0.72$ against ζ, the stream-wise distribution of suction of fluid through the surface. The results illustrate the different behavior that occurs when these parameters are varied.

Formulation of the problem

A two-dimensional steady free convective flow of a viscous incompressible fluid along a permeable vertical flat plate in the presence of soluble species is considered. The coordinate system and the flow configuration are shown in Figure 1.

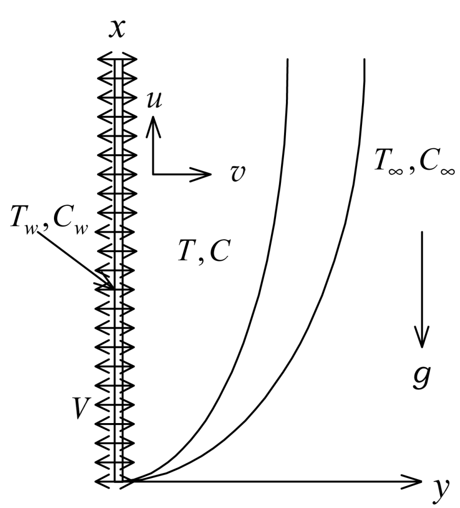

Figure 1.
Physical configuration and coordinate system

Under the usual Boussinesq approximation the flow, the heat and mass transfer processes are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g(\beta_T\theta+\beta_C\phi) \tag{2}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\alpha\frac{\partial^2 \theta}{\partial y^2} \tag{3}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D\frac{\partial^2 \phi}{\partial y^2} \tag{4}$$

$$\begin{aligned} &u(x,y)=0, v(x,y)=-V, \theta=\theta_0 x^n, \phi=\phi_0 x^n \text{ at } y=0\\ &u(x,y)=0, \theta=0, \phi=0 \text{ at } y=\infty \end{aligned} \tag{5}$$

where u and v are the x- and y- components of the velocity field, respectively, g is the acceleration due to gravity, β_T and β_C are the volumetric expansion coefficients for temperature and concentration, respectively, α is the thermal diffusivity and D is the molecular diffusivity of the species concentration. Further, $\theta = T - T_\infty$ where T and T_∞ are the temperature of the fluid and the ambient temperature, and $\phi = C - C_\infty$ is the difference between species concentration in the boundary layer and the ambient concentration. In equation (5), V represents the suction velocity of fluid through the surface of the plate. In this study we shall consider only the suction case (rather than blowing) and therefore V is taken as positive throughout. θ_0 and ϕ_0 are respectively the constant temperature and constant species concentration at the surface of the plate. In this study we have neglected stratification, viscous dissipation and other additional effects.

As suggested by Gebhart and Pera (1971a; 1971b), the cross-diffusion effects (i.e. Soret and Dufour effects) are assumed to be negligible compared with the direct effects, modeled by Fourier's law and Fick's law.

We now define the following group of transformations to reduce the above equations to convenient form:

$$\begin{aligned} &\psi(x,y)=\nu Gr_x^{1/4}[f(\zeta,\eta)+\zeta], \quad \theta(x,y)=g(\zeta,\eta,)\ ,\\ &\phi(x,y)=h(,\zeta,\eta), \quad \eta=\frac{y}{x}Gr_x^{1/4}, \quad \zeta=\frac{Vx}{\nu}Gr_x^{1/4} \end{aligned} \tag{6}$$

where ψ is the stream function, defined by:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

which satisfies the continuity equation (1). Here f, g, h, are the non-dimensional stream-function, temperature and concentration functions, respectively; η is the pseudo-similarity variable and ζ is the transpiration parameter which may also be interpreted as being a scaled stream-wise variable. We define:

$$Gr_x = \frac{g[\beta_T \theta_w + \beta_C \phi_w]}{\nu^2} x^3 = Gr_{x,T} + Gr_{x,C} \tag{8}$$

as the modified local Grashof number, with $Gr_{x,T}$ being the local Grashof number for thermal diffusion and $Gr_{x,C}$ the local Grashof number for mass diffusion.

Now, substituting the above group of transformations given in equation (6) into equations (2)-(5) one obtains the following non-similarity equations:

$$\begin{aligned} f''' + \frac{n+3}{4} ff'' - \frac{n+1}{2} f'^2 + \zeta f'' + (1-w)g + wh \\ = \frac{1-n}{4} \zeta \left[f' \frac{\partial f'}{\partial \zeta} - f'' \frac{\partial f}{\partial \zeta} \right] \end{aligned} \tag{9}$$

$$\frac{1}{\Pr} g'' + \frac{n+3}{4} fg' + \zeta g' = \frac{1-n}{4} \zeta \left[f' \frac{\partial g}{\partial \zeta} - g' \frac{\partial f}{\partial \zeta} \right] \tag{10}$$

$$\frac{1}{Sc} h'' + \frac{n+3}{4} fh' + \zeta h' = \frac{1-n}{4} \zeta \left[f' \frac{\partial h}{\partial \zeta} - h' \frac{\partial f}{\partial \zeta} \right] \tag{11}$$

In these equations, primes denote differentiation of the functions with respect to η.

The boundary conditions appropriate to the above equations are:

$$\begin{aligned} & f(\zeta, 0) = f'(\zeta, 0) = 0, \quad g(\zeta, 0) = h(\zeta, 0) = 1 \\ & f'(\zeta, \infty) = g(\zeta, \infty) = h(\zeta, \infty) = 0 \end{aligned} \tag{12}$$

In equation (9), $w = N/(1 + N)$, where $N = Gr_{x,C}/Gr_{x,T}$ and w measures the relative importance of solutal and thermal diffusion in causing the density changes which drive the flow. It is to be noted that $N = 0$ corresponds to no species diffusion and infinity to no thermal diffusion.

Once we know the solutions of the above equations, the physical quantities of interest are the skin-friction, the Nusselt number and the Sherwood number which may be calculated from:

$$C_{fx} Gr_x^{3/4} = f''(\zeta, 0), \quad Nu_x Gr_x^{-1/4} = -g'(\zeta, 0), \quad Sh_x Gr_x^{-1/4} = -h'(\zeta, 0) \tag{13}$$

For an impermeable plate (i.e. with $\zeta = 0$), the set of similarity equations (9)-(12) take the form of the equations investigated by Gebhart and Pera (1971a; 1971b), by simply converting w to N. Solutions obtained by the aforementioned authors for the skin-friction, the Nusselt number or Sherwood number may be reproduced simply by dividing or multiplying the present values by the factor $\sqrt{2}$ respectively.

The purpose of the calculations given here is to assess the effects of the parameters Sc, w or n and ζ upon the nature of the flow and transport. Results are limited to the case of Prandtl number Pr = 0.72, representing air at 200°C at 1 atmosphere. The effect of different Prandtl number, Pr, has already been discussed by Gebhart and Pera (1971a) for the flow past an impermeable plate.

Solution methodologies

In the present investigation we shall integrate the equations (9)-(12) for small and large values of ζ by the perturbation method, and for all ζ values by the implicit finite difference method of Keller (1978) as well as the local non-similarity method of Minkowycz and Sparrow (1978).

Extended series solutions (ESS) for small ζ

Near the leading edge, or equivalently for small ζ, we expand the functions f, g and h in powers of ζ as given below:

$$f(\eta,\zeta) = \sum_{i=0}^{\infty} \zeta^i f_i(\eta), \quad g(\eta,\zeta) = \sum_{i=0}^{\infty} \zeta^i g_i(\eta), \quad h(\eta,\zeta) = \sum_{i=0}^{\infty} \zeta^i h_i(\eta) \qquad (14)$$

On substituting these into equations (9)-(12) and equating the terms of like powers of ζ to zero, the leading order equations are obtained as follows:

$$f_0''' + \frac{n+3}{4} f_0 f_0'' - \frac{n+1}{2} f_0'^2 + (1-w) g_0 + w h_0 = 0 \qquad (15)$$

$$\frac{1}{\mathrm{Pr}} g_0'' + \frac{n+3}{4} f_0 g_0' = 0 \qquad (16)$$

$$\frac{1}{Sc} h_0'' + \frac{n+3}{4} f_0 h_0' = 0 \qquad (17)$$

and the boundary conditions are:

$$\begin{aligned} &f_0 = f_0' = 0, \quad g_0 = 1, \quad h_0 = 1 \quad \text{at} \quad \eta = 0 \\ &f_0' \to 0, \quad g_0 \to 0, \quad h_0 \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \qquad (18)$$

The higher order equations, for $i \geq 1$, are obtained as given below:

$$f_i''' + f_{i-1}'' + (1-w)g_i + wh_i = \sum_{r=0}^{i}\left[\left(\frac{n+1}{2} + \frac{r(1-n)}{4}\right)f_r'f_{i-r}' - \left(\frac{n+3}{4} + \frac{r(1-n)}{4}\right)f_r f_{i-r}''\right] \quad (19)$$

$$\frac{1}{\Pr}g_i'' + g_{i-1}' = \sum_{r=0}^{i}\left[\frac{r(1-n)}{4}g_r f_{i-r}' - \left(\frac{n+3}{4} + \frac{r(1-n)}{4}\right)f_r g_{i-r}'\right] \quad (20)$$

$$\frac{1}{Sc}h_i'' + h_{i-1}' = \sum_{r=0}^{i}\left[\frac{r(1-n)}{4}h_r f_{i-r}' - \left(\frac{n+3}{4} + \frac{r(1-n)}{4}\right)f_r h_{i-r}'\right] \quad (21)$$

In the above f_0, g_0 and h_0 are the well-known free convection similarity solutions for flow around a constant temperature semi-infinite vertical plate, and the functions f_i, g_i and h_i ($i = 1, 2, 3, \ldots$) are effectively first and higher order corrections to the flow due to the effect of the transpiration of fluid through the surface of the plate. Further, the equations (14)-(20) (for each $i \geq 1$) are linear, but coupled, and may be found by pair-wise sequential solution. These pairs of equations have been integrated using an implicit Runge-Kutta-Butcher (Butcher, 1964) initial value solver together with the iteration scheme of Nachtsheim and Swigert (1965). In the present investigation, solutions of 13 pairs of equations have been obtained.

The solution of the above equations enables the calculation of various flow parameters near the leading edge, such as the skin-friction, C_{fx}, Nusselt number, Nu_x, and the Sherwood number, Sh_x. Using the relations given in equation (13), the quantities C_{fx}, Nu_x and Sh_x can now be calculated respectively from the following expressions:

$$C_{fx}Gr_x^{-3/4} = f''(\zeta, 0) = \zeta^{1/4}(f_0'' + \zeta f_1'' + \zeta^2 f_2'' + \ldots) \quad (22)$$

$$Nu_x Gr_x^{-1/4} = -g'(\zeta, 0) = -(g_0' + \zeta g_1' + \zeta^2 g_2' + \ldots) \quad (23)$$

$$Sh_x Gr_x^{-1/4} = -h'(\zeta, 0) = -(h_0' + \zeta h_1' + \zeta^2 h_2' + \ldots) \quad (24)$$

Asymptotic solutions (ASS) for large ζ

In this section attention is given to the solution of equations (9)-(12) when ζ is large. The order of magnitude analysis of various terms in (9)-(11) shows that the largest are f''' and $\zeta f''$ in equation (9), g'' and $\zeta g'$ in equation (10), and h'' and $\zeta h'$ in equation (11). In the respective equations both the terms have to be balanced in magnitude and the only way to do this is to assume that η is small and hence derivatives are large. Given that $g = O(1)$ and $h = O(1)$ as $\zeta \to \infty$, it is essential to find appropriate scaling for f and η. On balancing the f''', g, h and $\zeta f''$ term in equation (9), it is found that $\eta = O(\zeta^{-1})$ and $f = O(\zeta^{-3})$ as $\zeta \to \infty$. Therefore, the following transformations may be introduced:

$$f = \zeta^{3}\bar{f}(\varsigma, \eta), \quad \eta = \zeta\bar{\eta}, \quad g = \bar{g}(\zeta, \eta), \quad h = \bar{h}(\zeta, \eta) \tag{25}$$

Equations (9)-(12) together with the transformations given in equation (25) then become:

$$\bar{f}''' + \bar{f}'' + (1-w)\bar{g} + w\bar{h} + n\zeta^{-4}\bar{f}\bar{f}'' - n\zeta^{-4}\bar{f}'^{2} = \frac{1-n}{4}\zeta^{-3}\left[\bar{f}'\frac{\partial \bar{f}'}{\partial \zeta} - \bar{f}''\frac{\partial \bar{f}}{\partial \zeta}\right] \tag{26}$$

$$\frac{1}{\Pr}\bar{g}'' + \bar{g}' + n\zeta^{-4}\bar{f}\bar{g}' = \frac{1-n}{4}\zeta^{-3}\left[\bar{f}'\frac{\partial \bar{g}}{\partial \zeta} - \bar{g}'\frac{\partial \bar{f}}{\partial \zeta}\right] \tag{27}$$

$$\frac{1}{Sc}\bar{h}'' + \bar{h}' + n\zeta^{-4}\bar{f}\bar{h}' = \frac{1-n}{4}\zeta^{-3}\left[\bar{f}'\frac{\partial \bar{h}}{\partial \zeta} - \bar{h}'\frac{\partial \bar{f}}{\partial \zeta}\right] \tag{28}$$

For sufficiently large ζ we expand the functions in powers of ζ^{-4}, considering the terms up to $O(\zeta^{-4})$. Equations for zeroth order then take the form:

$$\bar{f}_0''' + \bar{f}_0'' + (1-w)\bar{g}_0 + w\bar{h}_0 = 0 \tag{29}$$

$$\frac{1}{\Pr}\bar{g}_0'' + \bar{g}_0' = 0 \tag{30}$$

$$\frac{1}{Sc}\bar{h}_0'' + \bar{h}_0' = 0 \tag{31}$$

These equations satisfy the following boundary conditions:

$$\begin{gathered} \bar{f}(0) = \bar{f}'(0) = 0, \quad \bar{g}(0) = \bar{h}(0) = 1 \\ \bar{f}'(\infty) = \bar{g}(\infty) = \bar{h}(\infty) = 0 \end{gathered} \tag{32}$$

The equations for the order of ζ^{-4} are:

$$\bar{f}_1''' + \bar{f}_0\bar{f}_0'' - \bar{f}_0'^{2} + \bar{f}_1'' + (1-w)\bar{g}_1 + w\bar{h}_1 = 0 \tag{33}$$

$$\frac{1}{\Pr}\bar{g}_1'' + \bar{g}_1' + n\bar{f}_0\bar{g}_0' = 0 \tag{34}$$

$$\frac{1}{Sc}\bar{h}_1'' + \bar{h}_1' + n\bar{f}_0\bar{h}_0' = 0 \tag{35}$$

$$\begin{aligned} &\bar{f}_1 = \bar{f}_1' = 0, \quad \bar{g}_1 = \bar{h}_1 = 0 \text{ at } \eta = 0 \\ &\bar{f}_1' \to 0, \quad \bar{g}_1 \to 0, \quad \bar{h}_1 \to 0 \text{ as } \eta \to \infty \end{aligned} \tag{36}$$

The solutions of the equations (29)-(36) enable us to calculate various flow parameters. The skin friction, $C_{fx}Gr_x^{-3/4}$, the rate of heat transfer in terms of the local Nusselt number $Nu_xGr_x^{-1/4}$ and the rate of mass transfer in terms of the local Sherwood number $Sh_xGr_x^{-1/4}$ at the surface are as given below:

$$\begin{aligned} C_{fx}Gr_x^{-3/4} = {} & \zeta^{-1}\left(\frac{1-w}{\Pr} + \frac{w}{Sc}\right) + \varsigma^{-5}\left[nA_9 - \frac{1}{2\Pr}\{(B_1 - nA_5 + E_3)\right. \\ & + 2(nA_6 - E_1)\} - \frac{1}{2Sc}\{(B_3 - nA_7 + F_3) + 2(nA_8 - F_1)\} \\ & - \frac{3}{2}(B_5 - nA_1) - \frac{(nA_2 - B_2 - E_2)}{\Pr + 1} - \frac{(nA_3 - B_4 - F_2)}{Sc + 1} \\ & \left. - \frac{(E_4 - nA_4 + F_4)}{\Pr + Sc}\right] + \ldots \end{aligned} \tag{37}$$

$$\begin{aligned} Nu_xGr_x^{-1/4} = {} & \zeta\Pr + \zeta^{-3}n\left[\frac{(1-w)(1+\Pr - 2\Pr^2)}{2\Pr(1-\Pr^2)}\right. \\ & \left. + \frac{w\Pr^2(1 + \Pr - \Pr Sc - Sc^2)}{Sc^2(1-Sc)(1+\Pr)(\Pr + Sc)}\right] + \ldots \end{aligned} \tag{38}$$

$$\begin{aligned} Sh_xGr_x^{1/4} = {} & \zeta Sc + \zeta^{-3}n\left[\frac{w(1 + Sc - 2Sc^2)}{2Sc(1-Sc^2)} + \right. \\ & \left. \frac{Sc^2\{w(1+Sc) - (1-w)\Pr(\Pr + Sc)\}}{\Pr^2(1-\Pr)(\Pr + Sc)(1+Sc)}\right] + \ldots \end{aligned} \tag{39}$$

where:

$$A_1 = \frac{(1-w)^2}{\Pr^2(1-\Pr)^2} + \frac{2w(1-w)}{\Pr Sc(1-\Pr)(1-Sc)} + \frac{w^2}{Sc^2(1-Sc)^2}$$

$$A_2 = \frac{(1-w)^2(1+\Pr^2)}{\Pr^3(1-\Pr)^2} + \frac{w(1-w)}{Sc(1-\Pr)(1-Sc)} + \frac{w(1-w)}{\Pr^2 Sc(1-\Pr)(1-Sc)}$$

$$A_3 = \frac{w(1-w)}{\Pr Sc^2(1-\Pr)(1-Sc)} + \frac{w(1-w)}{\Pr(1-\Pr)(1-Sc)} + \frac{w^2(1+Sc^2)}{Sc^3(1-Sc)^2}$$

$$A_4 = \frac{w(1-w)}{Sc^2(1-\Pr)(1-Sc)} + \frac{w(1-w)}{\Pr^2(1-\Pr)(1-Sc)}, \quad A_5 = \frac{(1-w)^2}{\Pr^2(1-\Pr)^2}$$

$$A_6 = \frac{(1-w)^2}{\Pr^2(1-\Pr)} + \frac{(1-w)w}{Sc^2(1-\Pr)}, \quad A_7 = \frac{w^2}{Sc^2(1-Sc)^2}$$

$$A_8 = \frac{w(1-w)}{\Pr^2(1-\Pr)} + \frac{w^2}{Sc^2(1-Sc)}$$

$$A_9 = \frac{(1-w)^2}{\Pr^3(1-\Pr)} + \frac{(1-w)w}{\Pr Sc^2(1-\Pr)} + \frac{w(1-w)}{\Pr^2 Sc(1-Sc)} + \frac{w^2}{Sc^3(1-Sc)}$$

$$B_1 = \frac{(1-w)^2}{\Pr^2(1-\Pr)^2}, \quad B_2 = \frac{2(1-w)^2}{\Pr^2(1-\Pr)^2}, \quad B_3 = \frac{w^2}{Sc^2(1-Sc)^2}$$

$$B_4 = \frac{2w^2}{Sc^2(1-Sc)^2}, \quad B_5 = \frac{(1-w)^2}{\Pr^2(1-\Pr)^2} + \frac{w^2}{Sc^2(1-Sc)^2}$$

$$E_1 = \frac{n(1-w)^2}{1-\Pr}\left(\frac{1}{2\Pr^2} - \frac{\Pr}{1+\Pr}\right) + \frac{nw(1-w)\Pr^2}{Sc(1-Sc)}\left(\frac{1}{Sc^2(\Pr+Sc)} - \frac{1}{\Pr+1}\right)$$

$$E_2 = \frac{n(1-w)^2\Pr}{(1+\Pr)(1-\Pr)} + \frac{nw(1-w)\Pr^2}{Sc(1-Sc)(1+\Pr)}, \quad E_3 = \frac{n(1-w)^2}{2\Pr^2(1-\Pr)}$$

$$E_4 = \frac{nw(1-w)\Pr^2}{Sc^3(1-Sc)(\Pr+Sc)}$$

$$F_1 = \frac{nw^2}{1-Sc}\left(\frac{1}{2Sc^2} - \frac{Sc}{1+Sc}\right) + \frac{nw(1-w)Sc^2}{\Pr(1-\Pr)}\left(\frac{1}{\Pr^2(\Pr+Sc)} - \frac{1}{Sc+1}\right)$$

$$F_2 = \frac{nw^2 Sc}{(1+Sc)(1-Sc)} + \frac{nw(1-w)Sc^2}{\Pr(1-\Pr)(1+Sc)}, \quad F_3 = \frac{nw^2}{2Sc^2(1-Sc)}$$

$$F_4 = \frac{nw^2 Sc^2}{\Pr^3(1-\Pr)(\Pr + Sc)}$$

From the above solutions it can be seen that for large ζ the value of the local skin-friction, $C_{fx}Gr_x^{-3/4} \approx [(1-w)/\Pr + w/Sc]/\zeta$, the local Nusselt number, $Nu_x Gr_x^{-1/4} \approx \Pr\zeta$ and the local Sherwood number $Sh_x Gr_x^{-1/4} \approx Sc\zeta$, which shows that the value of all these quantities are independent of the surface temperature and the surface species concentration gradient n.

These asymptotic solutions obtained for different values of the pertinent parameters have been compared with finite difference solutions, discussed in Table I and Figures 2 and 3.

Finite difference solutions (FDS) for entire ζ regime

For entire ζ regime we now proceed to integrate the locally non-similar partial differential equations (9)-(11) subject to the boundary conditions (12) using the implicit finite difference method. The partial differential equations (9)-(11) are first converted into a system of first order equations with dependent variables $u(\zeta,\eta)$, $v(\zeta,\eta)$, $p(\zeta,\eta)$ and $q(\zeta,\eta)$ as follows:

ζ	$Nu_x/Gr_x^{1/4} = -g'(\zeta,0)$ ESS and ASS	FDS	LNS	$Sh_x/Gr_x^{1/4} = -h'(\zeta,0)$ ESS and ASS	FDS	LNS
n = 0.0						
0.0	0.3485 *s*	0.3485	0.3488	0.4031 *s*	0.4031	0.4033
0.2	0.4198 *s*	0.4264	0.4187	0.5028 *s*	0.5125	0.5011
0.4	0.4988 *s*	0.5078	0.4977	0.6152 *s*	0.6282	0.6128
0.6	0.5857 *s*	0.5961	0.5845	0.7399 *s*	0.7550	0.7364
0.8	0.6804 *s*	0.6920	0.6783	0.8761 *s*	0.8929	0.8725
1.0	0.7827 *s*	0.7954	0.7801	1.0230 *s*	1.0411	1.0191
2.0	1.4031 *a*	1.4180	1.3977	1.8835 *a*	1.9034	1.8777
4.0	2.8000 *a*	2.8155	2.8000	3.7600 *a*	3.7806	3.7599
6.0	4.2000 *a*	4.2142	4.1999	5.6400 *a*	5.6589	5.6399
8.0	5.6000 *a*	5.6128	5.5859	7.5200 *a*	7.5370	7.5011
10.0	7.0000 *a*	7.0116	6.9719	9.4000 *a*	9.4153	9.3623
n = 0.5						
0.0	0.35711 *s*	0.3571	0.3572	0.4132 *s*	0.4132	0.4134
0.2	0.4318 *s*	0.4375	0.4318	0.5177 *s*	0.5260	0.5175
0.4	0.5146 *s*	0.5228	0.5143	0.6347 *s*	0.6468	0.6343
0.6	0.6050 *s*	0.6149	0.6049	0.7636 *s*	0.7779	0.7633
0.8	0.7030 *s*	0.7140	0.7036	0.9032 *s*	0.9191	0.9026
1.0	0.8081 *s*	0.8220	0.8085	1.0525 *s*	1.0697	1.0522
2.0	1.4109 *a*	1.4380	1.4227	1.8984 *a*	1.9237	1.9028
4.0	2.8694 *a*	2.8179	2.8029	3.7581 *a*	3.7830	3.7629
6.0	4.3165 *a*	4.2146	4.1868	5.6394 *a*	5.6593	5.6220
8.0	5.7585 *a*	5.6127	5.5863	7.5197 *a*	7.5371	7.5015
10.0	7.1992 *a*	7.0112	7.0002	9.3998 *a*	9.4152	9.3601

Notes: Here *s* stands for series solutions and *a* stands for asymptotic solutions

Table I. Comparison of the solutions obtained by different methods for local heat transfer and mass transfer against the transpiration parameter ζ for different n for $\Pr = 0.72$, $Sc = 0.94$ and $w = 1/3$

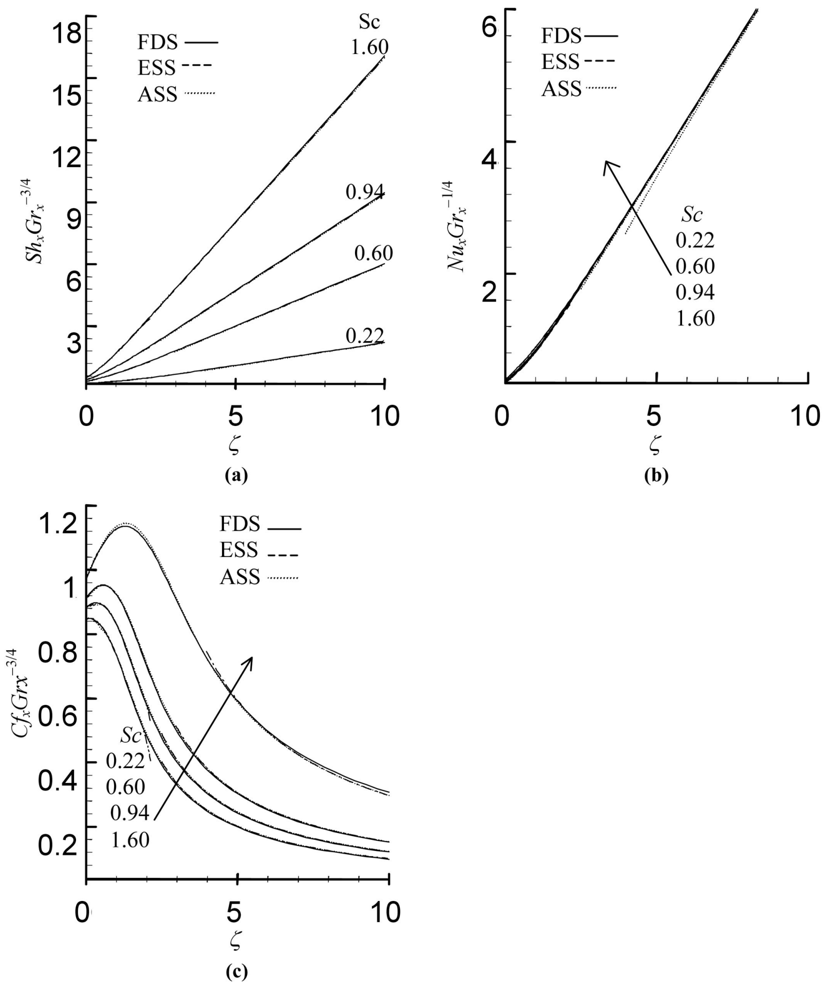

Figure 2.
Values of: (a) local mass transfer; (b) local heat transfer; and (c) local skin-friction against ζ for Pr = 0.72, w = 0.5, n = 0.5 for different Schmidt number, i.e. Sc =1.60, 0.94, 0.60 and 0.22

$$f' = u \tag{40}$$

$$u' = v \tag{41}$$

$$g' = p \tag{42}$$

$$h' = q \tag{43}$$

Equations (9)-(12) then take the forms:

$$v' + p_1 fv - p_2 u^2 + \zeta v + p_4 g + p_5 h = p_3 \zeta \left[u \frac{\partial u}{\partial \zeta} - v \frac{\partial f}{\partial \zeta} \right] \tag{44}$$

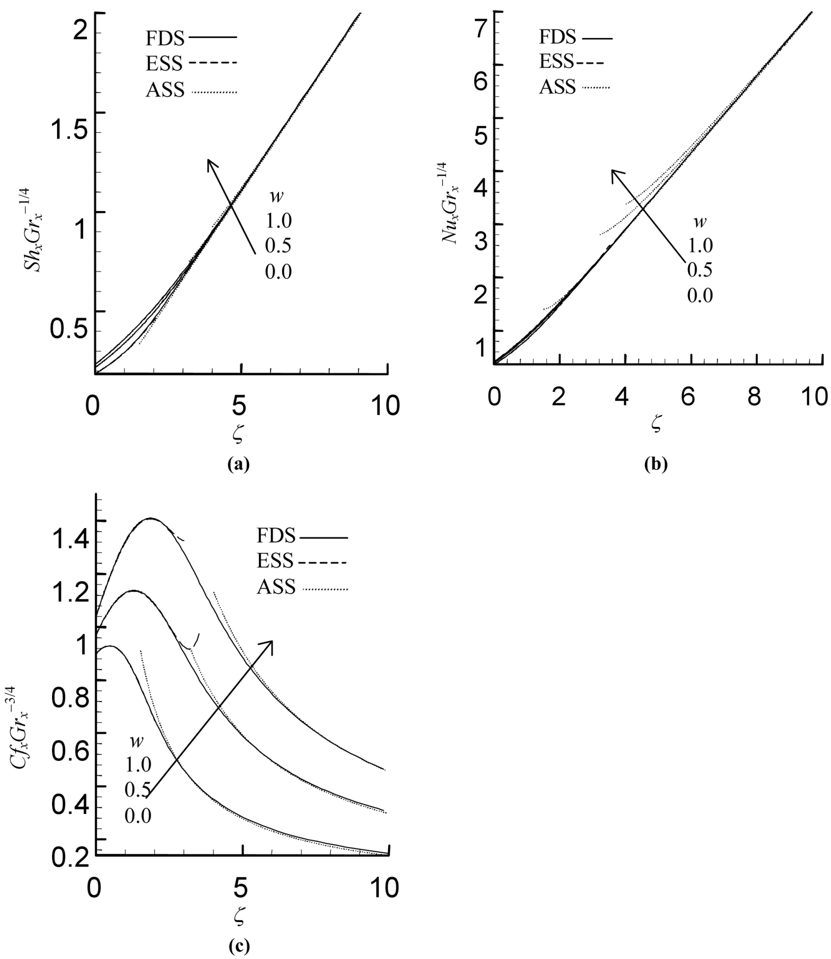

Figure 3.
Values of: (a) local mass transfer; (b) local heat transfer; and (c) local skin-friction against ζ for Pr = 0.72, Sc = 0.22 and n = 0.5 for different w

$$\frac{1}{\mathrm{Pr}}p' + p_1 f v + \zeta p = p_3 \zeta \left[u \frac{\partial g}{\partial \zeta} - p \frac{\partial f}{\partial \zeta} \right] \tag{45}$$

$$\frac{1}{Sc}q' + p_1 f v + \zeta q = p_3 \zeta \left[u \frac{\partial h}{\partial \zeta} - q \frac{\partial f}{\partial \zeta} \right] \tag{46}$$

$$\begin{aligned} &f = u = 0, \quad g = h = 1 \quad \text{at} \quad \eta = 0 \\ &u = 0, \quad g = h = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{47}$$

where

$$p_1 = \frac{n+3}{4}, \quad p_2 = \frac{n+1}{4}, \quad p_3 = \frac{1-n}{4}, \quad p_4 = 1 - w, \quad p_5 = w \tag{48}$$

We now consider the net rectangle on the (ζ,η) plane and denote the net points by:

$$\zeta^0 = 0,\ \zeta^i = \zeta^{i-1} + k_i \quad i = 1, 2, \ldots, M \tag{49}$$

$$\eta_0 = 0,\ \eta_j = \eta_{j-1} + l_j \quad j = 1, 2, \ldots, J,\ \eta_j = \eta_\infty \tag{50}$$

Here i and j index points on the (ζ,η) plane, and k_i and l_j give the variable mesh width.

We approximate the quantities (f, u, v, g, p, h, q) at points (ζ^l,η_j) of the net by $(f_j^i, u_j^i, v_j^i, g_j^i, p_j^i, h_j^i, q_j^i)$ which we call the net function. The notation m_j^i is also employed for any net function quantities midway between the net points as follows:

$$\zeta^{i-1/2} = \frac{1}{2}\left(\zeta^i + \zeta^{i-1}\right) \tag{51}$$

$$\eta_{j-1/2} = \frac{1}{2}\left(\eta_j + \eta_{j-1}\right) \tag{52}$$

$$m_j^{i-1/2} = \frac{1}{2}\left(m_j^i + m_j^{i-1}\right) \tag{53}$$

$$m_{j-1/2}^i = \frac{1}{2}\left(m_j^i + m_{j-1}^i\right) \tag{54}$$

We now write the difference equations that are to approximate equations (40)-(43) by considering one mesh rectangle. We start by writing the finite difference approximation to equations (40)-(43) using central difference quotients and average about the mid point $(\zeta^i,\eta_{j-1/2})$ to obtain:

$$\frac{f_j^i - f_{j-1}^i}{l_j} = u_{j-1/2}^i \tag{55}$$

$$\frac{u_j^i\ u_{j\ 1}^i}{l_j} = v_{j\ 1/2}^i \tag{56}$$

$$\frac{g_j^i - g_{j-1}^i}{l_j} = p_{j-1/2}^i \tag{57}$$

$$\frac{h_j^i - h_{j-1}^i}{l_j} = q_{j-1/2}^i \tag{58}$$

Similarly the equations (44)-(46) can be expressed in finite difference form, by approximating the functions and their derivatives by central differences about the midpoints $(\zeta^{i-1/2},\eta_{j-1/2})$, giving the following non-linear difference equations:

$$h_j^{-1}\left(v_j^i+v_{j-1}^i\right)+\alpha_1(fv)_{j-1/2}^i-\alpha_2\left(u^2\right)_{j-1/2}^i+\zeta_{j-1/2}^i v_{j-1/2}^i+p_4 g_{j-1/2}^i + p_5 h_{j-1/2}^i+\alpha_0\left(v_{j-1/2}^{i-1} f_{j-1/2}^i-f_{j-1/2}^{i-1} v_{j-1/2}^i=R_{j-1/2}^{i-1}\right) \tag{59}$$

$$\frac{1}{\Pr}\left[l_j^{-1}\left(p_j^i-p_{j-1}^i\right)\right]+\alpha_1(fp)_{j-1/2}^i+\zeta_{j-1/2}^i p_{j-1/2}^i+\alpha_0\left[u_{j-1/2}^{i-1} g_{j-1/2}^i - u_{j-1/2}^i g_{j-1/2}^{i-1}+p_{j-1/2}^i f_{j-1/2}^{i-1}-p_{j-1/2}^{i-1} f_{j-1/2}^i\right] = T_{j-1/2}^{i-1} \tag{60}$$

$$\frac{1}{Sc}\left[l_j^{-1}\left(q_j^i-q_{j-1}^i\right)\right]+\alpha_1(fq)_{j-1/2}^i+\zeta_{j-1/2}^i q_{j-1/2}^i+\alpha_0\left[u_{j-1/2}^{i-1} h_{j-1/2}^i - u_{j-1/2}^i h_{j-1/2}^{i-1}+q_{j-1/2}^i f_{j-1/2}^{i-1}-q_{j-1/2}^{i-1} f_{j-1/2}^i\right] = X_{j-1/2}^{i-1} \tag{61}$$

where:

$$R_{j-1/2}^{i-1}=-L_{j-1/2}^{i-1}+\alpha_0\left[(fv)_{j-1/2}^{i-1}-\left(u^2\right)_{j-1/2}^{i-1}\right] \tag{62}$$

$$L_{j-1/2}^{i-1}=\left[l_j^{-1}(v_j-v_{j-1})+\alpha_1(fv)_{j-1/2}.-\alpha_2\left(u^2\right)_{j-1/2} + \zeta_{j-1/2}^i\left(v^i\right)p_4 g_{j-1/2}+p_s h_{j-1/2}\right]^{i-1} \tag{63}$$

$$T_{j-1/2}^{i-1}=-M_{j-1/2}^{i-1}+\alpha_0\left[(fp)_{j-1/2}^{i-1}-(ug)_{j-1/2}^{i-1}\right] \tag{64}$$

$$M_{j-1/2}^{i-1}=\left[\frac{1}{\Pr}l_j^{-1}\left(p_j-p_{j-1}\right)+p_1(fp)_{j-1/2}+\zeta p_{j-1/2}\right]^{i-1} \tag{65}$$

$$X_{j-1/2}^{i-1}=-Y_{j-1/2}^{i-1}+\alpha_0\left[(fq)_{j-1/2}^{i-1}-(uh)_{j-1/2}^{i-1}\right] \tag{66}$$

$$Y_{j-1/2}^{i-1}=\left[\frac{1}{Sc}l_j^{-1}\left(q_j-q_{j-1}\right)+p_1(fq)_{j-1/2}+\zeta q_{j-1/2}\right]^{i-1} \tag{67}$$

$$\alpha_0 = p_3 k_i^{-1} \zeta^{i-1},\ \alpha_1 = p_1^i + \alpha_0,\ \alpha_2 = p_2^i + \alpha \tag{68}$$

The wall and the edge boundary conditions are:

$$\begin{aligned} &f_0^i = 0,\ u_0^i = 0,\ g_0^i = 1,\ h_0^i = 1 \\ &u_j^i = 0,\ g_j^i = h_j^i = 0 \end{aligned} \tag{69}$$

If we assume f_j^{i-1}, u_j^{i-1}, v_j^{i-1}, g_j^{i-1}, p_j^{i-1}, w_j^{i-1}, q_j^{i-1} to be known for $0 \leq j \leq J$, equations (59)-(61) are a system of $7J$+7 nonlinear algebraic equations for $7J + 7$ unknowns $(f_j^i, u_j^i, v_j^i, g_j^i, p_j^i, w_j^i, q_j^i)$, $j = 1, 2, \ldots, J$. This non-linear system of algebraic equations is linearized by means of Newton's method and solved in a very efficient manner by using the Keller-box method (see Cebeci and Bardshaw, 1984). For a given ζ,the iterative procedure was stopped to give the final velocity, temperature and concentration distribution when the diference in computing these functions in the next procedure becomes less than 10^{-5}, i.e. $|\delta f^k| \leq 10^{-6}$, where the superscript k denotes iteration number. For these computations, a non-uniform grid in the η direction has been used, with $\eta_j = \sinh((j-1)/a)$, where $j = 1, 2, 3, \ldots, J$. Here, $J = 351$ and $a = 100$ had been chosen in order to obtain quick convergence and thus save computational time and space. It should be mentioned that convergent solutions at every ζ stations had been found within three iterations only. In the present integration scheme, values of ζ are increased with the increament $\Delta\zeta = 0.05$ until the asymptotic values for the skin-friction, heat transfer and mass transfer were reached for every variation of the pertinent parameters, such as w, Sc and n, for Pr = 0.72.

Local non-similarity method (LNS)

The local non-similarity method was developed by Sparrow and Yu (1971) and has been developed and applied by many investigators, for example Minkowycz and Sparrow (1978) and Chen (1988) and Hossain (1992), to solve various non-similar boundary layer problems. This method embodies two essential features. First, the nonsimilar solution at any specific stream-wise location is found (i.e each solution is locally autonomous). Second, the local solutions are found from differential equations. These equations can be solved numerically by well-established techniques, such as forward integration (e.g a Runge-Kutta scheme) in conjunction with a shooting procedure to determine the unknown boundary conditions at the wall. The method also allows some degree of self-checking for accuracy of the numerical results.

In the local non-similarity method, all the terms in the transformed conservation equations are retained, with the ζ derivatives distinguised by the introduction of the new functions $f_1 = \partial f/\partial\zeta$, $g_1 = \partial g/\partial\zeta$ and $h_1 = \partial h/\partial\zeta$. These represent three additional unknown functions, therefore it is necessary to deduce three further equations to determine f_1, g_1 and h_1. This is accomplished by creating subsidiary equations by differentiation of the transformed conservation equations and boundary conditions (i.e. f, g, h system of equations) with respect to ζ. The subsidiary equations for f_1, g_1 and h_1 contain terms $\delta f_1/\delta\zeta$, $\delta g_1/\delta\zeta$, $\delta h_1/\delta\zeta$, and their η derivatives. If these terms are neglected, the system of equations for f, g, h, f_1, g_1 and h_1 reduces to a system of ordinary differential equations that provides locally autonomous solutions in the stream-wise direction. This form of the local non-similarity method is referred to as the second level of truncation, because approximations are made by dropping terms in the second level equation (the f, g, h equations being the first level equations).

To carry the local non-similarity method to the third level of truncation, all terms are retained in both the f, g, h and the f_1, g_1, h_1 equations. The ζ derivatives appearing in the f_1, g_1, h_1 are now distinguised by introducing $f_2 = \partial f_1/\partial\zeta = \partial^2 f/\partial\zeta^2$, $g_2 = \partial g_1/\partial\zeta = \partial^2 g/\partial\zeta^2$, $h_2 = \partial h_1/\partial\zeta = \partial^2 h/\partial\zeta^2$. The f_1, g_1, h_1 and their boundary conditions are then differentiated with respect to ζ to obtain three additional equations for the functions $f_2(\zeta,\eta)$, $g_2(\zeta,\eta)$ and $h_2(\zeta,\eta)$. In these new equations, terms involving $\partial f_2/\partial\zeta$, $\partial g_2/\partial\zeta$ and $\partial h_2/\partial\zeta$ and their η derivatives are neglected, so that once again a locally autonomous system of ordinary differential equations for f, g, h, f_1, g_1, h_1, f_2, g_2 and h_2 can be derived.

The procedure as described above in the formulation of the local non-similarity method can result in a large number of ordinary differential equations that may require simultaneous solution. For example, at the third level of truncation there will be nine equations involving f, g, h, f_1, g_1, h_1, f_2, g_2 and h_2. It is expected that the accuracy of the local non-similarity method results will depend upon the truncation level. Below we give only the equations valid up to the third level of truncation:

$$f''' + \frac{n+3}{4}ff'' - \frac{n+1}{2}f'^2 + \zeta f'' + (1-w)g + wh = \frac{1-n}{4}\zeta\left[f'f_1' - f''f_1\right] \tag{70}$$

$$\frac{1}{\Pr}g'' + \frac{n+3}{4}fg' + \zeta g' = \frac{1-n}{4}\zeta[f'g_1 - g'f_1] \tag{71}$$

$$\frac{1}{Sc}h'' + \frac{n+3}{4}fh' + \zeta h = \frac{1-n}{4}\zeta[f'h_1 - h'f_1] \tag{72}$$

$$f_1''' + \frac{n+3}{4}ff_1'' - \frac{3n+5}{4}f'f_1' + f_1f'' + f'' + \zeta f_1'' + (1-w)g_1 + wh_1 = \frac{1-n}{4}\zeta\left[f_1'^2 + ff_2' - f_1f_1'' - f''f_2\right] \tag{73}$$

$$\frac{1}{\Pr}g_1'' + f_1g' + \frac{n+3}{4}fg_1' - \frac{1-n}{4}f'g_1 + g' + \zeta g_1' = \frac{1-n}{4}\zeta\left[f'g_2 + g_1f_1'\ g_1'\ g'f_2\right] \tag{74}$$

$$\frac{1}{Sc}h_1''+f_1h'+\frac{n+3}{4}fh_1'-\frac{1-n}{4}f'h_1+h'+\zeta h_1' = \frac{1-n}{4}\zeta\left[f'h_2+h_1f_1'-h_1'-h'f_2\right] \tag{75}$$

$$f_2'''+\frac{n+3}{4}ff_2''\frac{3n+5}{4}f'f_2'-\frac{1-n}{2}ff_2'+\frac{5-n}{4}f_2f''+2f_1f_1''-\frac{n+3}{2}f_1'^2 + 2f_1''+\zeta f_2''+(1-w)g_2+wh_2 = \frac{1-n}{4}\zeta\left[2f_1'f_2'+f_1f_2'-f_1f_2''-2f_1''f_2\right] \tag{76}$$

$$\frac{1}{\Pr}g_2''+\frac{5-n}{4}f_2g'+2f_1g_1'+\frac{n+3}{4}fg_2'-\frac{1-n}{2}f_1'g_1-\frac{1-n}{2}f'g_2 + 2g_1'+\zeta g_2' = \frac{1-n}{4}\left[2f_1'g_2+g_1f_2'-g_2'f_1-2g_1'f_2\right] \tag{77}$$

$$\frac{1}{Sc}h_2''+\frac{5-n}{4}f_2h'+2f_1h_1'+\frac{n+3}{4}fh_2'-\frac{1-n}{2}f_1'h_1-\frac{1-n}{2}f'h_2 + 2h_1'+\zeta h_2' = \frac{1-n}{4}\zeta\left[2f_1'h_2+h_1f_2'-h_2'f_1-2h_1'f_2\right] \tag{78}$$

$$\begin{aligned} &f(\zeta,0)=f'(\zeta,0)=g(\zeta,0)=h(\zeta,0)=1\\ &f_1(\zeta,0)=f_1'(\zeta,0)=f_2(\zeta.0)=f_2'(\zeta,0)=0\\ &g_1(\zeta,0)=g_2(\zeta,0)=h_1(\zeta,0)=h_2(\zeta,0)=0\\ &f'(\zeta,\infty)=1,\ f_1'(\zeta,\infty)=f_2'(\zeta,\infty)=0\\ &g_1(\zeta,\infty)=g_2(\zeta,\infty)=h_1(\zeta,\infty)=h_2(\zeta,\infty)=0 \end{aligned} \tag{79}$$

At the third level of truncation, equations (78)-(80), the terms with $\partial f_2/\partial\zeta$, $\partial g_2/\partial\zeta$, and $\partial h_2/\partial\zeta$ have been neglected. It can be seen that equations (72)-(81) form a coupled non-linear system of ordinary differential equations taking ζ as a parameter. Equations (72)-(81) are solved numerically, employing here the Nachtsheim-Swigert iteration technique. Here, solutions of these equations are obtained, up to the third level of truncation, for different values of n and Sc, for Prandtl number equal to 0.72 and with ζ values starting from 0.0 to 10.0. Results for surface heat transfer and mass transfer are given in Table I. Comparison between the non-similarity solutions and the finite difference solutions shows that consideration of the above equations up to the third level of truncation is sufficient for the present case.

Results and discussion

Natural convection flows driven by a combination of thermal and solutal diffusion effects from a permeable surface are very important in many technological applications. The foregoing formulations may be used to analyze the interaction of the various contributions to buoyancy forces in addition to surface mass flux. These contributions may aid or hinder each other and be of different magnitudes, as characterized by the value of N, representing the ratio of heat and species concentration gradients. When the thermal and solutal effects are opposed, the value of N is negative in order to ensure that the flow is in the positive x direction. For example, Gebhart and Pera (1971a) used Pr = Sc for which $N = -1$ (i.e. $w = \infty$). It is to be noted that $N = 0$ corresponds to no species diffusion and infinity to no thermal diffusion. Positive values of N correspond to both effects combining to drive the flow, whereas negative values correspond to opposing effects from the two diffusing components. We further see that when $N = 0$, $w = 0$ and as $N \to \infty$, , $w \to 1$. The physical extent (η) of the two effects in the convection region is governed by the values of the Prandtl number and Schmidt numbers and by their relative magnitudes. It may be noted that the present problem with $w = 0$ (i.e. $N = 0$) has been discussed by Merkin (1975).

For steady flows, other authors have discussed at length the effects of varying the parameters Pr, Sc, w on the nature of the fluid flow and heat and mass transport, but in the absence of the effect of transpiration. Here we restrict our discussion to the aiding (or favorable) case only for a fluid with Prandtl number Pr = 0.72, which represents air at 200°C at 1 atmosphere. Diffusing chemical species of most common interest in air have Schmidt numbers in the range from 0.1 to 10.0; the present investigation considers a range from 0.2 to 2.0. These values of Sc are chosen to represent the presence of the species Benzene (Sc = 1.60), carbon dioxide (Sc = 0.94), water vapor (Sc = 0.60) and hydrogen (Sc = 0.22). The conjugate buoyancy parameter w equals 0.0, 0.5 and 1.0. Values of the parameter n are chosen to be 0.0, 0.5 and 1.0.

Computed values of the local heat flux, $Nu_x/Gr_x^{1/4}$ and mass flux, $Sh_x/Gr_x^{1/4}$, at the surface of the plate, obtained by the methods mentioned above for $Sc = 0.94$ with $w = 1/3$ (that corresponds to $N = 0.5$) and $n = 0.0, 0.5, 1.0$ are shown in Table I against values of ζ in [0, 10]. The comparison shows that the solutions for small and large ζ are in excellent agreement with those of the finite difference solutions as well as the local non-similarity solutions. From this table it is also observed that an increase in the value of the temperature and concentration gradient n leads to a small increase in the value of both the local heat and mass flux. But for the higher values of ζ both the heat and mass transfer start to decrease slowly. In this computation we have taken $w = 1/3$ which gives $N = 0.5$ in order to make a comparison with the results obtained by Gebhart and Pera (1971a). For $n = 0.5$ and $w = 1/3$ Gebhart and Pera (1971a) gave 0.66378, 0.49292, 0.57019 as the results for $Cf_x/Gr_x^{3/4}$, $Nu_x/Gr_x^{1/4}$, and $Sh_x/Gr_x^{1/4}$, respectively. From the present computations, the corresponding values of the above physical quantities are found to be 0.66874, 0.49294 and 0.57018. Undoubtedly it may be concluded that the present results are in excellent agreement with those of the earlier investigators. It should further be mentioned that the present results, obtained by the perturbation method for small ζ, the asymptotic solutions for large ζ and the finite difference solutions for all ζ, taking Pr =1 and $w = 0$ agree very well with those of Merkin (1975) (for the sake of brevity the comparison is not shown here).

The effect of varying Schmidt number, Sc, on the local Sherwood number, $Sh_x/Gr_x^{1/4}$, the local Nusselt number, $Nu_x/Gr_x^{1/4}$, and the local skin-friction, $Cf_x/\mathrm{Gr}_x^{3/4}$ are depicted in Figure 2(a)-(c), respectively, for the case Pr = 0.72 and $n = 0.5$. In these figures the dotted curves and the broken curves represent the solutions obtained, respectively, for the low and high values of the local transpiration parameter, ζ. It can be seen from Figure 2(a) that there is an increase in $Sh_x/Gr_x^{1/4}$ due to the increase in the value of the Schmidt number, Sc. This effect is most significant at large values of the surface transpiration parameter, ζ. From Figure 2(b) it can be seen that the value of $Nu_x/Gr_x^{1/4}$ increases very slowly with the increase of Sc. Thus, we may consider that the effect of foreign species on the surface heat transfer rate is negligible. From Figure 2(c) we see that the value of the local skin-friction, $Cf_x/Gr_x^{3/4}$, increases as the value of the Schmidt number, Sc, decreases. Here we also observe that for each value of Sc, there attains a local maxima of $Cf_x/Gr_x^{3/4}$ near the leading edge and then its value decreases to the asymptotic value as ζ increases. The numerical values show that, for $Sc = 1.60$ the maximum value of the local skin-friction is just at the leading edge, i.e. the maximum value is 0.8482 at $\zeta = 0.0$. The maximum value of skin-friction for $Sc = 0.94$ is 0.8973 and occurs at $\zeta = 0.35$ (at the vicinity of the leading edge). For $Sc = 0.60$, the maximum value is 0.9521 at $\zeta = 0.5$. Finally, the maximum value of skin-friction for $Sc = 0.22$ is found to be 1.1360 at $\zeta = 1.30$.

Effects of the combined buoyancy parameter, w, on $Sh_x/Gr_x^{1/4}$, $Nu_x/Gr_x^{1/4}$, and $Cf^x/Gr_x^{3/4}$, taking $n = 0.5$ and $Sc = 0.22$ are depicted, respectively, in Figure 3(a)-(c). From these figures we again see excellent agreement between the results obtained for low and high values of the local transpiration parameter, ζ, with those obtained by the finite difference method. It can be seen from Figure 3(a) that, near the leading edge, the value of $Sh_x/Gr_x^{1/4}$, increases, owing to increase in the value of the combined buoyancy parameter, w. On the other hand, one can see, from Figure 3(b), that the increase in the value of $Nu_x/Gr_x^{1/4}$, due to the increase of w, is very small. Figure 3(c) shows that an increase in the value of the combined buoyancy parameter, w, leads to an increase in the value $Cf_x/Gr_x^{3/4}$. Figure 3(c) also shows that there exist local maxima of skin-friction for different values of w. For $w = 0.0$ the maximum value of $Cf_x/Gr_x^{3/4}$ is 0.9291 which occurs at $\zeta = 0.45$. For $w = 0.5$ and 1.0 the corresponding maxima exist at $\zeta = 1.30$ and 1.86, respectively and the corresponding maximum values are found to be 1.1383 and 1.4102, respectively.

Attention is now given to the effect of pertinent parameters on the dimensionless velocity $f'(\eta,\zeta)$ $(= xu/\nu Gr_x)$, the dimensionless temperature, $g(\eta,\zeta)$ $(= (T - T_\infty)/(T_w - T_\infty))$, and the dimensionless concentration $h(\eta,\zeta) = (C - C_\infty)/(C_w - C_\infty)$ distributions in the flow field, computed only by the finite difference method. Values of the dimensionless velocity, temperature and concentration distributions are shown graphically in Figure 4(a)-(c),

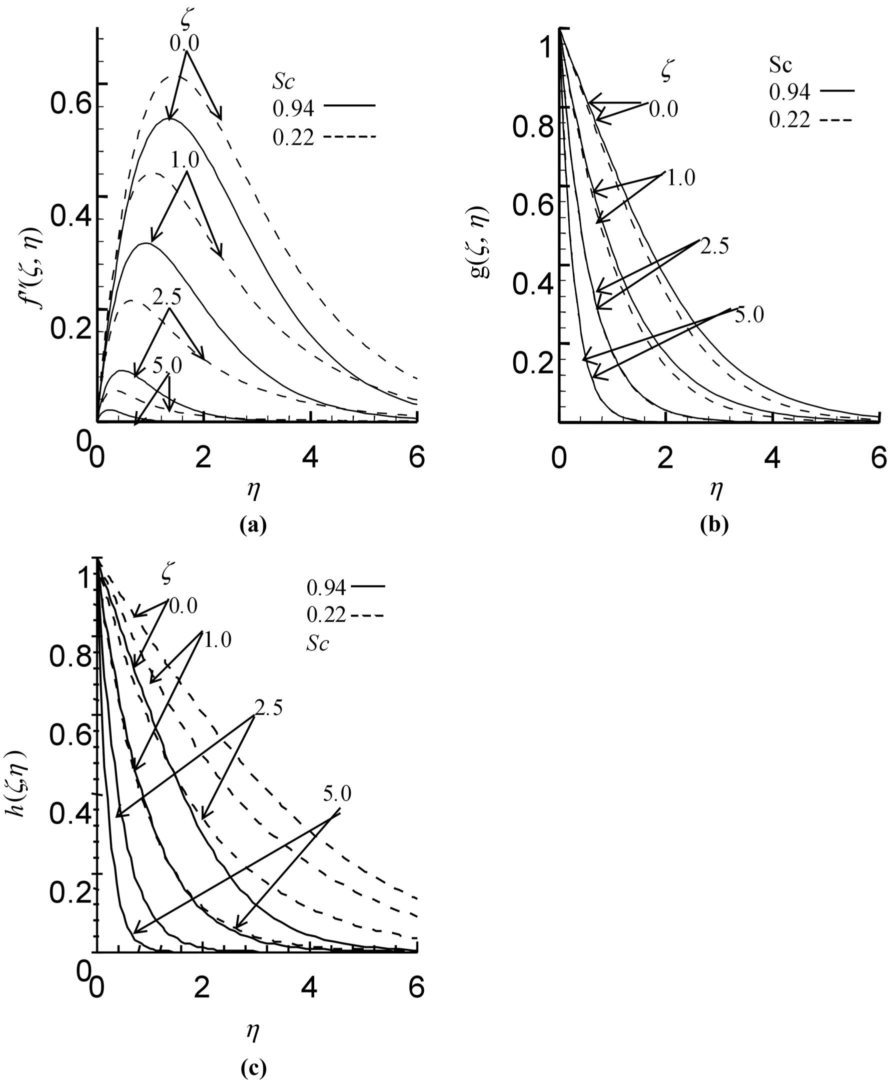

Figure 4.
(a) Velocity profiles, (b) temperature profiles and (c) concentration profiles against η for different values of ζ for Sc =0.94 and 0.22 for Pr = 0.72, w = 0.33 and n = 0.0

respectively, against the pseudo-similarity variable η for values of the transpiration parameter ζ = 0.0, 1.0, 2.5 and 5.0 while Pr = 0.72, w = 1/3 and n = 0.0 for two values of Sc = 0.94 (representing the presence of CO_2 as the chemical species) and 0.22 (representing the presence of H_2).

From Figure 4(a) it can be observed that the velocity profiles decrease with the increase of transpiration parameter ζ. It can also be seen that at each value of the transpiration parameter ζ there exist local maximum values of velocity in the boundary layer region. These maximum values are found to be 0.538, 0.317, 0.091 and 0.020 at η = 1.37, 0.89, 0.45 and 0.24, respectively, while Sc = 0.94. For Sc = 0.22 the velocity reaches to the maximum values 0.6149, 0.4422, 0.2151 and

0.0555, respectively, at $\eta = 1.44, 1.06, 0.64$ and 0.33. For an eventual experimental verification it is interesting to indicate the percentage decrease in the maximum velocity. Thus when $Sc = 0.94$, the relative maximum values of the velocity decrease by 41, 83 and 96 percent for $\zeta = 1.0, 2.5$ and 5.0, respectively, obtained with respect to $\zeta = 0.0$. Due to the presence of H_2 (i.e. for $Sc = 0.22$), the corresponding percentage decrease in the maximum values of the velocity are found to be 28, 65 and 90 percent, respectively, for $\zeta = 1.0, 2.5$ and 5.0.

Now from Figure 4(b) and (c) we see that owing to increase of the value of the transpiration parameter ζ both the temperature and concentration decrease. With $Sc = 0.94$ and at $\eta = 1.51$, the numerical values of temperature of the fluid are obtained at $\zeta = 0.0, 1.0, 2.5$ and 5.0 as 0.49638, 0.27372, 0.07367 and 0.00411, respectively. The corresponding values of the species concentration are obtained as 0.43391, 0.19017, 0.03320 and 0.00071. As before, the calculated values show that the temperature of the fluid decreases by 44.85, 79.11 and 99.17 percent, respectively, for $\zeta = 1.0, 2.5$ and 5.0 relative to $\zeta = 0.0$. The corresponding decreases in the species concentration are by 56.17, 92.34 and 99.83 percent. For $Sc = 0.22$, corresponding decrease of the temperature and the species concentration are 48.00, 84.65 and 99.01 percent and 14.16, 36.64 and 72.29 percent, respectively. Finally, from the Figure 4(a)-(c) it may be concluded that, as the value of Schmidt number, Sc, decreases both the velocity and the species concentration distribution in the boundary layer regime increase; whereas, the temperature decreases. The momentum boundary layer and the concentration boundary layer thickness increase, and the thermal boundary layer thickness decreases, with decrease in the value of the Schmidt number. We may further observe that increase of transpiration parameter ζ leads to decrease in all the boundary layer thicknesses.

Conclusions

In this paper we have sought to determine how the presence of non-uniform species concentration affects the natural convection boundary layer flow from a non-uniformly heated permeable surface with uniform withdrawal (or suction) of fluid. Solutions of the governing local non-similarity equations are obtained by three distinct methodologies, namely, the extended series solution method for lower values of ζ, the asymptotic solutions for higher values of ζ, the local non-similarity method with third level of truncation and the finite difference method for all $\zeta \in [0, \infty]$. The presence of species concentration serves to introduce two extra parameters into the problem, namely w and Sc. The detailed effect of varying n and w are complicated and selected results have been presented for a Prandtl number of 0.72, representing air at 20°C and at 1 atmosphere with the presence of CO_2 and H_2 for which the values of the Schmidt number are 0.60 and 0.22. Detailed numerical calculations have been carried out and presented in terms of local Nusselt number, Sherwood number and skin-friction. In general it is seen that the asymptotic solutions for small and large values of the transpiration parameter are in excellent agreement with the finite difference as well as the local similarity solutions.

From the above investigations we may also draw the following conclusions:

- an increase in the value of *Sc* serves to increase the momentum boundary layer thickness and to decrease the concentration and thermal boundary layer thickness;
- the values of the local Nusselt number, Sherwood number and skin-friction increase due to increase of the combined buoyancy parameter w;
- an increase in the value of the transpiration parameter, ζ, leads to decrease in the momentum, thermal as well as species concentration boundary layer thickness.

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