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STABILITY OF FLOW AND HEAT TRANSFER IN A ROTATING CAVITY WITH A STATIONARY OUTER CASING

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ABSTRACT

This paper describes the computation of flow and heat transfer in a rotating cavity with a stationary outer casing, using both a steady−state, axisymmetric finite volume solver and a time−dependent, axisymmetric direct numerical simulation (DNS) procedure. The geometric configuration represents the turbine disc cooling−air system used in some gas turbine−engines. Steady flow computations, using a low Reynolds number k−ε turbulence model, give reasonably good predictions of Nusselt numbers, however convergence histories suggest that some of the flows computed may in reality be unstable. DNS reveals a highly unstable secondary flow structure, and gives rise to time−averaged velocity distributions in better agreement with experimental data than were obtained with the k−ε turbulence model. There is some evidence that the effects of unsteadiness on heat transfer may be comparatively small.

NOMENCLATURE

a, b	inner, outer radius of disc
C_w	nondimensional mass flow rate ($= \dot{m}/\mu b$)
G	gap ratio (= s/b)
k	turbulent kinetic energy, thermal conductivity
$\dot{m}$	mass flow rate
Nu	Nusselt number ($= qr/k(T_o - T_{in})$)
q	heat flux from heated disc to air
r	radial coordinate
Re_ϕ	rotational Reynolds number ($= \rho\Omega b^2/\mu$)
s	axial gap between discs
S	standard deviation
t	time
T_o	surface temperature of heated disc
T_{in}	inlet air temperature
V_r, V_ϕ, V_z	time−averaged velocities in r,ϕ,z directions
x	nondimensional radius (= r/b)
z	axial coordinate
ε	turbulent energy dissipation rate
μ	dynamic viscosity
ρ	density
ϕ	tangential coordinate
Ω	angular speed of discs

1 INTRODUCTION

A rotating cavity, formed by co−rotating discs with a rotating inner cylinder (or hub), and confined by a stationary outer casing, is a configuration which has been studied in relation to two practical engineering situations. Abrahamson et al (1989) described an experimental study carried out in order to provide information on the air flow in computer disc drives, and Chang et al (1990) carried out axisymmetric, steady calculations for this application. Gan et al (1996) and Mirzaee et al (1997) carried out experiments and computations for flow and heat transfer in a similar rotating cavity, used as a simplified model for co−rotating turbine discs in a gas−turbine engine. For both applications, knowledge of the fluid dynamics of the cavity flow is important in understanding the transport of heat within the system, and the heat transfer at the disc surfaces. A comprehensive review of flow and heat transfer in rotating cavities is given by Owen and Rogers (1995).

Abrahamson et al (1989) carried out flow visualisation in a rotating cavity for which the inner rotating surface at radius r = a was located at a/b = 0.5, and the axial spacing, s, between the discs was varied to give gap ratios G = s/b between 0.013 and 0.1. The rotational Reynolds number Re_ϕ (based on the outer radius b of the discs) was varied between 1.5×10^5 and 1.5×10^6. Orderly circumferential asymmetries were observed in the flow, with a number of vortices in the outer part of the cavity precessing relative to the discs, and near solid−body rotation occurred in the inner region towards the hub. The number of vortices decreased (and the level of relative motion in the inner region increased) when the axial separation between the discs was increased. Ensemble−averaged tangential velocities on the mid−plane between the discs (inferred from flow visualisation results) were presented for $Re_\phi = 4.5 \times 10^5$ and G=0.05. These showed a clear distinction between free−vortex−type flow in the outer region, near the casing, and forced−vortex−type flow in the inner part of the cavity.

Gan et al (1996) made laser−Doppler anemometry (LDA) velocity measurements, and carried out turbulent flow computations, for a cavity also with a/b = 0.5 but with gap ratio G = 0.3, wider than was considered in the above work. The range of rotational Reynolds numbers tested was (coincidentally) the same as that studied by Abrahamson et al. In addition to the flow due to the stationary casing, the effect of a peripheral flow of air, simulating the cooling of gas−turbine discs, was also investigated. Heat transfer for these superposed flow cases was measured and computed by

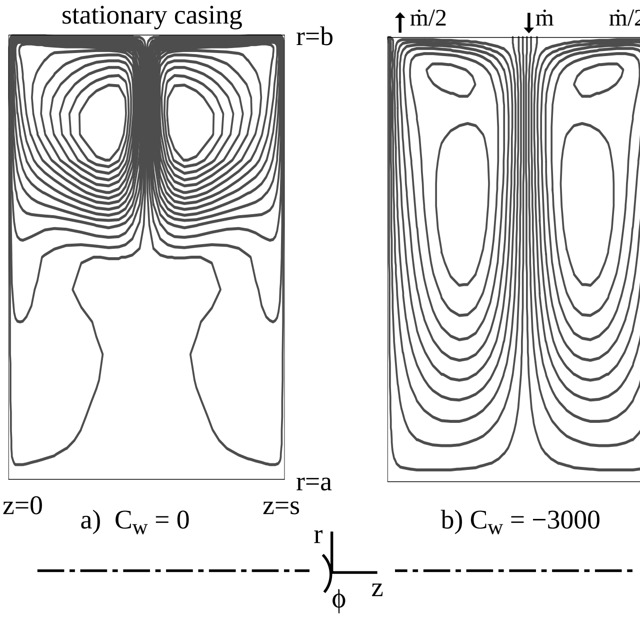


Fig. 1 The rotating cavity and computed streamlines at $Re_\phi = 3.75\mathrm{x}10^5$ (from Gan et al (1996))

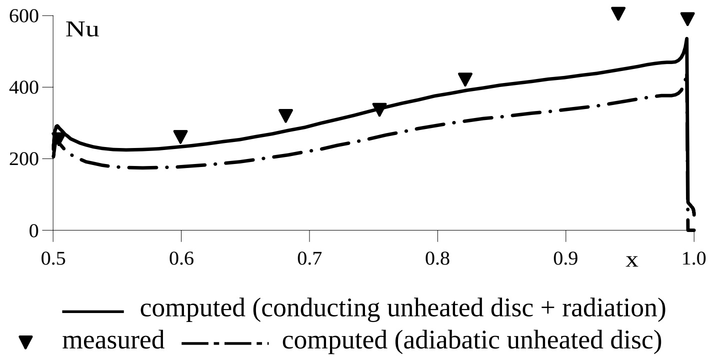


Fig. 2 Variation of heated disc Nusselt numbers with x: $Re_\phi = 3.75\mathrm{x}10^5$, $C_w = -3000$ (from Mirzaee et al (1997))

Mirzaee et al (1997).

For the "sealed" cavity (i.e. with no superposed flow), Fig. 1a shows the axisymmetric, steady flow prediction of Gan et al (1996) for secondary flow recirculations, symmetric about the axial mid−plane between the discs, with radial outflow in the boundary layers on the discs and radial inflow at the mid−plane. (Abrahamson et al (1989) also noted this structure). The penetration of these recirculations, radially inward from the outer casing, reduced with increasing rotational Reynolds number. LDA measurements of velocity profiles showed that, although these recirculations were predicted reasonably well using a low Reynolds number k−ε turbulence model, inflow velocities near the mid−plane were overpredicted. Measured tangential velocities on the mid−plane between the discs (z/s = 0.5) followed a Rankine (combined free and forced) vortex structure, which was not well predicted using this turbulence model.

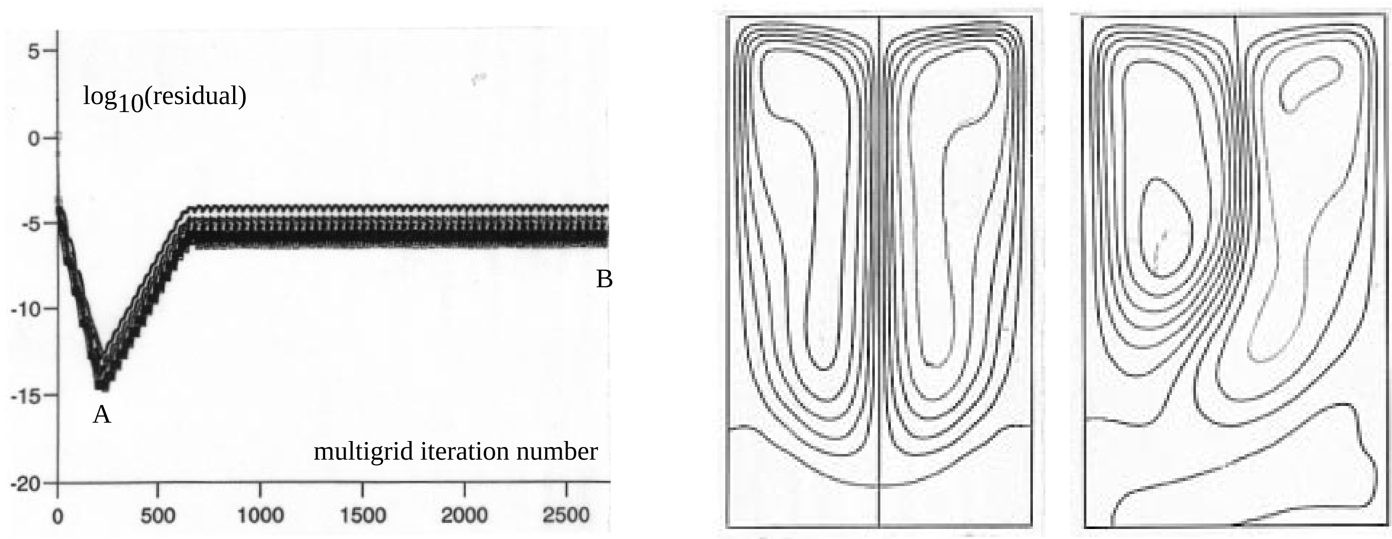
A superposed radial inflow (entering the cavity with zero swirl) increases the inward penetration of the secondary flow recirculations, as shown in Fig. 1b (the negative sign for the dimensionless flow−rate C_w is used to indicate radial *inflow*). Gan et al (1996) found that, with increasing flow−rate, the tangential velocity field tended towards a free vortex structure. Heat transfer tests were conducted with one disc heated and one unheated, and with insulated inner and outer cylindrical surfaces. Reasonably accurate computations of measured Nusselt numbers were obtained, although with consistent underprediction in the outer part of the system, when conduction through and radiation to the unheated disc were accounted for computationally. A typical result is given in Fig. 2 for the case $Re_\phi = 3.75\mathrm{x}10^5$, $C_w = -3000$. Measurements showed that heat transfer from the heated disc increased with increasing flow rate and increasing rotational Reynolds number, and these trends were correctly reproduced in computations.

This paper describes work which re−examines the flow computations described by Gan et al (1996). Axisymmetric, steady computations have been repeated with more demanding tolerances applied for convergence of the numerical method, and a direct numerical simulation (DNS) procedure under development at Bath has been used to predict the axisymmetric, *time−dependent* flow for sealed cavity cases. Section 2 describes the two computational methods employed, the numerical stability of steady−flow predictions is discussed in Section 3, and the results of time−dependent simulations are described and discussed in Section 4. Conclusions appear in Section 5.

2 COMPUTATIONAL METHODS

Axisymmetric steady−flow solutions for the Reynolds−averaged Navier−Stokes (RANS) and energy equations were obtained using the code (and computational grid) described by Gan et al (1996) and Mirzaee et al (1997). Briefly, a full−storage approximation, V−cycle multigrid scheme was used to accelerate the SIMPLEC pressure−correction technique for solution of the equations in primitive variables, using a staggered−grid finite−volume discretisation. Hybrid−upwind differencing was used for the convective terms. The turbulence closure was the low Reynolds number k−ε turbulence model due to Launder and Sharma (1974). Grid dependence studies suggested that a 91x91 nonuniform grid, contracted to solid surfaces, was required for sealed cavity calculations. Details of convergence criteria are given in Section 3 as these are pertinent to the discussion of new results.

Direct numerical simulations were carried out using an axisymmetric version of a time−dependent solver for the full Navier−Stokes equations, in a streamfunction−vorticity formulation expressed in a rotating frame of reference. (The RANS computations described above were performed in a stationary frame). In this formulation, the pressure field is eliminated using the vorticity definition. The equations were discretised using finite differences on a non−uniform collocated grid, with coefficients evaluated directly in preference to the use of a grid transformation. Hybrid upwinding was also used in this method for the non−linear convective terms, and a first−order−accurate forward difference in time was employed. While this formulation would be inadequate for the simulation of highly turbulent flow, it may be sufficiently realistic for the problem studied here where turbulence levels (according to the RANS



a) convergence history for normalised absolute residuals (for momentum and continuity equations)

b) secondary flow solution at minimum residual point A

c) secondary flow solution at final point B

Fig. 3 Numerical instability in steady laminar flow computations: $Re_\phi = 10^4$, $C_w = 0$

computations) are relatively low: the maximum value of turbulent viscosity was around 50 times the laminar viscosity for the case to be discussed. The DNS code is being developed primarily for the study of 3D buoyancy−driven flow in rotating cavities. The problem described here has been computed as part of its verification, in view of the availability of experimental data and interest in this configuration at Bath.

The time−dependent vorticity and tangential velocity equations were solved explicitly using the Du−Fort Frankel method, and the Poisson equation for the streamfunction was solved using a multigrid scheme with V−cycling and line−relaxation smoothing. Multigrid convergence at each time−step required the total absolute residual on the mesh to fall below 10^{-6} for the Poisson equation solution (for which the maximum streamfunction value was around 200). Values of vorticity on the boundaries were updated using the streamfunction solution at the new time level.

In these simulations, the fluid is initially at rest relative to a stationary frame of reference, and the discs then take up a prescribed speed instantaneously at time t = 0. In order to perturb the subsequent solution from a symmetric twin recirculation structure, as shown in Fig. 1, a small axially varying perturbation to the tangential velocity would normally be introduced. It was found during these studies that amplification of round−off errors within the code itself was sufficient to perturb the solution, however the results presented here do include the use of an axially varying perturbation.

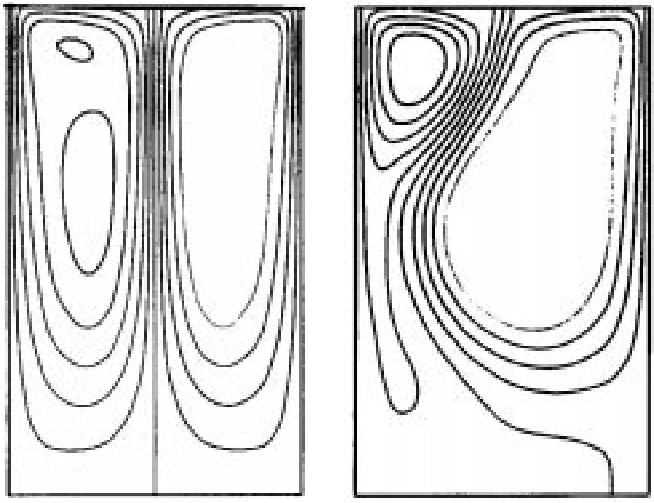
3 RESULTS OF STEADY FLOW COMPUTATIONS

Convergence difficulties were experienced in obtaining the steady, axisymmetric results presented by Gan et al (1996): asymmetric solutions were obtained in the course of grid optimisation studies. The asymmetry was attributed to the sensitivity of solutions to the grid resolution near the stationary outer casing. With suitable resolution in this region, symmetric results were obtained for a convergence tolerance of 10^{-3} on the absolute total residuals, and root−mean−square (rms) changes in solution variables between completed multigrid iterations fell below $2x10^{-5}$.

Arnold (1996) repeated the sealed cavity computations of Gan et al (1996) prior to carrying out new heat transfer computations for superposed flow cases. It was determined that, even using the same grid distribution as used in the earlier work, absolute solution residuals (and rms changes) tended to *increase*, with further iterations, after reaching a minimum point near the convergence criterion adopted for the original computations. An example of this behaviour is shown in Fig. 3a, for a laminar flow computation at $Re_\phi = 10^4$. The predicted streamlines (for the secondary flow in the axial−radial plane) at the "minimum−residual" point (point A in Fig.3a) and at the end of the computation (point B in Fig. 3a) are shown in Fig. 3b and Fig. 3c respectively. The oscillations observed in the residuals as the final point B is approached correspond to transition between the left−biased flowfield (Fig. 3c) and a similar but right−biased structure (not shown). At $Re_\phi = 10^3$, this type of behaviour was not observed; convergence was essentially monotonic, and symmetric secondary flows were predicted.

Oscillatory behaviour for residuals similar to that described above was also observed in turbulent sealed cavity computations, and in computations involving peripheral flow. The symmetric solutions reported by Gan et al (1996), shown in Fig. 1a and Fig. 1b for $Re_\phi = 3.75x10^5$, correspond to the "minimum−residual" points in the solutions.

The "minimum−residual" and "final" point solutions obtained for $Re_\phi = 3.75x10^5$ and Cw = −3000 are illustrated in Fig. 4a and Fig. 4b respectively. These computations



a) minimum residual point   b) final point solution

Fig. 4 Numerical instability in steady turbulent flow computation: $Re_\phi = 3.75x10^5$, $C_w = -3000$

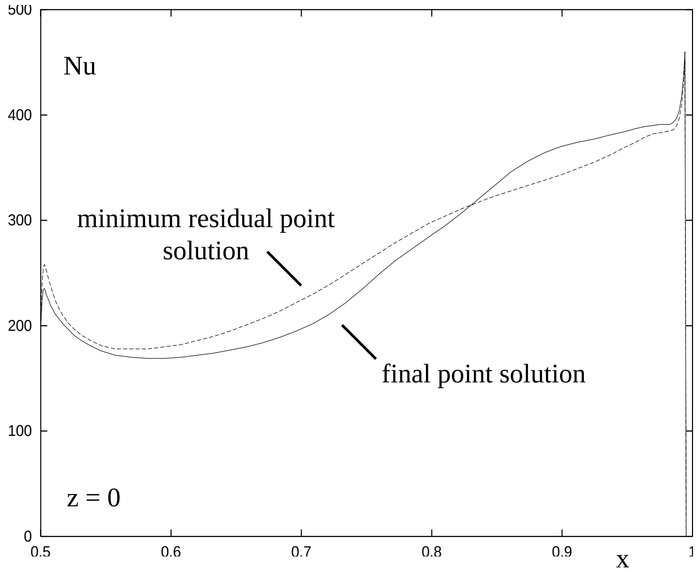


Fig. 5 Computed heated disc Nusselt numbers: $Re_\phi = 3.75x10^5$, $C_w = -3000$

require additional boundary conditions to account for the inflow and outflow of air, and it was found that the choice of outflow conditions could affect the convergence history and final solution obtained. The solution illustrated in Fig. 4b resulted from the use of prescribed tangential velocities at the outlets. The predicted Nusselt numbers shown in Fig. 5 correspond to the two flow solutions shown in Fig. 4 (the minimum point solution agrees with that shown in Fig. 2). These results suggest that the effect of the instability on heat transfer may be relatively small, and this is further discussed below.

At the higher rotational Reynolds numbers tested by Gan et al ($Re_\phi = 7.5x10^5$ and $Re_\phi = 1.5x10^6$) an intermediate minimum point for residuals did not occur and, although some oscillatory behaviour was observed, symmetric secondary flows were obtained subsequent to the convergence tolerances described above being reached. The numerical stability of superposed flow cases at lower rotational Reynolds numbers also improved with increasing flow−rate.

4 RESULTS OF TIME−DEPENDENT COMPUTATIONS

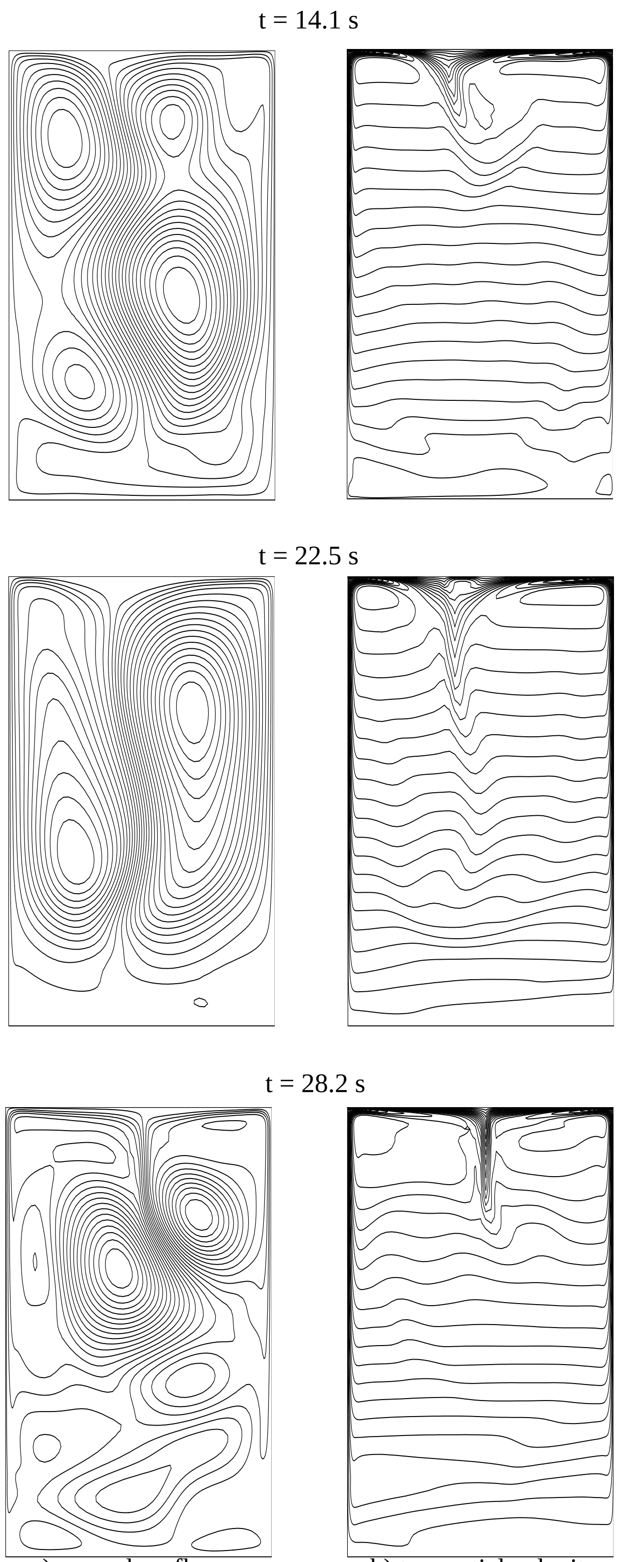
The DNS code described in section 2 was applied to the calculation of axisymmetric flow in a sealed cavity for $Re_\phi = 10^3$, 10^4 and $1.46x10^5$. The latter was the lowest rotational Reynolds number at which experimental data were obtained using the Bath rig, and only results for this case will be described in detail here. From the other cases computed, it was concluded that steady flow (with symmetric secondary flow recirculations) developed at $Re_\phi = 10^3$, while unsteady flow occurred at $Re_\phi = 10^4$. (The latter case was computed using a uniform grid and higher−order differencing for convective terms than that described in section 2). These findings agree with the conclusions drawn from steady finite volume predictions on the likely onset of unsteady flow.

Time−dependent computations for $Re_\phi = 1.46x10^5$ were carried out on a nonuniform 97x129 computational grid using a time step (determined by numerical stability tests) of $\Delta t = 2.8x10^{-4}$ seconds The computation was allowed to progress for 200,000 time steps, so simulating a total time of 56 seconds. This calculation required around 24 CPU hours using an R10000 processor on an SGI Power Challenge machine.

Fig. 6 shows instantaneous predictions of secondary flow streamlines, and tangential velocity contours, at times t = 14.1 seconds, t = 22.5 seconds and t = 28.2 seconds. The computed flow is unsteady and asymmetric. The number, location and strength of large−scale recirculations in the secondary flow change with time, and no periodic behaviour was observed during the course of the simulation. The tangential velocity contours show, in general, only small axial variations in the core flow between the boundary layers on the discs, and the circumferential flow within the disc boundary layers does not appear to be strongly affected by the secondary flow activity. The region of strongest radial inflow, formed where the boundary layers meet on the stationary outer casing, can clearly be identified, and this causes modest disturbances to the circumferential velocity field in the outer part of the cavity.

Instantaneous velocity distributions taken from the computations described above have been compared with experimental data, but it is also appropriate to consider time−averaged results. Fig. 7 shows the dimensionless tangential velocity distribution, $V_\phi/\Omega r$, at the axial location z/s = 0.8, time−averaged over the second half of the simulation (a total time of 28 seconds). There is very good agreement between this result and the experimental data. The standard deviation, S, of the computed velocities about the mean over the period is also illustrated, confirming the modest variations suggested in Fig. 6b for earlier times. In terms of computed flow structure, the time−averaged DNS result in Fig. 7 is superior to the computation, also shown, obtained with the steady finite volume code using the Launder−Sharma turbulence model. The axes for Fig. 7 are chosen to illustrate the Rankine−vortex nature of the flow (Owen and Rogers, 1995) for which $V_\phi/\Omega r = Ax^{-2} + B$, where A and B are constants.

The wide variation of the secondary flows with time (Fig. 6a) complicates the comparison between computed radial velocity distributions and the LDA data. Fig. 8a and Fig. 8b show time−averaged dimensionless radial velocity profiles



a) secondary flow streamlines

b) tangential velocity contours

Fig. 6 Instantaneous results from time−dependent simulation: $Re_\phi = 1.46x10^5$, $C_w = 0$

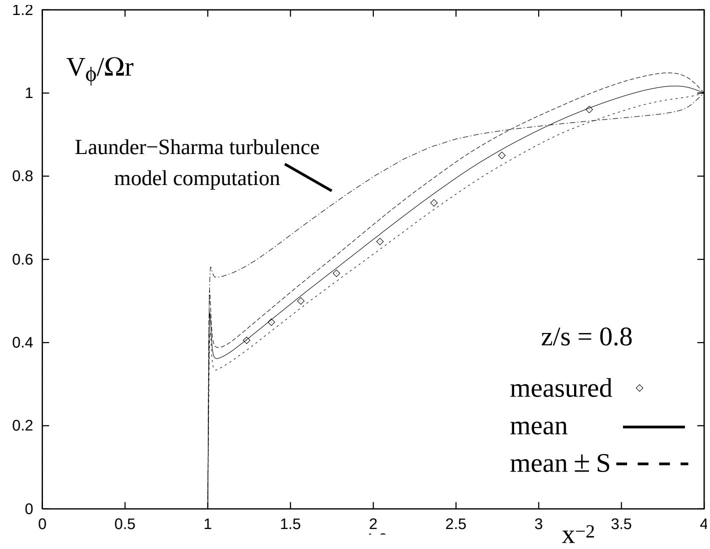


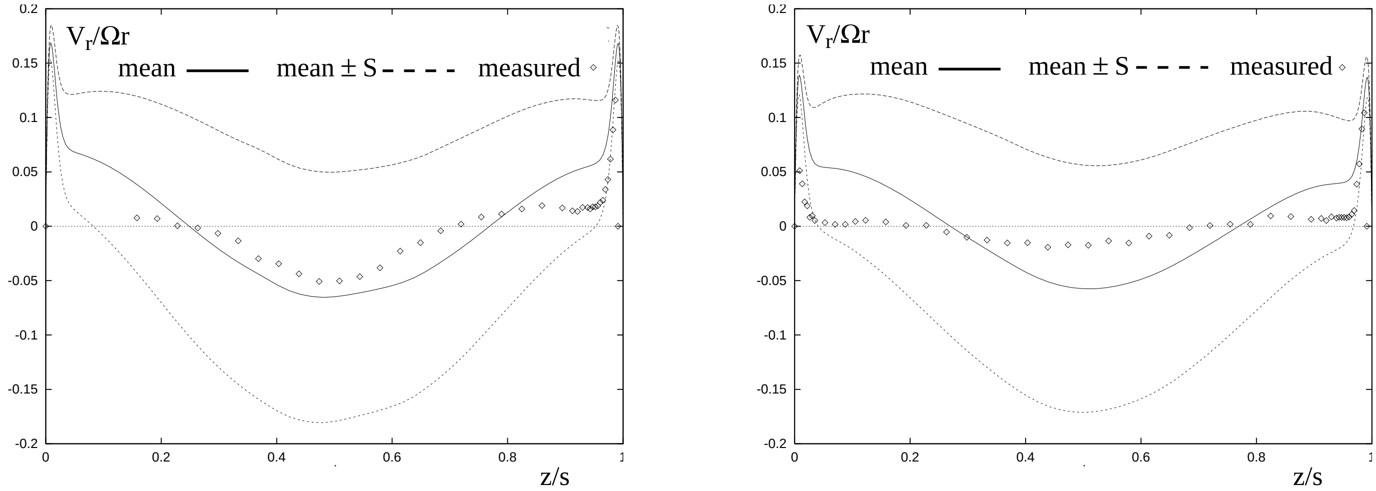
Fig. 7 Time−averaged tangential velocity distribution and comparison with experimental data: $Re_\phi = 1.46x10^5$, $C_w = 0$

(over the same interval as described above) in comparison with data at the dimensionless radial locations x = 0.85 and x = 0.75, and standard deviations are also shown. There is good agreement for the width of the boundary layers and peak radial velocity. The computed radial inflow is in better agreement with the data than the steady flow finite volume results, which further overpredicted inflow at the mid−plane $z/s \approx 0.5$ (Arnold, 1996). Overpredicted radial velocities, close to the edges of the boundary layers, may be an indication that the axisymmetric assumption is an oversimplification for this flow: the physical flow is almost certainly nonaxisymmetric and unsteady. (This is supported by the visualisation of Abrahamson et al (1989), as described in the introduction, which showed the existence of circumferential asymmetries for cases with smaller gap ratios.)

5 CONCLUSIONS AND FUTURE WORK

Steady−flow finite−volume techniques for the RANS equations, and a DNS procedure, have been used to predict axisymmetric flow in a sealed rotating cavity with a stationary outer casing. Steady−flow computations suggest that the flow is in fact unsteady, and this has been confirmed for some cases by DNS. The DNS results give rise to time−averaged tangential velocity predictions which agree well with LDA measurements for the case considered. The Rankine vortex structure of the flow, and the magnitude of the velocities, are more accurately predicted by the DNS than by steady computations employing a low Reynolds number $k-\varepsilon$ turbulence model. Reasonable agreement is obtained between the DNS results and experimental data for radial velocity distributions, but temporal variations are very large. In view of experimental observations made by other workers, fully three−dimensional calculations may be required for more realistic computations of the secondary flows.

Axisymmetric steady RANS computations for heat transfer show reasonably good predictions of measured Nusselt numbers for cases with superposed flow. The large variations



a) x = 0.85 b) x = 0.75

Fig. 8 Time−averaged radial velocity distributions and comparison with experimental data: $Re_{\phi} = 1.46x10^5$, $C_w = 0$

in secondary−flow structure may have a relatively small effect on the near−wall flows, as the larger tangential velocities in the disc boundary layers appear to remain largely undisturbed. Similarly, steady heat transfer computations suggest that disc heat transfer rates may be only modestly affected by flow instabilities, although time dependent heat transfer simulations have still to be carried out.

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