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**NUMERICAL AND THEORETICAL STUDY OF FLOW AND HEAT TRANSFER IN A
 PRE-SWIRL ROTOR-STATOR SYSTEM**

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ABSTRACT

In a "direct-transfer" pre-swirl supply system, cooling air flows axially across the wheel space from stationary pre-swirl nozzles to receiver holes located at a similar radius in the rotating turbine disc. This paper describes a combined numerical and theoretical study of flow and heat transfer in such a system. As 3D computations involve long computing times and large resources, a simplified axisymmetric model has been used to study the effects of flow parameters on flow and heat transfer in a rotor-stator pre-swirl system. This allows the effects of the main non-dimensional parameters on the flow and heat transfer in the system to be studied with computing times reduced by a factor of around 7 compared with 3-dimensional computations. The computed results are compared with available measured data. An expression has been derived for calculating the adiabatic effectiveness of the system, and this has been compared with computed values. The computations show that, due to mixing losses, there is a significant drop in angular momentum. The computed flow structure is compared with free vortex behaviour between the pre-swirl inlet and the receiver outlet. The computed moment coefficient decreases as pre-swirl ratio increases and increases as non-dimensional flow rate increases. The computed average Nusselt number decreases with inlet swirl ratio up to a critical value and then increases again.

Keywords pre-swirl systems, effectiveness

NOMENCLATURE

a	inner radius of disc
b	outer radius of disc
C_M	moment coefficient for one side of the disc ($= \frac{M}{1/2\rho\Omega^2 b^3}$)
C_p	specific heat at constant pressure
C_w	non-dimensional mass flow rate ($= \frac{\dot{m}}{\mu b}$)
k	turbulence kinetic energy or thermal conductivity of air
M	moment on one side of disc ($= -2\pi \int_a^b r^2 \tau_{\phi,r} dr$)
\dot{m}	mass flow rate
Nu	Nusselt number ($= \frac{rq_r}{k(T_r - T_{r,ad})}$)
Nu_{av}	average Nusselt number
$q_{r,av}$	average heat flux from disc to air
\dot{Q}_{ab}	total heat transfer rate from disc
r, ϕ, z	radial, tangential and axial directions
R	recovery factor ($= Pr^{1/3}$)
Re_ϕ	rotational Reynolds number ($= \frac{\rho\Omega b^2}{\mu}$)
s	axial gap between discs
T	temperature
$T_{0,1}$	inlet total temperature in stationary of frame reference
U	total velocity
V_r, V_ϕ, V_z	velocity components in a stationary frame
\dot{W}	rate of work done on the air
x	ratio of nondimensional radial coordinate (r/b)

y	distance normal to the wall
y^+	nondimensional distance ($=\rho y U_\tau / \mu$)
Θ_b	adiabatic effectiveness
Ω	angular speed of disc
β	swirl ratio ($=\frac{V_\phi}{\Omega r}$)
ε	turbulence energy dissipation rate
λ_T	turbulent flow parameter ($=C_w / Re_\phi^{0.8}$)
τ	shear stress
μ	viscosity
ρ	density

Subscripts

0	total value in stationary frame reference
1	downstream of the pre-swirl nozzles
2	downstream of the receiver holes
ad	value for adiabatic system
av	radially weighted average value
b	blade-cooling air
$crit$	critical value
eff	effective value
id	ideal value
p	pre-swirl inlet
r	rotating surface
s	stationary surface
t	total value in rotating frame of reference
w	wall surface
∞	mid plane

INTRODUCTION

A rotor-stator system, as shown in Fig. 1, provides a simplified model for the flow and heat transfer that occurs in the wheel-space between an air-cooled turbine disc and adjacent stationary casing. In this model pre-swirl nozzles are located at a low radius on the stator and the cooling air flows radially outward to the receiver holes.

Benim et al [1] studied "direct transfer" pre-swirl systems using the commercial multi-purpose CFD code Fluent. The computations were carried out using a steady-state 3D method and a so-called frozen rotor approach for treating the interface between the stationary and rotating domains.

Geis et al [2] measured the cooling efficiency of a pre-swirl rotor-stator system equipped with a small number of pre-swirl nozzles of circular shape, located on a radius equal to that of the receiver holes. They compared their experimental data with a simple theoretical model, which predicted air temperatures in an "ideal" pre-swirl system. It was found that the pre-swirl system performed worse, in terms of cooling air temperature reduction, than was expected for isentropic flow. Laurello et al [3] studied ways to reduce the rotor pumping work and to increase the

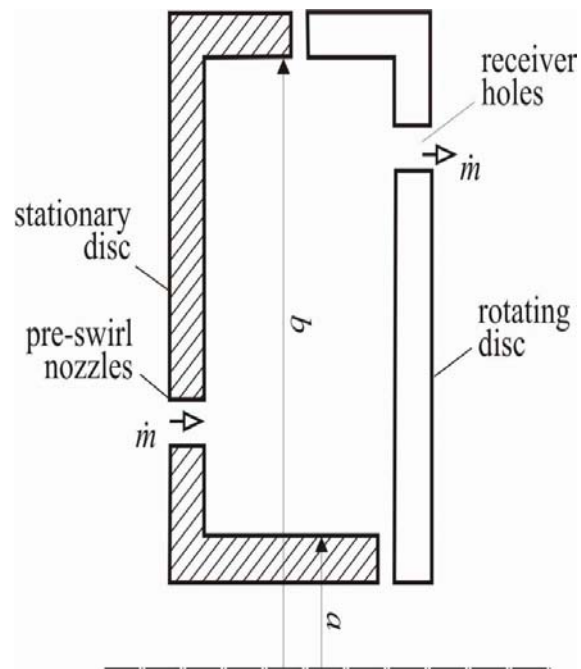


Figure 1. A schematic diagram of a direct-transfer pre-swirl rotor-stator system

efficiency of the pressure augmentation.

Yan et al [4] carried out measurements and three dimensional computations for the flow structure in an idealised pre-swirl rotor-stator system, and Farzaneh et al [5] described an investigation of the heat transfer in the same system. The results obtained show that the flow in the pre-swirl system has some similarities with that found in classical rotor-stator systems. The measurements and computations showed that significant losses in total pressure occurred between the inlet nozzles and the mid-axial plane between the rotor and stator (where pitot-tube measurements were made). These mixing losses, which were caused by a momentum exchange between the primary pre-swirl flow and the recirculating secondary flow, increased as the inlet pre-swirl ratio increased.

Karabay et al [6] carried out a combined experimental and computational study of flow in a "cover-plate" pre-swirl system. The cooling air from the stationary pre-swirl nozzles flowed radially outward (to the receiver holes) in a rotating cavity formed by the rotating disc and a cover-plate attached to it. Free vortex flow was found to occur for this system, and a theoretical analysis was used to show that there was an optimal value of the pre-swirl ratio, for which the average Nusselt number for a heated rotating disc would be a minimum.

Pilbrow et al [7] presented experimental and computational results for heat transfer in the same cover-plate system. Local Nusselt numbers, Nu , for the heated disc were found to depend

principally on Re_ϕ , λ_T and β_p . Nu increased with increasing Re_ϕ , and λ_T affected the shape of the Nu distribution; for low values of λ_T , non-entraining Ekman-type boundary layers formed on the corotating discs, giving rise to lower heat transfer rates than were measured (and computed) at higher values of λ_T . Axisymmetric computations, carried out using low-Reynolds-number $k - \epsilon$ turbulence models, were in reasonably good agreement with the measured Nu and reproduced the parametric variations observed.

As found by Karabay et al and other research workers, Yan et al also concluded that the flow structure in the pre-swirl chamber is controlled principally by the pre-swirl ratio, β_p , and the turbulent flow parameter, $\lambda_T = C_w/Re_\phi^{0.8}$ (The turbulent flow parameter, which combines the effects of the nondimensional preswirl flow rate, C_w , and the rotational Reynolds number, Re_ϕ , for the disc, is related to the entrainment of fluid into the boundary layer on a rotating disc; for turbulent flow, $\lambda_T \approx 0.22$ for the entrainment due to an unconfined disc.)

Owen and Rogers [8] described early work on pre-swirl systems, and Owen and Wilson [9] gave a brief review of more recent heat transfer research.

As the 3D computations described by Farzaneh et al [5] and Yan et al [4] incur long computing times, the model of an axisymmetric inlet and outlet is investigated in this paper. This allows computation times to be reduced by a factor of around 7, compared with corresponding 3D computations. Also, a theoretical expression has been derived for calculating adiabatic effectiveness of the system, and the computed results are compared with this.

Computational model

The solver described by Yan et al [4] and Farzaneh et al [5] was used here to investigate parametrically axisymmetric flow and heat transfer in the pre-swirl rotor-stator system. Brief details of the computational method are given here.

The governing equations were discretised using the finite-volume method with hybrid-differencing used for the convection terms. The SIMPLE pressure-correction scheme was adopted within a staggered grid arrangement, and the discretised equations were solved using the tri-diagonal matrix algorithm. For improving convergence performance, The Gosman damping factor [10] was used. The Yap correction term was used to reduce unrealistically large levels of near-wall turbulence that are returned by the Launder-Sharma low-Reynolds-number $k - \epsilon$ turbulence model in regions of flow separation.

A schematic diagram of the geometry modeled is shown in Fig. 1 It is based on an experimental rig in which there were 24 pre-swirl nozzles in the stator, inclined at an angle of 70 degrees to the axial direction, and 60 axial receiver holes in the rotor. The radial location of the nozzles, for which $x_p = r_p/b = 0.74$, is less than the centreline radius of the holes, $x_b = r_b/b = 0.93$ (see Yan et al [4] and Farzaneh et al [5] for more details).

To satisfy requirements for the turbulence model ($y^+ < 0.5$), a large number of grid points was used in the near-wall regions. Equivalent-area axisymmetric annular rings were used to represent the pre-swirl nozzles and receiver holes.

The mesh is illustrated in Fig. 2 and contains 40% more points in the axial-radial plane than were used by Yan et al [4] for 3D computations, mainly in order to resolve the smaller annular inlet required for the axisymmetric model and to keep the grid-spacing expansion/contraction ratios lower than 1.2. Computations were carried out using 139×295 (axial \times radial) points, determined by grid-distribution tests. Three tangential planes were required for the axisymmetric computations using the cyclic-symmetry boundary conditions described in Farzaneh et al [5].

No-slip boundary conditions were used for the velocity components at solid surfaces. Uniform axial velocity, deduced from the specified mass flow rate, and tangential velocity components were prescribed to give the angled pre-swirl flow at inlet. At the outlet, a prescribed uniform axial velocity was used to ensure continuity, and tangential velocities were computed from a zero normal derivative condition. The radial velocity component was zero at inlet and outlet. Flows through the clearance between rotating and stationary surfaces were taken to be zero, Fig. 1.

For heat transfer computation, adiabatic thermal boundary conditions were used for solid surfaces other than for the rotating disc for which a constant disc temperature, 20°C , was used. The inlet flow total temperature was kept constant at 55°C , and a zero normal-derivative condition was used for the outlet. Fluid properties were calculated at 20°C , as in the experiments [4].

The computations were carried out for values of the turbulent flow parameter $\lambda_T = 0.1, 0.14, 0.20, 0.26$ and 0.30 , with $9500 \leq C_w \leq 27200$ and $0.5 \leq \beta_p \leq 1.88$, and with Re_ϕ kept constant at 1.18×10^6 . The cases computed have been arranged in groups for which C_w varies while β_p remains constant.

Results and Discussion

Flow structure

Swirl ratio An ideal free vortex (see [6]) is one where $\beta_{\infty, id} = V_\phi/\Omega r \propto x^{-2}$ and $V_\phi = \Omega r$ at $x = x_1$ such that

$$\beta_{\infty, id} = (\beta_p x_1^2) x^{-2} \quad (1)$$

It is instructive to plot β_∞ v. x^{-2} to show how the actual swirl ratio compares with the ideal: any straight line passing through the origin represents free-vortex flow, but owing to losses the gradient of the line will be less than the ideal value of $\beta_p x_1^2$.

Fig. 3 shows the computed and measured variation of β_∞ v. x^{-2} for $\beta_p = 1.44$ and 1.88 ; $\beta_{\infty, id}$ is shown for comparative purposes. It can be seen that, despite some scatter, most of the measured values approximate to free-vortex flow with, for the

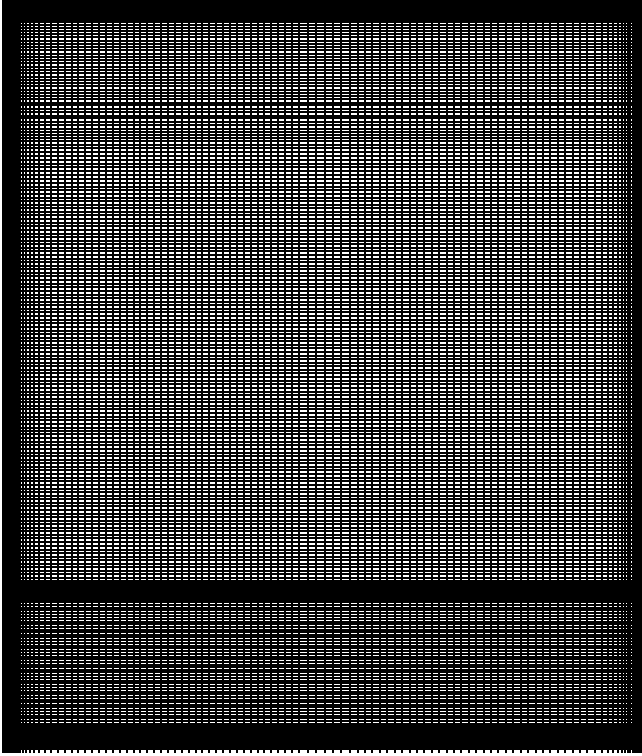


Figure 2. 139 × 295 (axial × radial) computation grid

reason given, a gradient less than the ideal value: $\beta_\infty/\beta_{\infty,id} \approx 0.7$ and 0.6 for $\beta_p = 1.44$ and 1.88 respectively. The reduction of 'effective swirl ratio' with increasing β_p was also observed in [6].

The computed values of β_∞ in Fig. 3, which display similar trends to the measurements, show that β_∞ increases as C_w increases. This increase becomes small for higher values of C_w (i.e. for $C_w \geq 23300$). The spike in the computations at $x^{-2} \approx 1.8$ occurs where $x = x_p$, the location of the narrow annular inlet slot used in the computational model. (Yan et al [4] and Farzaneh et al [5] obtained slightly better agreement between computations and measurements using a 3D model that involved a discrete inlet nozzle of simplified geometry.)

Moment coefficients The moment coefficient, C_M , for one side of the rotating disc is defined as:

$$C_M = -2\pi \int_a^b r^2 \tau_{\phi,r} dr / \frac{1}{2} \rho \Omega^2 b^5 \quad (2)$$

where $\tau_{\phi,r}$ is the shear stress on the rotating disc. As $y^+ < 1$, the first grid point is always inside the viscous sublayer, so:

$$\tau_{\phi,r} = \mu \left(\frac{\partial V_\phi}{\partial z} \right)_{z=0} \quad (3)$$

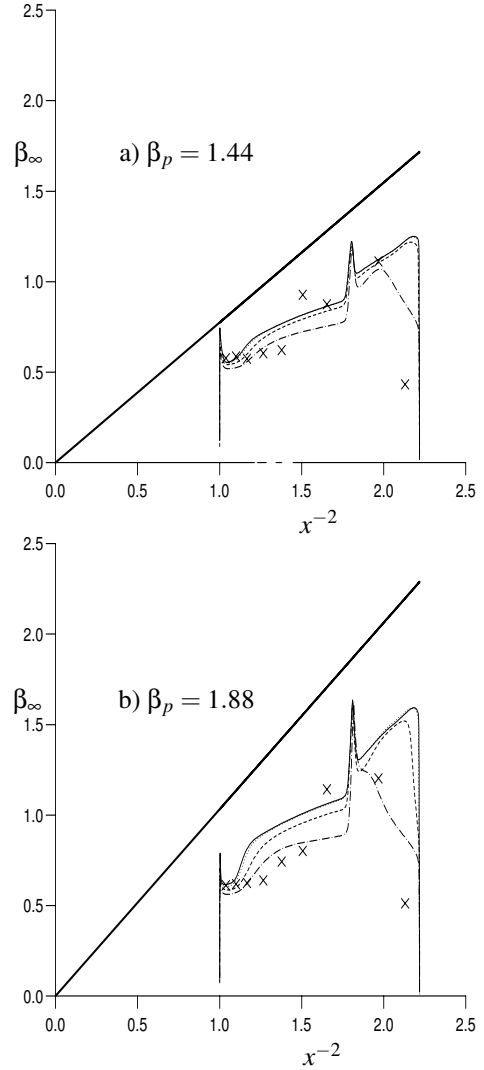


Figure 3. Computed and measured distribution of $\beta_\infty (= V_\phi / \Omega r)$ (mid plane) ($Re_\phi = 1.18 \times 10^6$)

- × measurement (a) $C_w = 27200$ (b) $C_w = 18100$)
- - - Computation ($C_w = 12500$)
- · - Computation ($C_w = 18100$)
- · · Computation ($C_w = 23300$)
- Computation ($C_w = 27200$)
- Ideal free vortex

The moment coefficients, computed for the rotating disc using equation 2, are shown in Fig. 4 .

Considering Fig. 4, for constant C_w , C_M decreases as β_p increases. For all cases, $C_M < 0.001$ when $\beta_p \geq 1.3$. Karabay at al [6] analysed the free vortex flow in a "cover-plate" pre-swirl system . Computations showed that C_M also reduced as β_p increased in that system, and that $C_M = 0$ at some "critical" value of the inlet swirl ratio, which depended only on the system geome-

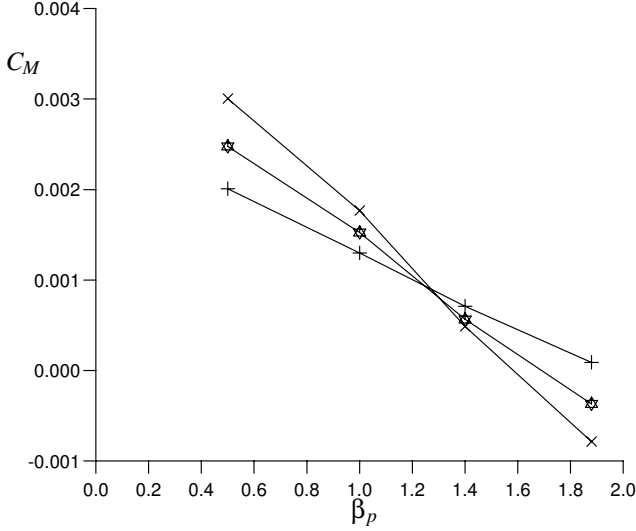


Figure 4. Computed effect of varying β_p and C_w on C_M ($Re_\phi = 1.18 \times 10^6$)

$$\begin{aligned} +C_w &= 12500 & *C_w &= 18100 \\ \times C_w &= 23300 \end{aligned}$$

try. Using the Reynolds analogy, they showed that:

$$\beta_{p,crit} = \frac{a^2 + b^2}{2r_p^2} \quad (4)$$

For the present rotor-stator system, the Reynolds analogy assumptions leading to the above result do not apply, and $C_M=0$ does not occur at a unique value of β_p . However, the rotating-disc moment is balanced by that on the other surfaces and, for the geometry of the pre-swirl chamber ($a = 145mm, b = 216mm, r_p = 160mm$) the above expression gives

$$\beta_{p,crit} = 1.317 \quad (5)$$

It is interesting to note that this result is close to the value $\beta_p \approx 1.3$ shown in Fig. 4 where computed values of C_M are independent of C_w .

Heat transfer

Adiabatic effectiveness By applying the first law of thermodynamics for an adiabatic open system, the rate of work done on the air is equal to the rate of increase of its total enthalpy.

$$-\dot{W}_{12} = \dot{m}c_p(T_{0,2} - T_{0,1}) \quad (6)$$

where T_o is total temperature in a stationary frame and \dot{W}_{12} is the work done on the air, such that:

$$\dot{W}_{12} = M_r \Omega \quad (7)$$

where M_r is the momentum exerted by the air on the rotating surface. Also as the air moves from the pre-swirl nozzles, station 1, to the receiver holes, station 2, the moment exerted by the rotating and stationary surfaces equals the rate of change of the angular momentum of the air. Hence,

$$M_r + M_s = -\dot{m}(r_2 V_{\phi,2} - r_1 V_{\phi,1}) \quad (8)$$

where M_s is the moment exerted by the air on the stationary surface. Now $\beta_1 = V_{\phi,1}/\Omega r_1$ and $\beta_2 = V_{\phi,2}/\Omega r_2$ hence:

$$c_p(T_{0,2} - T_{0,1}) = \Omega^2 r_2^2 \left[\beta_2 - \beta_1 \left(\frac{r_1}{r_2} \right)^2 \right] + \frac{\Omega M_s}{\dot{m}} \quad (9)$$

The total temperature at the receiver holes (station 2) in a stationary frame of reference can be written as

$$c_p T_{0,2} = c_p T_2 + \frac{1}{2}(V_r^2 + V_\phi^2 + V_z^2)_2 \quad (10)$$

Here, it is convenient to use $T_{t,2}$ as the total-temperature in the rotating frame, which is the value that would be measured by a total-temperature probe inside a blade-cooling passage (it is this temperature that controls the heat transfer from the blade to the air). By definition, when $V_{\phi,2} = \Omega r_2$,

$$c_p T_{t,2} = c_p T_2 + \frac{1}{2}(V_r^2 + V_z^2)_2 \quad (11)$$

where, for an axial blade-cooling passage, $V_{r,2} = 0$.

The adiabatic effectiveness, $\Theta_{b,ad}$, (see Karabay et al [6]) is defined as:

$$\Theta_{b,ad} = \frac{c_p(T_{o,1} - T_{t,2})}{1/2\Omega^2 r_2^2} \quad (12)$$

This is the non-dimensional difference in the total temperature between the stationary pre-swirl nozzles and the rotating receiver holes. A positive value of $\Theta_{b,ad}$ indicates that the pre-swirl system is effective in reducing the total-temperature of the blade-cooling air entering the holes. Hence from equation 9,

$$\Theta_{b,ad} = \beta_2^2 - 2(\beta_2 - \beta_1 \left(\frac{r_1}{r_2} \right)^2) - \frac{M_s}{1/2\dot{m}\Omega r_2^2} \quad (13)$$

For an ideal system, where solid-body rotation is achieved in the receiver outlet, $V_{\phi,2}/\Omega r_2 = 1$, and knowing $\beta_1 = \beta_p$, it follows

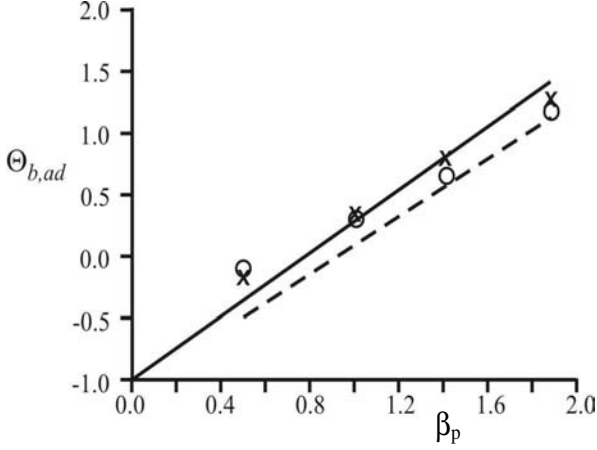


Figure 5. Comparison between the computed and theoretical variation of $\Theta_{b,ad}$ with β_p ($C_w = 18100$, $Re_\phi = 1.18 \times 10^6$)

- theoretical (equation 14) × computation (equation 12)
 - - theoretical (equation 15) o computation (equation 13)

that the ideal adiabatic effectiveness, $\Theta'_{b,ad}$ is given by

$$\Theta'_{b,ad} = 2\beta_p \left(\frac{r_1}{r_2} \right)^2 - 1 - \frac{M_s}{1/2\dot{m}\Omega r_2^2} \quad (14)$$

For the cover-plate case (see Karabay et al [6]), $M_s=0$ and equation 14 reduces to

$$\Theta'_{b,ad} = 2\beta_p \left(\frac{r_1}{r_2} \right)^2 - 1 \quad (15)$$

For the results presented here, where $r_1/r_2 = 0.8$, it follows from equation (15) that $\Theta'_{b,ad} = -1, 0$ and $+1$ when $\beta_p = 0, 0.78$ and 1.56 respectively, and $\Theta'_{b,ad} = 0.28$ when $\beta_p = 1$.

The two theoretical curves for $\Theta'_{b,ad}$ are plotted in Fig. 5, and the difference between them indicates the magnitude of the computed M_s term in equation (14). This term, which depends on the swirl inside the wheel-space, increases in magnitude as β_p increases.

Also shown in Fig. 5 are the values of $\Theta_{b,ad}$ computed using equations (12) and (13). (It should be noted that in the axisymmetric computations, the computed value of β_2 is always less than unity, which results in an underestimate in the computed value of $T_{i,2}$; the correction used by Karabay et al. [6] was used here to correct this underestimate in $T_{i,2}$.) It can be seen that the two sets of computed values of $\Theta_{b,ad}$ are in close agreement, although they both overestimate the theoretical curves for $\beta_p < 1$. As β_p increases, the computed results converge with the theoretical curve based on equation (14).

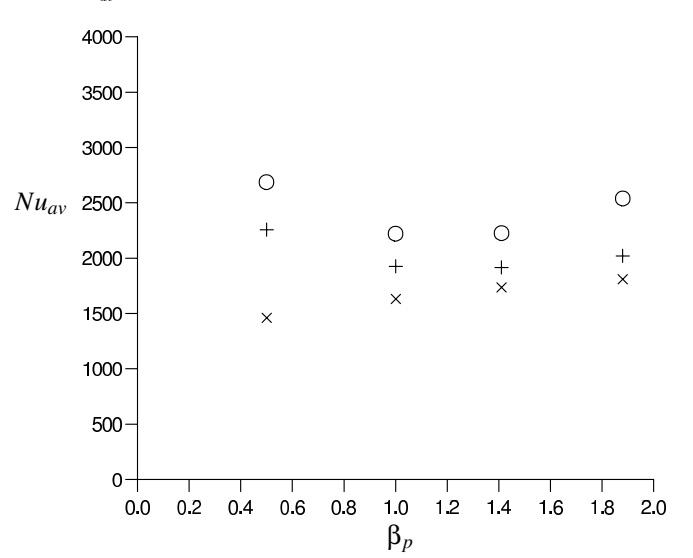


Figure 6. Computed effect of varying β_p and C_w on average Nusselt number ($Re_\phi = 1.18 \times 10^6$)

- × $C_w = 12500$ + $C_w = 18100$
 o $C_w = 23300$

Average Nusselt number Average Nusselt numbers were computed using the following equations

$$Nu_{av} = \frac{q_{r,av}b}{k(T_r - T_{r,ad})_{av}} \quad (16)$$

where $q_{r,av}$ and $T_{r,ad}$ are calculated as:

$$q_{r,av} = \frac{\dot{Q}_{ab}}{\pi(b^2 - a^2)} \quad (17)$$

$$T_{r,ad} = T_\infty + \frac{1}{2c_p}(R(\Omega r - V_{\phi,\infty})^2 - V_{\phi,\infty}^2) \quad (18)$$

a and b being the inner and outer radii of the heated disc, and \dot{Q}_{ab} the total heat transfer rate.

Fig. 6 shows the computed variation of Nu_{av} with β_p for $Re_\phi = 1.18 \times 10^6$ and for $C_w = 12500, 18100$ and 23300 ($\lambda_T = 0.10, 0.14$ and 0.20 respectively). For the two higher values of C_w , Nu_{av} first decreases then increases as β_p increases, and Nu_{av} reaches a minimum at $\beta_p \approx 1.2$. This behaviour, which does not occur for $C_w = 12500$, was also found in the cover-plate pre-swirl system (see [6]). In that paper, the Reynolds analogy was used to explain how this phenomenon can occur when there is free-vortex flow over a rotating disc.

Conclusions

A combined computational and theoretical study has been carried out to investigate the effects of flow rate and swirl ratio on the flow and heat transfer in a pre-swirl rotating-disc system, representative of that found in gas-turbine cooling systems. Measurements of swirl ratio have been compared with results obtained using a simplified axisymmetric model of the system. Computed and measured values of swirl ratio show that there are losses of angular momentum in the system and that these losses increase as β_p increases. The computations show that the moment coefficient reduces as inlet swirl ratio increases and, for $\beta_p \approx 1.3$, moment coefficients are independent of inlet flow rate.

A theoretical expression has been developed to show how the adiabatic effectiveness, $\Theta_{b,ad}$, varies with β_p . Computed values of $\Theta_{b,ad}$ for $0.5 < \Theta_{b,ad} < 1.88$ overestimate the theoretical values, but the difference decreases as β_p increases.

The variation of Nu_{av} with β_p has been computed for three values of C_w . For the higher two values, Nu_{av} first decreased then increased as β_p was increased, and a minimum value of Nu_{av} occurred at $\beta_p \approx 1.2$. This behaviour is believed to be caused by free-vortex flow over a rotating disc, as found in cover-plate systems. For the smallest value of C_w , Nu_{av} increased monotonically as β_p was increased and no minimum was observed.

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