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Semi-Empirical Compressor Performance Analysis and Prediction Using Only Basic Characteristic Information

Mark A Johnson
Dyson Ltd
Tetbury Hill
Malmesbury
SN16 0RP, UK

Stuart A MacGregor
Department of Mechanical Engineering
University of Bath
Bath BA2 7AY, UK

Michael Wilson
Department of Mechanical Engineering
University of Bath
Bath BA2 7AY, UK

ABSTRACT

The measured performance of a small turbocharger compressor has been analysed in order to develop a method of calculating the internal aerodynamic performance where only the basic compressor characteristic and geometry is available. Through the application of impeller and diffuser efficiency boundaries, the compressor internal component performance, such as impeller discharge angle and impeller relative velocity ratio, can be calculated within a small range of possible magnitudes. Impeller efficiency set equal to the square root of the stage efficiency offers a good approximation where impeller discharge static pressure measurements are not available.

A procedure is devised so that the full compressor characteristic can be predicted using the known basic performance at one impeller speed, using either a fan law approach to predict the stage characteristic, or correlations for stage efficiency and an impeller slip factor. Where a vaned diffuser is used, it is found that correlation should be made against impeller discharge conditions, while in the case of a vaneless diffuser an average of impeller inlet and discharge conditions is found to be more suitable.

The accuracy of the prediction procedures is shown to be satisfactory, however the ability to predict the onset of surge has not been addressed.

Keywords: *centrifugal compressors, performance prediction*

NOMENCLATURE

a	speed of sound
b	impeller blade height
A	area
C	absolute velocity
C_m	absolute radial component of velocity
C_u	absolute tangential component of velocity
h	enthalpy
M	Mach number
M_u	non-dimensional velocity ($= U_2/a_{01}$)
$\dot{m}$	mass flow rate
P_{RS}	stage pressure ratio
P_o	stagnation pressure
r	radius
T_o	stagnation temperature
U	blade speed
α	absolute flow angle
β	relative flow angle
ϕ	flow coefficient
γ	ratio of specific heats
η	efficiency
μ	slip factor
ρ	density
θ	non-dimensional mass flow rate
ψ	head coefficient

Subscripts

AV	average
b	blade
D	discharge
E	entry
max	maximum value
o	stagnation condition

RMS root mean square
s stage or shroud
1 , 2 respectively, impeller inlet and impeller discharge

1 INTRODUCTION

A method for calculating the internal aerodynamic performance of a compressor, where only basic geometry and performance characteristics of pressure ratio and stage efficiency are available, can be of value to industrial users of packaged compressor installations. Additional measurements (such as of impeller discharge pressure) can be difficult or expensive to obtain, and it is often required either to assess the compressor performance beyond that given by the basic characteristic or to modify the installation, for example by changing the vaned diffuser in order to obtain a required stage performance. In this paper, an analysis procedure based on knowledge of only the basic geometry and performance characteristics of the compressor is described, and is assessed using principally the measured performance of a small turbocharger compressor with a vaned and a vaneless diffuser. The results of the method could be used to assess the need for modifications or design changes prior to undertaking more detailed work.

The modelling techniques used to analyse compressor performance are described fully by Japikse[1]. Models rely on the measurement of the basic compressor performance data, the inlet and discharge stagnation pressure and temperature, the gas mass flow rate and the impeller rotational speed, together with the measurement of a parameter at the impeller discharge, usually the static pressure. Rodgers[2] and Swain[3] presented performance prediction methods based on limited input information. As described by Whitfield et al[4], both the modelling procedures and the experimental measurements (particularly of the impeller discharge parameters) can only be carried out with a degree of uncertainty.

Whitfield et al[4] described an analysis procedure based on knowledge of basic compressor pressure ratio and efficiency characteristics, the inlet stagnation conditions and the compressor geometry. The method allows compressor discharge conditions and discharge flow angles to be calculated within an acceptable range where no internal pressure measurements are available, and has been found to be useful in reducing development time for new compressor stages. The main features of the analysis are given below.

2 ANALYSIS PROCEDURE

The stagnation temperature can be found through the isentropic efficiency definition:

$$\frac{T_{oD}}{T_{o1}} = 1 + \frac{P_{RS}{}^{(\gamma-1)/\gamma} - 1}{\eta_S} \qquad (1)$$

The impeller discharge stagnation temperature, T_{OE}, attributable to the change in angular momentum across the impeller, was set equal to the discharge stagnation temperature T_{OD} (ignoring the effects of disc friction). The Euler Turbomachinery equation was then used to provide the tangential component of velocity, C_{u2}:

$$\frac{T_{oE}}{T_{o1}} = 1 + (\gamma - 1)\frac{U_2}{a_{o1}}\frac{C_{u2}}{a_{o1}} \qquad (2)$$

C_{u2} is also given through the velocities and the slip factor μ:

$$C_{u2} = \mu U_2 + C_{m2} \tan\beta_{b2} \qquad (3)$$

where β_{b2} is the impeller discharge blade angle (which is negative for a normal backsweep design).

Where no measurements at impeller discharge are available, it is necessary to estimate a further parameter in order to proceed. The absolute discharge flow angle, α_2, was selected as it leads to a relatively simple derivation of the other parameters.

Whitfield et al[4] assessed the uncertainty associated with the assumed flow angle at impeller discharge. A wide range of discharge flow angles could be considered, leading to a wide range of predicted impeller discharge conditions and as a consequence a range of possible impeller and diffuser efficiencies. However, the lowest possible impeller efficiency is that which gives rise to a predicted diffuser efficiency of 100%, with the lowest diffuser efficiency occurring at an impeller efficiency of 100%. Thus, restricting these efficiencies to a maximum of unity, limits the range of possible flow angles α_2. By specifying that the efficiencies cannot exceed 1.0 the range of possible absolute discharge flow angles for any mass flow rate was found to be small (Whitfield et al[4]). (As the stage efficiency reduces at high flow rates an experienced designer could reduce the upper limit for the impeller and diffuser efficiency from 100%, and thus the uncertainty in finding the flow angle improved.)

The absolute flow angle, together with the known tangential component of velocity, equation (3), made it possible to establish both the velocity vectors at impeller discharge and the impeller slip factor, again within a small range due to the efficiency limits used. As the impeller discharge conditions have been derived from the assumed absolute flow angle, the impeller discharge static pressure can be calculated by application of the continuity condition. The range of possible discharge static pressures that could occur within the efficiency limits described above was found to be large. It was found to be acceptable however to estimate the impeller efficiency as the square root of the stage efficiency, known from the basic compressor data (Whitfield et al[4]). The discharge static pressure and density could then be found.

The impeller discharge conditions provide with reasonable accuracy the diffuser inlet flow angle and the absolute Mach number. This, therefore, provides the diffuser inlet conditions and can be used to assess the suitability of the impeller design. If the design incorporates a vaneless diffuser the inlet conditions can be used together with an analysis procedure, such as that by Stanitz[5], to calculate the discharge conditions. The impeller discharge flow angle provides the means to assess the performance of the vaneless diffuser and the collecting volute, Young[6]. If a vaned diffuser is incorporated the impeller discharge conditions provide the vaned diffuser incidence angle and approach Mach number and an assessment of the impeller/diffuser match can be made.

3 PERFORMANCE CORRELATION

Any procedure should predict accurately:

i) the mass flow rate at which peak efficiency occurs
ii) the peak efficiency
iii) the mass flow rate at which peak pressure ratio occurs
iv) the peak pressure ratio
v) the mass flow rate at which 'choke' occurs, or the flow rate at a specified lower efficiency limit
vi) the pressure ratio at a specified flow rate
vii) the mass flow rate at which surge will occur

For the present work, the peak efficiency at all impeller speeds was considered known. Eventually the peak efficiency must be specified from experience or derived through the application of empirical correlations.

Two prediction procedures have been developed. The first uses classical non-dimensional analysis leading to the fan laws for incompressible flow machines, developed to accommodate compressibility. The second uses the analysis of Whitfield et al[4], described in outline in section 2, to develop correlations for efficiency and slip factor with impeller discharge flow angle.

3.1 Fan law analysis

Application of non-dimensional analysis to incompressible flow in turbomachines leads to the flow coefficient, ϕ, and head coefficient, ψ (Sayers[7]). For compressible flow machines, the volume flow rate is not constant and the mass flow rate is used in the definition of the flow coefficient:

$$\phi_1 = \frac{\dot{m}}{\rho_{01} A_I U_2} \qquad (4)$$

The mass flow rate can be calculated through:

$$\dot{m} = \rho_1 A_1 C_1 \cos\alpha_1 \qquad (5)$$

and combined with equation (4) to give:

$$\phi_1 = \frac{\rho_1 A_1}{\rho_{01} A_I} \frac{C_1 \cos\alpha_1}{U_2} \qquad (6)$$

For zero inlet swirl, $\alpha_1 = 0$, this can be written as:

$$\phi_1 = \frac{A_1 r_1}{A_I r_2} \frac{\rho_1}{\rho_{01}} \frac{1}{\tan\beta_{1s}} \qquad (7)$$

For incompressible flow machines the flow coefficient, ϕ_1, provides a satisfactory parameter for the correlation of performance data. Equation (7) implies that the performance is a function of impeller inlet relative flow angle, β_{1s}, hence impeller incidence angle. For this to apply to compressible flow machines the incidence angle and density ratio ρ_1/ρ_{01} should not vary with impeller non-dimensional speed.

The head coefficient is often used as a direct alternative to the compressor stage pressure ratio, using the isentropic enthalpy rise across the impeller Δh_{0s} so that:

$$\psi = \Delta h_{os} / U_2^2 \qquad (8)$$

The head coefficient can be written, in terms of stage pressure ratio and non-dimensional speed, as:

$$\psi = \frac{1}{\gamma - 1} \frac{1}{M_u^2} \left[\left(P_{RS}\right)^{(\gamma-1)/\gamma} - 1 \right] \qquad (9)$$

With the inclusion of a vaned diffuser it is probable that the crucial flow rates are a function of diffuser incidence. A flow coefficient, ϕ_2, based on impeller discharge conditions may therefore be more appropriate, and can be defined as:

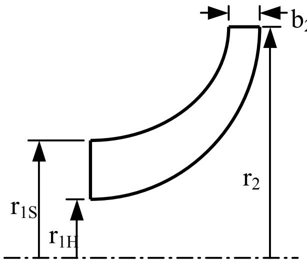
$$\phi_2 = \frac{\dot{m}}{\rho_2 A_I U_2} = \left(2 \frac{b_2}{r_2} \frac{a_{02}}{a_{01}} \frac{1}{M_u} \right) M_{02} \cos\alpha_2 \qquad (10)$$

In section 2 it was found that the impeller discharge flow angle α_2 and Mach number M_{02} could be predicted accurately, within the 100% efficiency limits for the impeller and diffuser. Equation (10) can therefore be used to calculate ϕ_2 for any reasonable impeller efficiency. As described above, the square root of the stage efficiency was used.

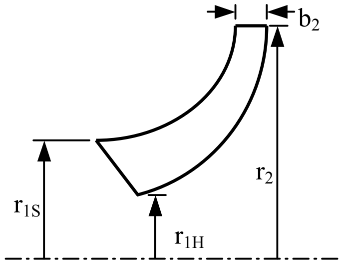
3.2 Analysis results and discussion

The analysis methods described above were applied to a gas pipeline compressor, compressor C, for which a measured performance characteristic was available, and to a small turbocharger compressor as tested by Eynon[8]. Compressor C incorporated an impeller with a non-axial inducer and a low solidity vaned diffuser having a vane leading edge angle of 84°. The turbocharger compressor was analysed in two configurations, referred to here as V and VL: compressor V incorporated a low solidity vaned diffuser having a vane leading edge angle of 70°, while compressor VL incorporated an alternative vaneless diffuser. Other details are shown in Fig. 1.

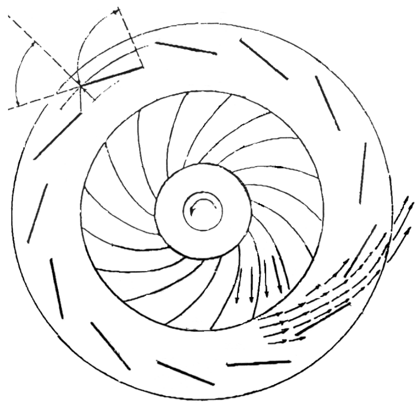
Fig. 2 shows head coefficient and stage efficiency plotted as a function of ϕ_2 for compressor C. The correlation for head coefficient is fair (although not entirely satisfactory), and a shift in peak efficiency with impeller speed is also apparent. This deficiency is attributed to uncertainty in the experimental data for the low speed efficiency curve used for the stage efficiency correlation, Johnson[9]. (Due to this uncertainty, other results for compressor C are not shown here.)



(a) axial inducer impeller, compressor V



(b) non-axial inducer impeller, compressor C



(c) schematic illustration of compressor C

Fig. 1: Compressor geometry details
(a) (r_{1H} = 10.3 mm, r_2 = 49 mm, b_2 = 5.2 mm)
(b) (r_{1H} = 236 mm, r_2 = 590.5 mm, b_2 = 47.625 mm)

Results for head coefficient and stage efficiency plotted as a function of ϕ_2 for compressor V are shown in Fig. 3. The correlation for head coefficient is reasonably good, with the variation with impeller speed being around 7%. For compressor VL however, having a vaneless diffuser, a variation with speed was found of around 24%.

The correlation for stage efficiency with discharge coefficient for compressor VL was found to be good at low flow rates, but unsatisfactory at flow rates greater than that at peak efficiency. Conversely, acceptable correlation with impeller inlet conditions was apparent only at high flow rates. As an intermediate solution, the use of an average flow coefficient, $\phi_{AV} = (\phi_1 + \phi_2)/2$, was assessed. Fig. 4 shows that both the head coefficient and stage efficiency correlate sufficiently well with this average of impeller inlet and discharge flow coefficients.

With satisfactory correlations found for each compressor, the known performance at a single impeller speed could be used to predict the mass flow rate, pressure ratio and efficiency characteristics at the remaining speeds. For any selected flow coefficient, ϕ_2, the efficiency and head coefficient is known through the correlation, and the mass flow rate and pressure ratio can be established, Johnson[9].

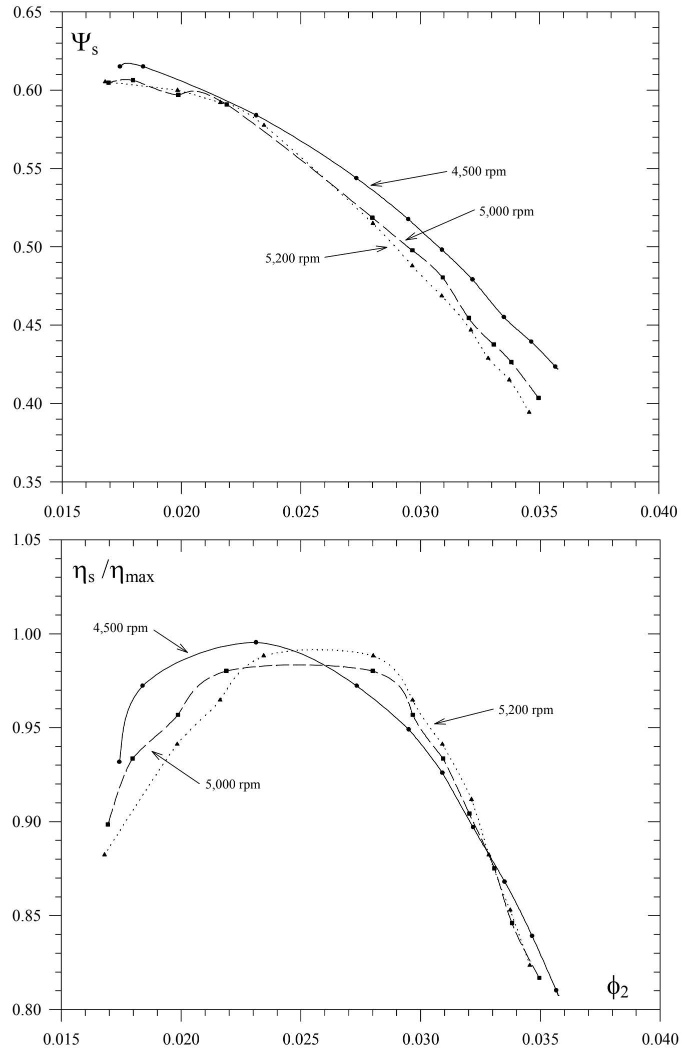


Fig. 2: Head coefficient and stage efficiency ratio as a function of ϕ_2 for compressor C

Specification of the average flow coefficient, ϕ_{AV}, for compressor VL does not provide direct access to the impeller inlet or impeller discharge conditions. The impeller discharge absolute flow angle was therefore estimated initially, leading to a calculated discharge flow coefficient, ϕ_2, from which the inlet flow coefficient, ϕ_1, could be found through continuity. The solution was converged to give the required average flow coefficient and the mass flow rate found, Johnson[9]. Performance prediction results are discussed in section 4.

3.3 Alternative correlation parameters

The above application of the fan laws shows that satisfactory correlations of efficiency can be obtained with ϕ_2, which is a function of impeller discharge flow angle. In section 2 it was found that the impeller discharge flow angle could be estimated, with good accuracy, for any given compressor performance.

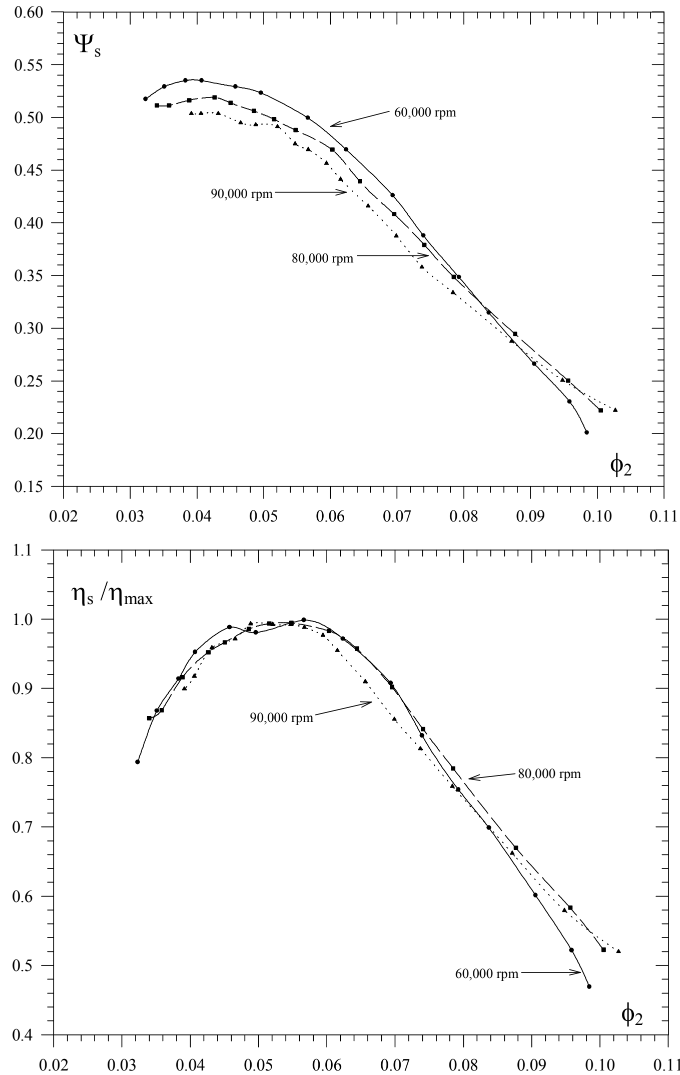


Fig. 3: Head coefficient and stage efficiency ratio as a function of ϕ_2 for compressor V

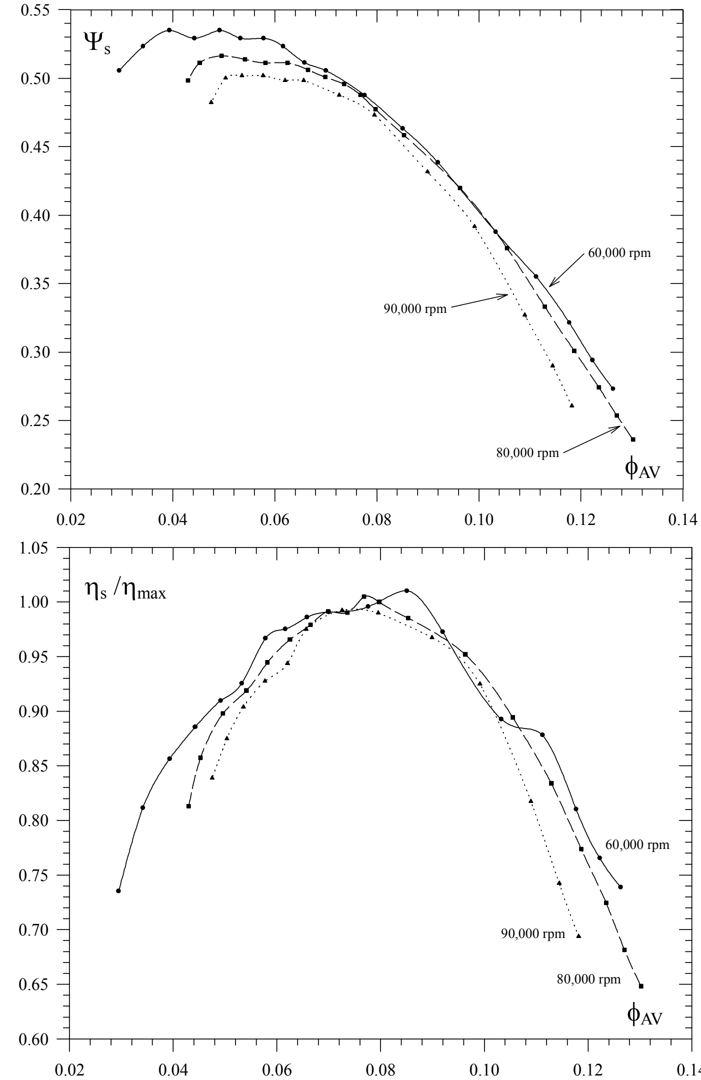


Fig. 4: Head coefficient and stage efficiency ratio as a function of $(\phi_1+\phi_2)/2$ for compressor VL

Where a vaned diffuser is installed, the incidence onto the vanes has a large impact on performance. The derived discharge flow coefficient, which is a function of impeller discharge flow angle α_2, should therefore be a suitable parameter to correlate both head coefficient and stage efficiency for compressors C and V. It is therefore appropriate to assess the direct application of the flow angle as a correlation parameter.

Through the application of the fan laws the stage pressure ratio is obtained directly from the head coefficient, equation (9). The slip factor, which is known to vary with flow rate, is not considered directly. For a compressor operating with zero inlet swirl the stage pressure ratio is given by:

$$P_{RS}^{\gamma-1/\gamma} = 1 + (\gamma - 1)\eta_s \frac{U_2}{a_{01}} \frac{C_{u2}}{a_{01}} \tag{11}$$

The tangential component of velocity C_{u2} is a function of the slip factor, equation (3). Using equations (11) and (3), the head coefficient equation (9) can be rewritten as:

$$\psi = \eta_s\left(\mu + \phi_2 A_I \tan\beta_{b2}/A_2\right) \tag{12}$$

where $\phi_2 = A_2 C_{m2}/A_I U_2$, as equation (10).

The head coefficient is therefore a function of slip factor μ as well as flow coefficient ϕ_2, and this was not included in the application of the fan laws.

To accommodate the slip factor, the pressure ratio, equation (11), can be written as:

$$\left(P_{RS}\right)^{(\gamma-1)/\gamma} = 1 + (\gamma - 1)\eta_s \frac{U_2^2}{a_{o1}^2} \frac{\mu \tan\alpha_2}{\tan\alpha_2 - \tan\beta_{b2}} \tag{13}$$

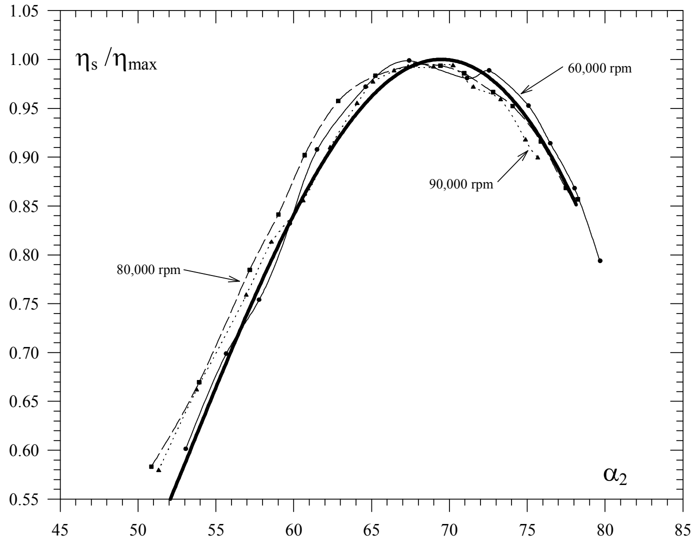


Fig. 5: Stage efficiency ratio as a function of impeller discharge flow angle α_2 for compressor V

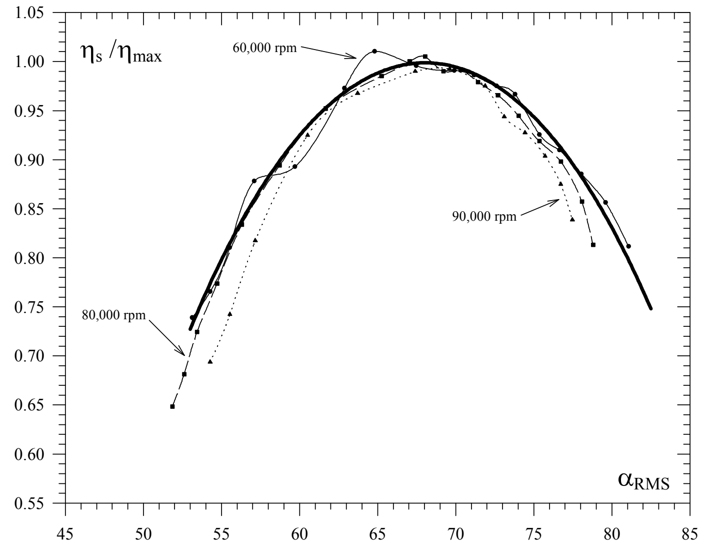


Fig. 6: Stage efficiency ratio as a function of the mean flow angle α_{RMS} for compressor VL

where the tangential component of velocity is given by

$$C_{u2} = \mu U_2 \frac{\tan \alpha_2}{\tan \alpha_2 - \tan \beta_{b2}} \tag{14}$$

The efficiency correlation for compressor V as a function of discharge flow angle α_2 is shown in Fig. 5. At low flow angles, corresponding to high flow rates, there is less than a 2° spread in flow angle, and this uncertainty will not lead to a large uncertainty in the derived mass flow rate, Whitfield et al[4]. For any specified flow angle the maximum uncertainty for efficiency was of the order of 3%.

For compressor VL, having a vaneless diffuser, it was found that a good correlation of efficiency could be obtained using a root mean square of the impeller inlet and discharge flow angles, $\alpha_{RMS} = \sqrt{\left(\alpha_2^2 + \beta_{1s}^2\right)/2}$, as shown in Fig. 6.

In order to predict pressure ratio, a further correlation for slip factor is required. A correlation allowing for a variation with flow rate was obtained using the slip factors derived from measurements. The slip factor was correlated against the same parameter as for the efficiency. For the three impeller speeds used, the results provided a broad correlation of the trends and the variation with flow rate for both compressors V and VL. The results for compressor V are shown in Fig. 7.

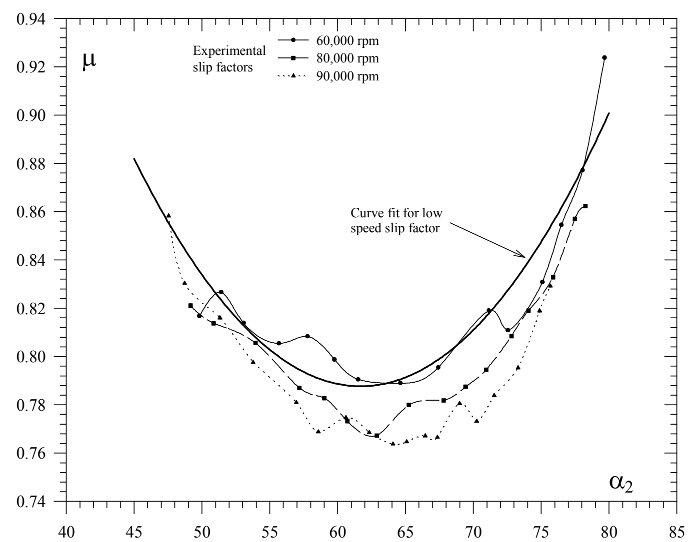


Fig. 7: Slip factor μ as a function of discharge flow angle α_2 for compressor V

4 PERFORMANCE PREDICTION

The correlations described above can be used to form a performance prediction procedure where limited experimental data are available. In order to assess the prediction method, experimental data available at the lowest impeller speed were used to predict the compressor performance at all other speeds.

4.1 Modified fan laws

For compressor V the correlation of both efficiency and head coefficient is good, see Fig. 3. The consequent prediction of efficiency is very good, as shown in Fig. 8, with the location of the peak efficiency predicted accurately. The maximum error observed for the peak pressure ratio is around 8%. The results shown assume that the peak efficiency is known, or can be derived, at speeds other than the test speed. If the peak efficiency measured at the test speed were used at all speeds the peak efficiency would be over estimated, see below.

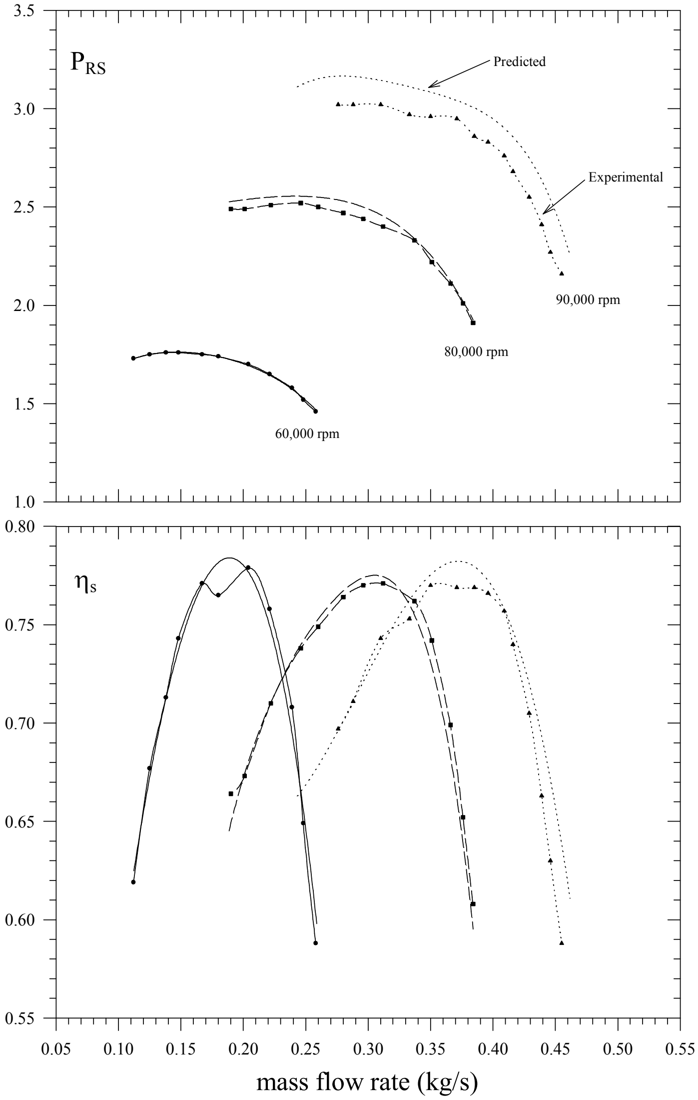


Fig. 8: Predicted performance characteristic using modified fan laws for compressor V

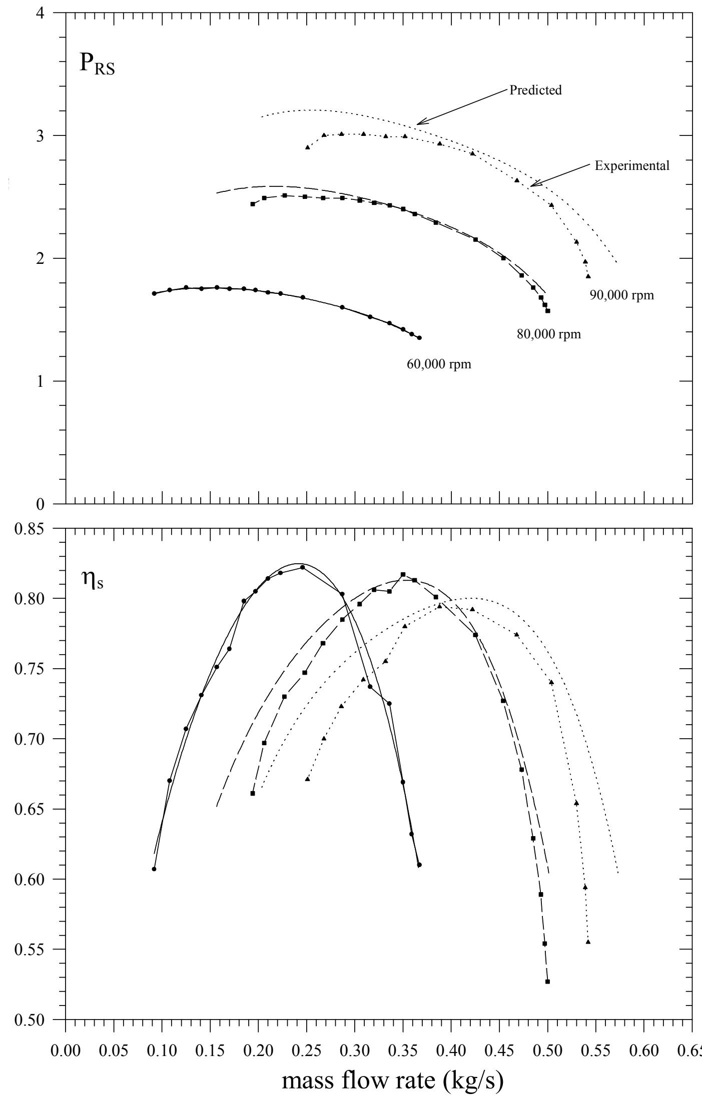


Fig. 9: Performance prediction using mean flow coefficient correlation for compressor VL

For compressor C the prediction of efficiency was poor due to the poor correlation of efficiency with discharge flow coefficient, see section 3.2. The predicted pressure ratio using the fan laws was satisfactory however (and is given by Johnson[9]), as this is based on the correlation for head coefficient shown in Fig. 2, which is sufficiently good.

For compressor VL, having a vaneless diffuser, the predicted efficiency is again good, Fig. 9, following from the good correlation with ϕ_{AV} shown in Fig. 4. Similarly, the peak efficiency pressure ratio is well predicted, being based on a good correlation for head coefficient. The correlation for head coefficient is not as good away from the peak efficiency flow rate, see Fig. 4. As with compressor V, this translates into an error in the predicted peak pressure ratio.

4.2 Application of the slip factor correlation.

For this alternative approach the predicted efficiency is similar to that described above, being based on satisfactory correlations with α_2 and α_{RMS}. The predicted performance for compressor VL, using the correlations for efficiency and slip factor as a function of mean flow angle, is shown in Fig. 10. The mass flow rate at which the peak stage efficiency occurs is predicted with good accuracy at all impeller speeds. As the efficiency reduces some divergence from the measured efficiency is present, due to the error in the stage efficiency correlation used, see Fig. 6.

The accuracy of the predicted pressure ratio is dependent upon the accuracy of both the predicted efficiency and the slip factor, equation (13). The slip factor correlation with flow angle exhibits a degree of uncertainty, as illustrated in Fig. 7.

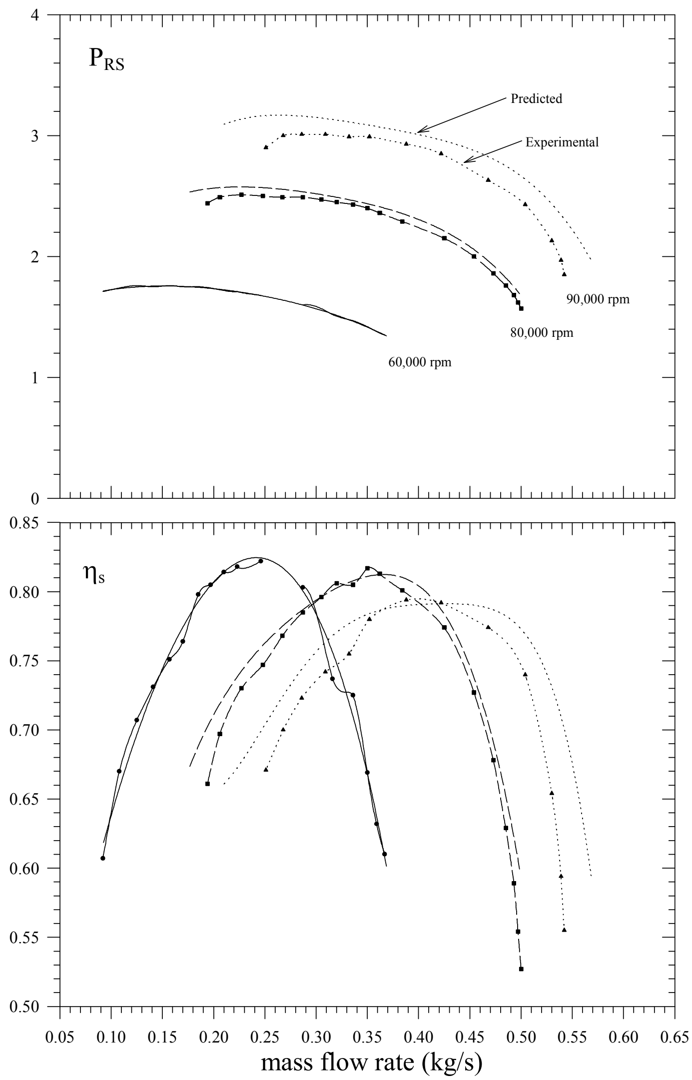


Fig. 10: Performance prediction using mean flow angle correlation for compressor VL

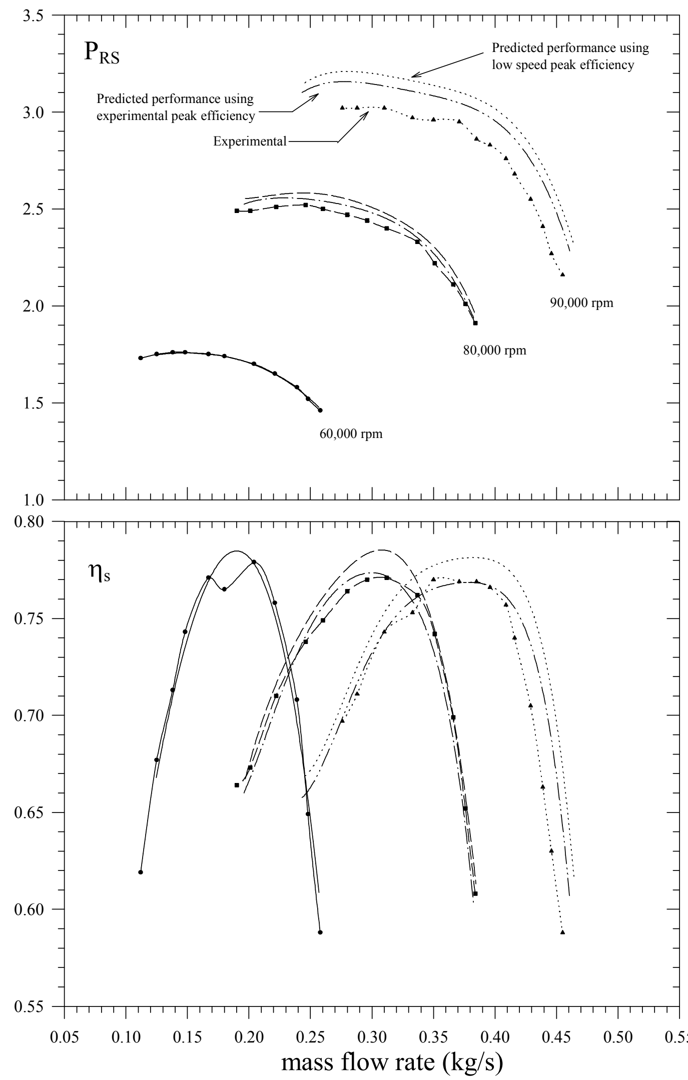


Fig. 11: Performance prediction using flow angle correlation for compressor V, using the low speed peak efficiency for all speeds.

Fig. 10 shows that the pressure ratio is predicted with reasonably good accuracy at all mass flow rates, with an error of about 7% being observed in the peak efficiency pressure ratio at 90,000rpm. At impeller speeds greater than 60,000 rpm the slip factor is consistently over-estimated, leading to the error detected in the impeller discharge pressure. The peak pressure ratio also is predicted with reasonably good accuracy. The larger error, when compared to the error in the peak efficiency pressure ratio, can again be traced to the efficiency correlation used (Fig. 6). The error was negligible in the case of the peak efficiency operating point but about 3% at the peak pressure ratio flow rate.

Due to uncertainty in the experimental data for the vaned pipeline compressor C, the use of the flow angle correlation to predict its performance was not assessed, however results for the vaned turbocharger compressor V are described below.

4.3 Derivation of the peak efficiency.

The above performance predictions were derived using a correlation based on an efficiency ratio, η/η_p, derived at the test speed. The peak efficiency was assumed to be known at all speeds, however, eventually this must either be derived, or the known peak efficiency at the test speed used. A motivation for the development of the method described here is the time and cost benefit of characterizing the stage at a single speed and then using the procedure to complete the performance map at other desired speeds.

The performance of compressor V was recalculated, for each speed, using the peak efficiency measured at the lowest speed. The results are shown in Fig. 11. The error in the predicted peak efficiency at the highest speed, although still acceptable, is higher than that obtained using the ratio η/η_p based on the measured peak efficiency at that speed. The increase in the error also affects adversely the accuracy of the predicted pressure ratio. The accuracy of the prediction is then similar to that found using the modified fan law approach (Fig. 8).

The significance of the use of the low speed peak efficiency will depend on how the peak efficiency varies for a given compressor. In the case of compressor VL, the peak efficiency decreases from 82.6% to 80% as the speed increases from 60,000 to 90,000rpm, representing an additional error in the efficiency correlation compared to that used previously. The increase in the error in the predicted peak efficiency pressure ratio at 90,000rpm was similar to that shown in Fig. 11 for compressor V.

From the data available it is not certain that surge was reached and therefore the ability of the routine to predict the onset of surge could not easily be assessed, although the mass flow rate at which peak pressure ratio occurred could be predicted with good accuracy. This area would benefit from further work to fully assess the accuracy of the procedures as surge is approached, as the ability to predict the onset would be beneficial to prevent compressor operation reaching this point.

5 CONCLUSIONS

An analysis procedure has been developed for the prediction of compressor performance using only basic compressor characteristic information. Using the method, the mass flow rate at which peak efficiency occurred could be predicted with good accuracy for a turbocharger compressor fitted with either a vaned diffuser or a vaneless diffuser.

Both a modified fan law and a flow angle method gave good overall predictions of performance. The maximum error observed for the peak pressure ratio using the modified fan law approach is around 8%. If the peak efficiency is not known at all impeller speeds, but only at the test speed, the maximum error observed for the modified fan law routine is not affected. For the alternative prediction method using correlations against flow angle, the efficiency is used in the prediction for pressure ratio. If the known peak efficiency at the test speed is applied at all impeller speeds the error observed in the pressure ratio increases.

The greater simplicity of the modified fan laws would be beneficial for online monitoring of compressor performance. If the peak efficiency is known or derived at all impeller speeds, however, the flow angle prediction method is more accurate than using the modified fan laws. Further application of the methods is being undertaken, and more work is needed to find possible correlations of compressor parameters with surge.

ACKNOWLEDGMENTS

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