Free convective heat transfer within rotating annuli

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Abstract

Numerical simulations have been performed to investigate free convective heat transfer inside sealed, air-filled rotating annuli with an imposed radial thermal gradient. Results for the cases computed are compared with published results for rotating annuli and other suitable stationary enclosures. We are interested in particular in the relationship between Nusselt number and Rayleigh number, and in identifying factors that either increase or decrease heat transfer. The study is concerned with fluids having Prandtl number Pr ≈ 0.7.

1. Introduction

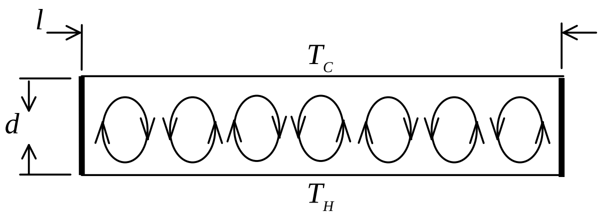

Figure 1 Horizontal layer with heating from below

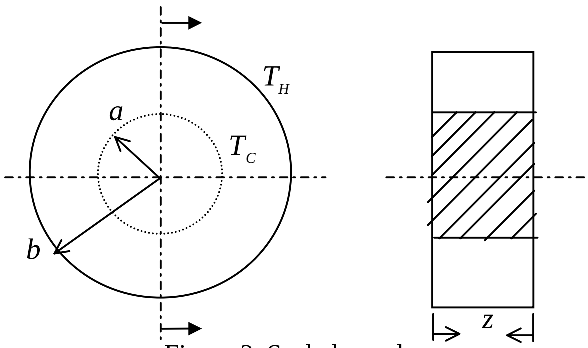

Figure 2 Sealed annulus

Free convective heat transfer in flat horizontal layers with heating from below (Figure 1), the so-called Rayleigh-Benard convection, has been extensively studied for decades (see Koschmieder, 1993, Siggia, 1994 and Grossmann & Lohse, 2000). This type of flow can be approximated by a rotating annulus with large value of the ratio $r_m/(b\text{-}a)$ as illustrated in Figure 2; the centrifugal acceleration $r_m\Omega^2$ directed radially away from the centre of the annulus replaces the gravitational acceleration. The Coriolis force may or may not be negligible (or it can be removed by eliminating artificially certain terms in the governing equations). Such solutions for Rayleigh-Benard type convection may assist our understanding of heat transfer in rotating annulus convection, such as that which occurs (for example) in the rotating cavities between co-rotating compressor or turbine discs in gas-turbine engines (see Owen and Wilson, 2000 and Bohn et al, 1995).

It can be suggested (from dimensional and physical considerations, King, 2002) that $Nu = C_1Ra^A + C_2Ra^B + \ldots$ in Rayleigh-Benard convection, where C_i and A, B etc are constants. It is assumed widely however that a single term, $Nu \propto Ra^\gamma$, is sufficient. Kraichnan (1962), by dimensional reasoning, suggested $\gamma = 1/2$; Malkus (1954), using other assumptions, calculated $\gamma = 1/3$. The former value ($\gamma = 1/2$) is believed to be the 'ultimate state' of turbulent thermal convection, in which the heat flux is independent of viscosity and thermal diffusivity as heat is transported by buoyant structures. The latter ($\gamma = 1/3$) assumes that the thermal field is well mixed in the bulk flow and thus heat flux is independent of the enclosure height.

Nomenclature

- a inner radius, b outer radius
- d distance between hot and cold wall(=b-a for annulus)
- Nu Nusselt number (heat flux from hot wall to cold wall) / (heat flux by conduction alone)
- Pr Prandtl number = ν/κ
- Ra Rayleigh number = $g\beta\Delta Td^3/\nu\kappa$
- Ra_b rotational Rayleigh number based on b = $\Omega^2\beta\Delta Tb^4/\nu\kappa$
- Ra_ϕ rotational Rayleigh number = $r_m\Omega^2\beta\Delta Td^3/\nu\kappa$
- Re_b rotational Reynolds number based on b = $b^2\Omega/\nu$
- Re_ϕ rotational Reynolds number = $r_m\Omega d/\nu$
- r radius , $r_m = (a+b)/2$
- T temperature, $\Delta T = T_H - T_C$ (shown in Fig. 1, Fig. 2)
- t time
- z axial gap
- β volumetric expansion coefficient
- κ thermal diffusivity, ν kinematic viscosity
- Ω angular velocity
- γ exponent in $Nu \propto Ra^\gamma$

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There are some variations in the reported values of γ at high Ra ($>10^6$), at which the bulk flow is turbulent. Values of γ between 0.25 and 0.33 have been deduced from measurements (see Grossmann & Lohse, 2000 for a summary of past results). Most experiments, beginning with those of Heslot *et al* (1987) and Castaing *et al* (1989), suggest $\gamma = 0.282 \pm 0.006 \approx 2/7$ as the most robust value of the exponent (Grossman & Lohse, 2000). A few experiments (e.g. Chavanne *et al*, 1997 and Cioni *et al*, 1997) suggested the possibility of transition to $\gamma = 1/2$. Grossmann & Lohse devised a theory that produced a relationship $Nu = 0.27\mathrm{Ra}^{1/4} + 0.038\mathrm{Ra}^{1/3}$ (the coefficients were determined from laboratory experiments) which resembles a $\gamma = 2/7$ variation. Recent reliable experiments by Glazier *et al* (1999) and Niemela *et al* (2000) gave $\gamma = 0.285$ (for $10^5 \leq Ra \leq 10^{11}$) and $\gamma = 0.309$ (for $10^6 \leq Ra \leq 10^{17}$) respectively. They found no evidence of a transition to the $\gamma = 1/2$ 'ultimate state' (described above). The theory of Grossmann & Lohse suggests that this regime may exist in fluids with very low Prandtl numbers (Pr << 0.1). The exponent γ is most probably independent of aspect ratio for aspect ratios above $l/d=1/2$, but the coefficients C_i do depend on the aspect ratio (Wu & Libchaber, 1992).

Figure 3 shows the *Nu-Ra* relationships from Grossmann & Lohse (2000), Niemela (2000) and the correlation derived, using an analytical model with adjustments using empirical data, by Hollands *et al* (1975) for $Pr = 0.7$:

$$Nu = 1 + 1.44\, max\{\, (1 - 1708/Ra)\, ,\, 0\, \} + max\{\, (Ra/5830)^{1/3} - 1\, ,\, 0\, \} \tag{1}$$

We will use these three correlations as the datum for comparisons.

Bohn *et al* (1995) carried out experiments, for $10^7 \leq Ra_\phi \leq 10^{11}$, for free convective heat transfer in sealed air-filled rotating annuli, for three different geometries and with differential radial heating. They reported $Nu \propto Ra_\phi^{\gamma}$, with γ close to 0.2. Their results are also shown in Figure 3. It is clear that the heat transfer rate in these experiments for rotating annuli is much lower than that for Rayleigh-Benard convection.

	Cavity A	Cavity B	Cavity C
a,b,z [mm]	125, 355,120	125,240,120	125,240,120
Correlations	$Nu=0.246Ra_\phi^{0.228}$ $Re_\phi=0.733Ra_\phi^{0.573}$	$Nu=0.317Ra_\phi^{0.211}$ $Re_\phi=1.441Ra_\phi^{0.557}$	$Nu=0.365Ra_\phi^{0.213}$ $Re_\phi=1.615Ra_\phi^{0.556}$

Table I. Rotating cavities studied experimentally by Bohn *et al* (1995)

Table I summarises the cavity geometries and the correlations obtained by Bohn *et al* (1995). Cavity C had eight equally-spaced radial separation walls, thus producing eight sealed sectors. The presence of these walls should decrease the radial component of the Coriolis force, which is believed to lower heat transfer in a rotating system. However, it is clear from their results that differences in geometry and weakening of the Coriolis force contribute only slightly to the reduction in heat transfer in a rotating annulus compared with that for Rayleigh-Benard convection.

The main factors that influence the differences in heat transfer between the rotating annuli and Rayleigh-Benard convection deserve further investigation. Using the technique described above, computations were performed for a rotating annulus which approximates Rayleigh-Benard convection. The effect of changing the radius ratio, *a/b*, was investigated. Flows inside a rotating annulus were then computed.

2. Governing equations and numerical procedures

The non-dimensional streamfunction-vorticity equations for 2-D (radial-tangential) flow and heat transfer, expressed in a rotating frame of reference and in cylindrical polar co-ordinates, are:

$$\frac{1}{\mathrm{Pr}}\left[\partial_{\bar{t}}\bar{\varpi} + \frac{1}{\bar{r}}J(\bar{\varpi},\bar{\psi})\right] + \frac{2Ra_b}{\bar{r}\,\mathrm{Re}_b\,\mathrm{Pr}}J(\bar{T},\bar{\psi}) + Ra_b\partial_\theta\bar{T} = \nabla^2\bar{\varpi}, \tag{2}$$

$$\partial_{\bar{t}}\bar{T} + \frac{1}{\bar{r}}J(\bar{T},\bar{\psi}) = \nabla^2\bar{T}, \tag{3}$$

$$\overline{\varpi} = \nabla^2 \overline{\psi} \ , \tag{4}$$

where the operator $J(A,B) = \partial_{\bar{r}} A \partial_\theta B - \partial_{\bar{r}} B \partial_\theta A$. Density, ρ, is assumed to depend linearly on temperature, this is the so-called Boussinesq approximation: $\rho = \rho_{ref}[1-\beta(T-T_C)]$. The overbars denote non-dimensional quantities, where the following substitutions have been made:

$$r = b\bar{r} \ , \ t = \frac{b^2}{\kappa}\bar{t} \ , \ \omega = \frac{\kappa}{b^2}\overline{\varpi} \ , \ \psi = \kappa\overline{\psi} \ , \ \overline{T} = \frac{T - T_C}{T_H - T_C} \tag{5}$$

where T_H and T_C are the temperatures at the hot and cold boundary surfaces respectively (see Fig. 1 and Fig. 2). Axial vorticity is defined as $\omega = \partial_r v + \frac{v}{r} - \frac{1}{r}\partial_\theta u$. The streamfunction is related to radial velocity u and tangential velocity v by the following definitions: $u = \frac{1}{r}\partial_\theta \psi$ and $v = -\partial_r \psi$.

No-slip boundary conditions were applied to all solid walls. At the inner cylinder surface, $\overline{T} = 0$ and at the outer cylinder surface $\overline{T} = 1$. The initial conditions correspond to solid-body rotation with a conduction profile for temperature. A point disturbance in temperature was used to perturb these initial conditions. Note that as side discs were not modelled the flow is assumed to be axially invariant.

To solve equations (2) to (4), second-order accurate finite-difference approximations are utilised in space on a collocated uniform grid. The nonlinear terms were approximated by the Arakawa (1966) formulation in order to aid numerical stability. Time-stepping is based on Du-Fort Frankel method where values of the dependent variable at the n^{th} timestep are exchanged for averages of new and old time values. Equation (4), which is a boundary value problem within each timestep, was solved using a V-cycling multigrid routine, incorporating a line relaxation scheme, to accelerate convergence (see Lewis (1999) for a more detailed description). The finest mesh used was 96 by 192 (radial by tangential) points with a non-dimensional time-step of $2\text{x}10^{-8}$.

3. Results and discussion

Using the technique described in section 1, Rayleigh-Benard (or R-B) convection approximated by (effectively) a non-rotating annulus with "gravity" (an imposed constant radial acceleration) directed away from the centre, was computed for two different radial gaps (giving radius ratio a/b equal to 0.7 and 0.5). The results are shown in Figure 3. (The results for a rotating annulus, shown in Figure 4, are discussed below). The computations produce very similar heat transfer rates, and both agree well with the three selected correlations for Rayleigh-Benard convection for low up to moderately high Rayleigh numbers. Axially invariant flow in an annulus has a horizontal layer counterpart whose aspect ratio *l/d* is large (see Figure 1). It is found by many (for example Catton & Edwards, 1967 and Catton, 1972) that for *l/d* greater than about 1/2, the effects from vertical side walls are insignificant. The aspect ratio for an annulus is defined as *z/d*.

Thermal conductivity of the lateral walls and the low aspect ratio of 'tall' cavities can delay the onset of convection and reduce heat transfer (*Nu*) after convection begins. Conducting walls dampen both velocity and temperature perturbations. However, the effects from both of these factors are negligible when the Rayleigh number is higher than about 10^7 or the aspect ratio is greater than ½ (Catton & Edwards, 1967).

The annulus appears to approximate Rayleigh-Benard convection very well, with the curvature of the radial gap having very little effect on the heat transfer. A local fluid region behaves as if in a "flat" environment with the surrounding gravity vectors parallel to each other. This implies that this convection process happens over a small length-scale in terms of the spatial geometry. Barenblatt (1996) showed that Rayleigh-Benard convection (in air and water) has a very large characteristic scale, so that in most laboratory or industrial convection layers, there is no effect of the layer thickness on heat transfer.

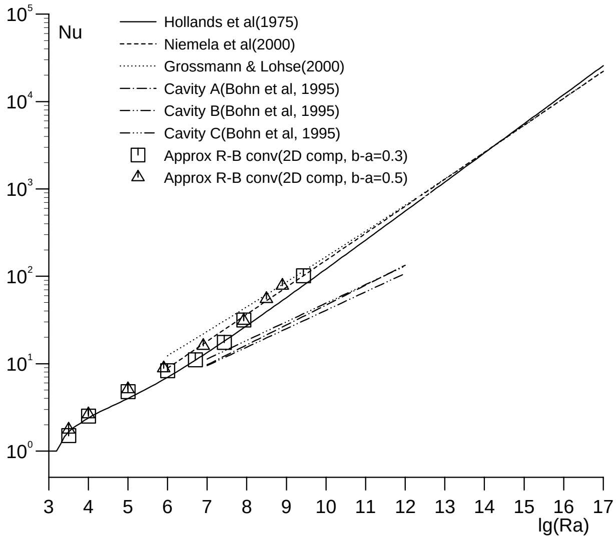

Figure 3 Stationary enclosures

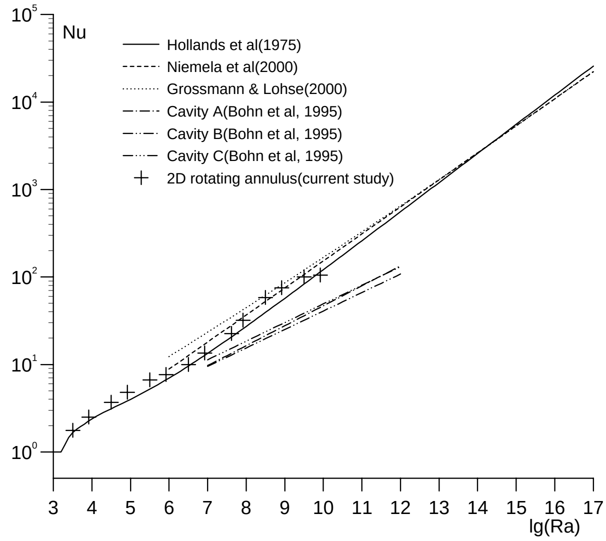

Figure 4 Rotating annuli, comparison with R-B convection

Figure 5 shows isotherms obtained for the stationary annulus with *d*=0.3. The convective plumes become stronger as Rayleigh number increases and impinge on the cylindrical surfaces, concentrating the isotherms. For Ra > 10^7 (approximately), similar to the value above which side wall effects are negligible, both the plumes and the fluid regions with high thermal gradients become unstable. The regular convective rolls break down to give instead a well-mixed turbulent flow with very thin thermal boundary layers at the cylindrical surfaces.

Rotating annulus flows were computed for Cavity B of Bohn *et al* (see Table I). Note that these are again time-dependent, 2-D (radial-tangential plane) computations. The side discs are not modelled as the cavity aspect ratio is (just) greater than unity. Moreover, despite a reported heat loss of 10% to 20% of the total heat input through the side discs, the resulting heat transfer and the result obtained from a computation carried out for cavity C with insulated side discs conditions agreed very closely (Bohn *et al*, 1995). Also, the range of Rayleigh numbers considered is above 10^7. This all suggests that the effects of the side discs are not significant.

The rotating annulus results are shown in Figure 4. The computed heat transfer agrees closely with the Rayleigh-Benard correlations, suggesting that the effects of the Coriolis force on turbulence are small compared to buoyancy effects for these high Rayleigh number cases. Cavity C, which had radial barriers that are expected to weaken the radial Coriolis force (which reduces heat transfer), produced higher values of Nu (Figure 4) but not a sufficient increase to explain the differences from that in Rayleigh-Benard convection. Extrapolating the correlated result for Cavity B to lower Rayleigh numbers agrees with the gradient for laminar Rayleigh-Benard convection at $10^4 \leq Ra \leq 10^5$, i.e. $\gamma \approx 0.2$. In the experiments of Bohn *et al* the flow may have remained laminar even at high Rayleigh number; Owen & Wilson (2000) made the same comment. Bohn *et al* did not make observations of the flow structures or velocity measurements.

The steady-state computation carried out by Bohn *et al* (1995) agreed with their experimental results. They also carried out an unsteady calculation and found that the flow showed no tendency to reach a steady state. The Nusselt number they obtained for a particular Rayleigh number was greater in the unsteady than in the corresponding steady computation (Bohn *et al*, 1995). Bohn & Gier (1998) carried out a steady turbulent simulation which produced almost a 20% increase in heat transfer from that in the experiment, with three-dimensionality having only a small impact on the overall heat transfer. Thus, it is concluded from these results and the present time-dependent simulations that both unsteadiness and turbulence are responsible for the higher computed heat transfer compared with the measurements of Bohn *et al*. A stable and steady laminar fow may have occurred in the experiments due to (unidentified) conditions in the rig, but the differences in the heat transfer between rotating annulus convection and Rayleigh-Benard convection have not yet been fully resolved. (The radial variation of the acceleration field may be expected to exert some influence on the stability of the flow, but a major effect is not observed in the computations described above.)

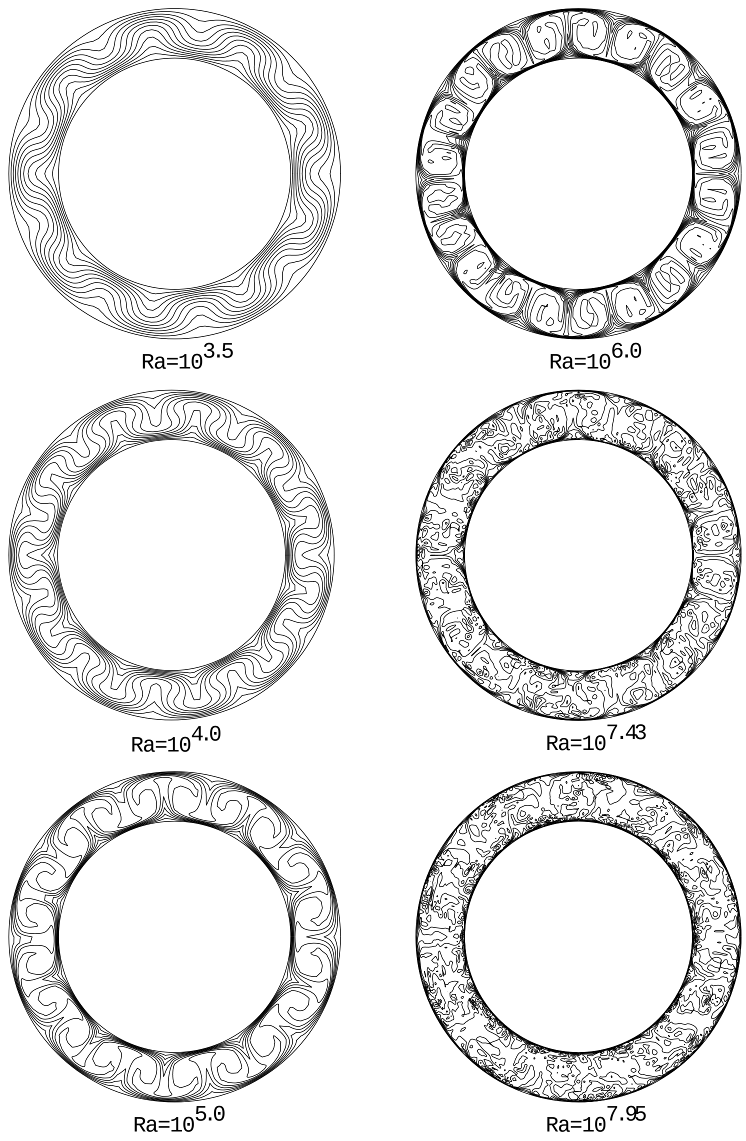

Figure 5 Computed isotherms, non-rotating annulus, *b*-*a*=0.3 (a/b=0.7)

4. Conclusions

Two-dimensional (radial-tangential), time-dependent numerical simulations were performed for air-filled, sealed annuli with an imposed radial thermal gradient. For stationary annuli, good agreement was obtained with correlations appearing in the literature for the *Nu* vs. *Ra* relationship for Rayleigh-Benard convection. The radial separation between the inner and outer cylinders forming the annulus had little effect on the heat transfer.

Heat transfer in a rotating annulus was also computed and it was found that the *Nu* vs. *Ra* relationship remained close to the Rayleigh-Benard correlations, which gives considerably higher heat transfer than the experimental results obtained by Bohn *et al* (1995). The flows in the laboratory experiments, for reason(s) still to be investigated, may have remained laminar even at high Rayleigh numbers, where the present computations showed well-mixed or turbulent bulk flow. More investigations are required to resolve these issues.

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