Extra questions for Chapter 3

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1. The early parts of this question will help you do the later parts. Let $w = e^{2\pi i/3}$.

(a) Show that every complex number can be uniquely written as $a + bw$ with a, b real.

(b) Show that $w^2 + w + 1 = 0$ very easily, by considering $w^3 - 1$.

(c) If $z = a+bw$ with a, b real, let $M(z) = (a+bw)(a+bw^2)$. Show that $M(z)$ is real and that for all z_1, z_2 we have $M(z_1)M(z_2)$.

(d) Show that the triangle with sides $3, 5, 7$ contains an angle of $2\pi/3$.

(e) Show that there are infinitely many pair-wise non-similar triangles with integral sides, and containing an angle of $2\pi/3$.

(f) Suppose that α is an angle occuring in a triangle with integral sides. Is it true that there are infinitely many pair-wise non-similar triangles with integral sides, and containing the angle α?