

Christmas Assessment Exercise, 2014 ©G.C.Smith

These problems are designed to be addressed by your guests over the Christmas season. Users are encouraged to construct league tables of their guests, and to publish the results in newspapers and on the internet. Only by making the Christmas process fully transparent can we ensure that stakeholders are fully informed concerning the merits of current and potential guests. The CAE 2014 may be shared and used for any non-profit making purpose.

1. The following incident happened in a Christmas morning Pirate Film. “Listen up shipmates!” cried Captain Silver, his parrot jumping excitedly on his shoulder. Pointing at the treasure map of Skull Island with his wooden leg, the notorious scoundrel continued: “It says here that if you draw three equally spaced parallel lines, one through Lepers’ Rock, one through Gibbet Point and one through Slaughter Tor, the middle line will pass over the location of the buried treasure. That’s not much of a clue is it, we’ll have to dig a trench!”. A small voice piped up at the back. It was Jack Hawkins, who had been taken aboard the pirate ship having been kidnapped at his local library while working on a sheet of UKMT mentoring problems. “No captain, the map tells us exactly where to dig”. Do you think that Jack will get an invite to the UK-Hungary IMO training camp if he manages to escape?
2. Five gold rings fell out of a Christmas song and into a plane containing a point X . The first ring did not pass through X but the other four did. Moreover each of these rings passing through X cuts the first circle at two opposite points of the first circle. What can you deduce about the centres of the rings passing through X ? A Euclidean solution would be much preferred.
3. At Christmastime, in odd corners of derelict churchyards, you may find celebrants of the old religion acting out their rites for the festival of *Yule*. The three witches stood stirring the giant pot containing

many unspeakable ingredients. The first one, Hazel, drew a perfect circle on the ground in bat's blood. The second, Endor, put five spots of toad vomit on the circle, trying to space them regularly, but she had been at the wormwood again so her accuracy was off. The third witch, Agnesi, was supposed to draw the pentagram, but she had eaten one mushroom over an omelette, and had entered a sort of geometric trance. "Sisters", the hag croaked, "each three of the five mystic points makes a triangle, so there are ten triangles hidden here. The choice of points our sister has made has the property that the ten straight lines each through the centroid of a mystic triangle and perpendicular to the line joining the other two mystic points, are concurrent". She slumped against the yew tree, and chewing on a twig of the fatal wood she continued "I think that if she did this for $2 \times 3^2 \times 37$ Yules, she could not do such a thing again". What do you think? *No animals were harmed during the construction of this question.*

4. Santa's home *Dungivin* is situated at some distance from his toy factory. He is considering building a visitor centre to strip cash from gormless children and their hapless parents who make the journey to Lapland to view his industrial complex. Santa sought planning permission from the Fairyland Planning Committee, and was told that in order to keep the Magic Kingdom in harmony, the visitor centre must be built on a particular ley line which happens to be parallel to the line through *Dungivin* and the toy factory. Santa couldn't make up his mind where to put the visitor centre on the ley line, and became convinced that he really needed to understand where the orthocentre (the intersection of the altitudes) of the triangle formed by the three buildings might be. What are the possibilities?