Group Theory: Math30038, Sheet 8

GCS

1. Let P be a sylow p-subgroup of the finite group G. Suppose that $N \trianglelefteq G$.
   (a) Show that $P \cap N \in \mathrm{Syl}_p(N)$.
   (b) Show that $PN/N \in \mathrm{Syl}_p(G/N)$.
2. Show that every group of order 15 must be abelian.
3. Show that every group of order 35 must be abelian.
4. Show that there is no non-abelian finite simple group of order less than 60.
5. Let p and q be distinct prime numbers.
   (a) Show that a group of order pq can not be simple.
   (b) Show that a group of order p^2q can not be simple.
   (c) Show that a group of order p^2q^2 can not be simple.