November Exam 2

1. Prove that the equation

$$\frac{x^{2000}-1}{x-1} = y^2$$

has no solutions in integers $x, y \geq 2$.

2. Let n be a positive integer. A *corner* is a finite set C of ordered n-tuples of positive integers such that if $a_1, a_2, \ldots, a_n$, $b_1, b_2, \ldots, b_n$ are positive integers with $a_k \geq b_k$ for $k = 1, 2, \ldots, n$ and $(a_1, a_2, \ldots, a_n) \in C$, then $(b_1, b_2, \ldots, b_n) \in C$. Prove that among any infinite collection S of corners there exist two corners, one of which is a subset of the other one.

3. In $\mathbb{R}^3$ a circle Γ is tangent to each of the three planes $P_x = \{(0, y, z) : y, z \in \mathbb{R}\}$, $P_y = \{(x, 0, z) : x, z \in \mathbb{R}\}$ and $P_z = \{(x, y, 0) : x, y \in \mathbb{R}\}$. What is the set of possible locations for the centre of Γ?

Solutions should be sent to Adrian Sanders, Trinity College, Cambridge CB2 1TQ for receipt on or before Wednesday 24 November.