

Practice Exam 1

Maximum time 4 hours

October 2004

Write on one side of the paper only. Use A4 size, plain or lined as you please. Pages should be clearly numbered, and each page should carry your initials. Page 1 of each solution should carry your name. This is not a handwriting contest, but nonetheless the markers would appreciate a serious attempt at clarity. An individual may be banned from using certain letters for variables if he or she cannot write them recognizably at speed. A good strategy is to write the solution in rough first, and then attempt a “write up”. The benefit to be gained from these exams is the struggle rather than the solution. You get the full benefit of this even if you solve nothing, but do spend all the time trying out ideas. This paragraph will be omitted from future exams, but it always applies.

1. We are given a family of disks in the plane, with pairwise disjoint interiors. Each disk is tangent to at least six other disks of the family. Show that the family is infinite.
2. Determine all functions defined on the positive reals and taking real values which satisfy

$$f(x + y) = f(x^2 + y^2) \text{ for all } x, y \in \mathbb{R} \text{ such that } x + y > 0.$$

3. Let a and b be integers. Is it possible to find integers p and q such that $p + na$ and $q + nb$ are coprime for all integers n ?

Solutions should be posted to *Dr G C Smith, 16 Bloomfield Drive, Odd Down, Bath BA2 2BG* to arrive on or before October 31st 2004. Include rough work unless it is complete rubbish.