Practice Exam 2

Maximum time 4 hours

October 2004

1. Find the smallest positive integer n such that

   (a) n has exactly 144 distinct positive divisors and

   (b) there are 10 consecutive positive integers among the divisors of n.

2. We are given six straight lines in space. Among any collection of three of those lines, at least one pair is perpendicular. Show that the given lines can be labelled $l_1, l_2, \ldots, l_6$ in such a way that l_1, l_2, l_3 are pairwise perpendicular, and l_4, l_5, l_6 are pairwise perpendicular.

3. Find the greatest possible value of the expression

$$(a + b)^4 + (a + c)^4 + (a + d)^4 + (b + c)^4 + (b + d)^4 + (c + d)^4$$

   given that the real numbers a, b, c and d satisfy

$$a^2 + b^2 + c^2 + d^2 \leq 1.$$