Christmas Assessment Exercise, 2005

These problems are designed to be addressed by your guests over the Christmas season. Users are encouraged to construct league tables of their guests, and to publish the results in newspapers and on the internet. Only by making the Christmas process fully transparent can we ensure that stakeholders are fully informed concerning the merits of current and potential guests. The CAE 2005 may be shared and used for any non-profit making purpose.

1. There are three pine trees near the toy factory on Lapland plain. Two tall ones, which have the same height, and one short one. The line joining the bases of two tall trees runs east-west. The quality assurance team from Oftoy recently tested the Christmas kill-o-matic laser blast ray by climbing each of the tall trees in turn, and fired just over the tip of the short tree to burn a pair of small holes in the tundra. Show that the line joining the two holes runs east-west. Incidentally, because of the efforts of Jack Frost, the great wind always blows from the direction of the Ice Palace, so trees don't grow vertically in Lapland, but they do grow parallel.

2. Captain Hook has captured Tinkerbelle, and holds her in a cage. In traditional fashion, she has asked him for one last chance, a riddle or problem which if she solved, she would be set free. Captain Hook had secret interest in mathematics, and asked Tinkerbelle to construct a perfect isosceles triangle by using a pencil, a coin and a straight edge. He pointed out that it was a trick coin, and would vanish if anyone tried to use it to draw more than two circles. Can she escape? *Note that drawing a tangent line to a circle 'by eye' does not constitute* ***construction****.*

3. Greedy Geoff sawed off a corner of a brick shaped block of Christmas cake, exposing a triangular fresh face of moist rich delicious gateau. He placed the tetrahedral fragment on the table, with its fresh face downwards. He mused through a port laden haze that it looked very stable, just like a mountain in fact, with its summit above a point inside its (not necessarily equilateral) triangular footprint ABC. He decided to decorate it, and took a UKMT pennant flying from a toothpick, and stuck it at the summit, with the flagpole perfectly vertical. Of course, the port was still at work and he is a bit clumsy, so he jammed the toothpick right through the cake, stabbing it into the tablecloth at a point X. Show that the circles ABX, BCX and CAX all have the same radius.

4. Labour laws in Lapland are similar to those in the 5th Republic. It is impossible to sack elves, and the Fairy Queen has long since cast her spell to make Santa's 'workers' immortal. These workers started their 'jobs' at various points a long time ago, and of course Santa never hires any new ones. The rule is that an elf gets the day off unless the number of days since his or her appointment to the Toy Factory is a power of a prime number.

   (a) Show that Jean-Pierre Elf will one day be able to take a year long holiday.

   (b) Show that it will happen that all 1729 of Santa's elves have the day off simultaneously.

5. The Ice Palace is 100 stories high. The Fairy Queen has a collection of baubles (beautiful identical glass spheres). She wants to know the highest floor of the Ice Palace from which it is possible to drop a bauble and not have it shatter on the ground. She presents the President of the Royal Elf Society with two such baubles for test purposes. What is the least number of experimental drops that the PRES must be prepared to make in order to determine the answer?

Thanks for mathematical suggestions from Ceri Fiddes, Noel Stephens and Marcus du Sautoy. Please pass this on to anyone who might be interested. Past CAEs are at `http://www.bath.ac.uk/~masgcs/xmas.html`; `say hello at G.C.Smith@bath.ac.uk`