

**FURTHER MATHEMATICAL TECHNIQUES (MA10193)**  
**EXAMPLES SHEET 2**

*I, or somebody, will look at work given to me at the end of the examples class at 12:15 on Wednesday, 9th March, or left before then in the folder on my office door (1W3.35). If you do not have a copy of this sheet, you can find one at <http://www.bath.ac.uk/~masgks/MA10193/sheet1.ps> (or .dvi or .pdf).*

1. Solve the following set of simultaneous equations using elementary row operations:

$$\begin{aligned}x + 2y - 8z &= 0 \\2x - 3y + 5z &= 0 \\3x + 2y - 12z &= 0\end{aligned}$$

2. Use the method of Gauss-Jordan elimination to solve the following set of simultaneous equations:

$$\begin{aligned}9x + 4y + 3z &= -1 \\5x + y + 2z &= 1 \\7x + 3y + 4z &= 1\end{aligned}$$

3. Use elementary row transformations to find the solution to the following simultaneous equations in the case where  $a_{11}a_{22} - a_{12}a_{21} \neq 0$ .

$$\begin{aligned}a_{11}x_1 + a_{12}x_2 &= b_1 \\a_{21}x_1 + a_{22}x_2 &= b_2\end{aligned}$$

4. Find the rank of  $A$  where

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 0 & 3 \\ 2 & -4 & 2 \end{pmatrix}.$$

Hence (without solving the system) state how many solutions, if any, the following linear system has:

$$\begin{aligned}2y - z &= 1 \\x + 3z &= 3 \\2x - 4y + 2z &= -7\end{aligned}$$

5. Solve the equation (remember that  $\det A$  or  $|A|$  means the determinant of  $A$ )

$$\begin{vmatrix} x+1 & x & x-4 \\ 2 & 1 & -4 \\ 3 & 5 & 1 \end{vmatrix} = 0$$

6. Consider the matrix

$$A = \begin{pmatrix} -1 & 3 & x \\ -2 & 2 & 1 \\ x & 4 & 0 \end{pmatrix}.$$

Calculate  $\det A$  in terms of  $x$  and hence determine those values of  $x$  (to 2 d.p.) for which the matrix has rank less than 3. If  $x = 2$  write down the rank of  $A$ .

GKS, 2/03/05