**PROGRAMMING AND DISCRETE MATHEMATICS (XX10190)**
**SEMESTER 2 MATHEMATICS: PROBLEM SHEET 0 – SOLUTIONS**

1. Suppose that $m|n$ (remember: that says "$m$ divides $n$" and it means that $\frac{n}{m}$ is an integer). Suppose that $G$ is a cyclic group of order $n$, generated by $g$. Write down an element of $G$ of order exactly $m$.
*Simply $g^{n/m}$. In general it is not true that a group of order $n$ contains a subgroup of order $m$ if $m|n$, let alone an element of order $m$. It is true if $m = p$ is prime, but that uses the Sylow theorems which are beyond this course. But for cyclic groups, it's easy.*

2. Compute $\varphi(24)$ and verify that $24 = \sum_{d|24} \varphi(d)$.
$\varphi(24) = 24(1 - \frac{1}{2})(1 - \frac{1}{3}) = 8$. *The divisors of* $24$ *are* $1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24$ *and these have* $\varphi(1) = \varphi(2) = 1$, $\varphi(3) = \varphi(4) = \varphi(6) = 2$, $\varphi(8) = \varphi(12) = 4$ *and* $\varphi(24) = 8$, *and* $1 + 1 + 2 + 2 + 2 + 4 + 4 + 8 = 24$.

3. Show that the group $(\mathbb{Z}/24)^*$, where

$$(\mathbb{Z}/24)^* = \{a \in \mathbb{Z}/24 \mid \text{hcf}(a, 24) = 1\}$$

with multiplication mod 24 as the group operation, is not cyclic.
*For instance because the elements* $5$, $7$ *and* $11$ *all have order* $2$ *– as indeed everything does except* $1$. *In the cyclic group* $\mathbb{Z}/8$ *(with addition) there is only one element of order* $2$ *(it is* $4$*), and of course there is an element of order* $8$.

GKS, 21/2/17