Problem class 4, 2019

1) Construct regular expression for language in $\{a, b\}$ containing exactly one b:

$$a^* b a^*$$

2) Two occurrences of b: $a^* b a^* b a^*$

3) All words in $\{a, b\}^*$ that do not end with ba

$$e \cup a \cup b \cup ((a \cup b)^* (aa \cup ab \cup bb))$$

4) All words in $\{a, b\}^*$ where every occurrence of a is followed immediately by a b

$$((ab) \cup b)^*$$

5) Prove that $L = \{b^k a b^{3k} \in \{a, b\}^* \mid k > 0\}$ is not regular.

Assume L is regular. $\Rightarrow$ there exists $n \dots$

Consider $b^n a b^{3n} = \underbrace{xy}_{\le n} z \Rightarrow xy = b^k \Rightarrow$

$xz = b^{<n} a b^{3n} \notin L$ – contradiction.

6) $L = \{a^{k^2} \mid k \ge 0\} \subset \{a\}^*$. Take n from PL, assume $n > 1$

Consider $a^{n^2} = xyz$, where $|xy| \le n \Rightarrow$

~~y~~ $|y| = m \le n$ $\quad xy \in L$

$|xz| = n^2 - m \ge n^2 - n \underbrace{>}_{n>1} n^2 - 2n + 1 = (n-1)^2$

So: $n^2 > |xz| > (n-1)^2 \Rightarrow xz \notin L$

we get contradiction.

7) Palindromes in $\{a, b\}$:

$L = \{ w \in \{a, b\}^* \mid w = w^R \}$

Assume L is regular, take n from PL.

Choose $a^n b a^n \in L$.

$a^n b a^n = xyz$, $|xy| \leq n \Rightarrow xy = a^l$

for some $l \leq n$. Then $xz = a^{<n} b a \notin L$.

We get contradiction.