Problem class 5, 2019

1) Last time proved that language

$\{b^k a b^{3k} \mid k > 0\}$ is not regular.

Prove that it is context-free:

$S \to bTbbb$, $T \to bTbbb$, $T \to a$

Why need T? Without T:

$S \to bSbbb$, $S \to a$ will generate

$\{b^k a b^{3k} \mid k \geq 0\}$.

2) $L = \{w \in \{a, b\}^* \mid$ number of occurrences of a = n. of. occ. of $b\}$

Grammar G:

R: $S \to aSbS$

$S \to bSaS$

$S \to e$

Proof that L is generated by this grammar:

$\underline{L(G) \subset L}$ - easy, since each time a rule is applied the number of a's and b's is either remains the same (last rule) or one a and one b is added (1st or 2nd rule)

$\underline{L \subset L(G)}$ - induction on length $|w|$, where $w \in L$.

Base of induction, for $|w| = 0$, i.e., $w = e$
– trivially true, because $S \to e$

Assume $|w| > 0$ and w begins with a
(case of b is symmetric).

So, $w = au$, where word u has one more b than a's. Thus, $u = u_1 b u_2$, where u_1 and u_2 are balanced (same number of a's and b's). By inductive hypothesis,

$$S \Rightarrow^* u_1 \quad \text{and} \quad S \Rightarrow^* u_2$$

Start with

$S \Rightarrow aSbS$ and then generate u_1 from first S and u_2 from second S.

$\boxed{\leqslant}$

3) Pushdown automaton for $\{a^k b^{3k} \mid k \geq 0\}$:

(1) $((s, e, e), (f, e))$
(2) $((s, a, e), (s, aaa))$
(3) $((s, b, a), (f, e))$
(4) $((f, b, a), (f, e))$

Example: aab^6

State	Input remaining	Stack	Transition
s	aab^6	e	—
s	ab^6	aaa	2
s	b^6	a^6	2
f	b^5	a^5	3
f	b^4	a^4	4
—	—	—	—
f	e	e	4

4) Alternatively (using translation from grammar to automaton).

$G: \; S \to aSbbb, \; S \to e$

(1) $(s, e, e)\ (f, S)$

(2) $(f, e, S)\ (f, aSbbb)$

(3) $(f, e, S)\ (f, e)$

(4) $(f, a, a), (f, e)$

(5) $(f, b, b), (f, e)$

Example: ab^3

State	Input remaining	Stack	Transition
s	abbb	e	-
f	abbb	S	1
f	abbb	aSbbb	2
f	bbb	Sbbb	4
f	bbb	bbb	3
f	bb	bb	5
f	b	b	5
f	e	e	5

~~Possible~~ Typical error when using PL for regular languages:

$$L = \{a^k b^m \mid k \neq m\} \subset \{a,b\}^*$$

Take n from PL and consider

$a^{n+1} b^n \in L$ then $a^{n+1} b^n = \underbrace{xy}_{\leq n} z$

So xy consists only of a's.

So far all is correct. But: „assume $y = a$"

then $xy^0 z = a^n b^n \notin L \Rightarrow$ „contradiction".

Can't assume anything about y except what is stated in PL.