

LMS Prospects in Mathematics 2020, Bath

## GEOMETRY

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# Geometry: Study of “Shapes”

**Topology** continuous properties of shapes (connectedness, compactness,...)

**Differential Geometry** approximating smooth shapes with Euclidean coordinates  
Applications: Einstein's relativity, economics,...

**Complex Geometry** differential geometry over complex coordinates

Applications: String theory,...

**Algebraic Geometry** shapes defined by algebraic equations (polynomials) over

...,  $\mathbb{C}$  ...,  $\mathbb{R}$ , ...,  $\mathbb{Q}$ , ...,  $\mathbb{Z}$

Applications: String theory, statistics, biology, combinatorics, number theory,...

**Arithmetic Geometry** algebraic geometry over non-closed fields (excluding  $\mathbb{R}$ ),  
e.g. number fields, finite fields, p-adic fields,... Applications: cryptography,...

**Discrete Geometry** relative position of points, lines, circles in the plane  
(combinatorics)

**Geometric Group Theory** geometric techniques in the study of finitely generated groups.  
Highlight: Perelman's proof of Poincaré conjecture

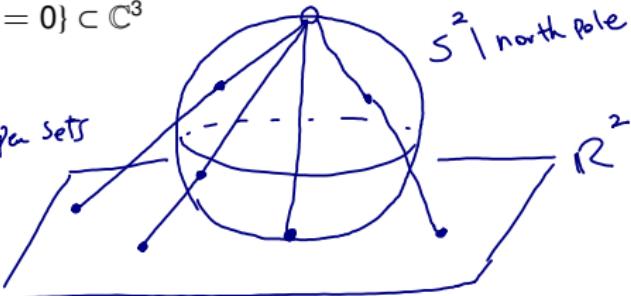
Chow's theorem

# Algebraic Geometry

Within algebraic geometry, most of my work fall within *Birational Geometry*. For example, Let  $S: \{x^2 + y^2 + z^2 - 1 = 0\} \subset \mathbb{C}^3$

$$S^2 \xrightarrow{\text{Bir'l}} R^2$$

- they have two iso (zariski) open sets
- there is a rat'l map  $S^2 \rightarrow \mathbb{R}^2$  with a rational inverse



Question: Let  $X$  be the space of solutions to a (random) degree 3 polynomial in 5 variables (i.e., a 4-dimensional subset of  $\mathbb{C}^5$ ). Is  $X$  birational to  $\mathbb{C}^4$ ?



Claire Voisin (Paris)

She offers a method that can tackle such problems (rationality problem) extensively. However we still don't have a solution to this particular question.

## Birational Geometry

Any given algebraic variety of dimension at least two is birational to infinitely many other varieties. The aim is to find a geometrically simple representative in each birational class.

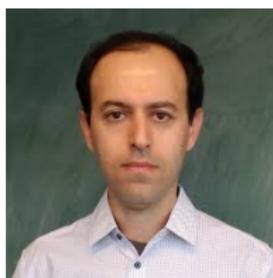
The two dimensional case is classical, and the three dimensional birational geometry was mainly developed in the 80s. Higher dimensional case is very advanced but with many open questions remaining.



Shigefumi Mori  
Fields medal 1990

- 3 types:
- Fano fibrations
  - Calabi-Yau
  - General type

There are finitely many families of Fano varieties in each dimensions.



Caucher Birkar  
Fields medal 2018

Most of my work concerns the study of birational geometry of Fano varieties, their rationality, birational classification, K-stability,...

# My PhD students

Current Students:



Laura Mallinson

PhD: October 2017 – Summer 2021

Project: Construction of flips in dimensions 3 and 4



Tiago Guerreiro

PhD: October 2017 – Summer 2021

Project: Birational models of Fano varieties with higher index

# My PhD students

Past and Future Students:



Erik Paemurru

PhD: October 2016 – July 2020

Project: Birational models of singular  
Fano varieties

Now: 4 months LMS fellowship at Imperial College London, then moving as a postdoc to Basel (Switzerland)



Errox Etxabarri

PhD: October 2020 – Summer 2024

Project: who knows? :)

# Algebraic Geometry in the UK

Bath  
Birmingham  
Cambridge  
Cardiff  
Edinburgh  
Glasgow  
Kent  
Liverpool  
London (Imperial College, UCL,...)  
Loughborough  
Manchester  
Nottingham  
Oxford  
Sheffield  
Warwick

+ Essex + more ---

The world is bigger than the UK



THANKS  
FOR  
LISTENING