Prospects in probability

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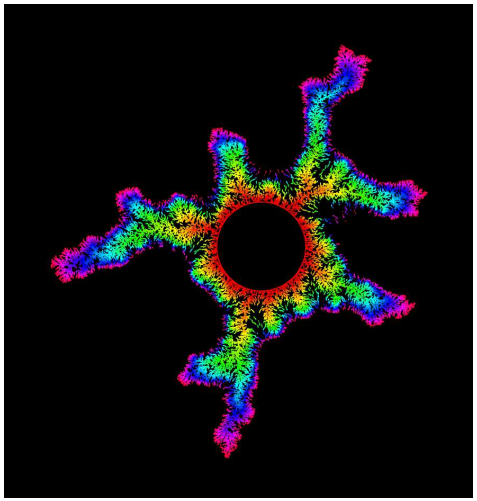


What is research in probability about?

- Probability provides a language for describing situations in which uncertainty is present.
- One of the main aims of a probabilist is to answer the question:
  "What does a *typical* X look like?"
  - X can be "epidemic", "queue", "investment", "social network", "ecosystem", ...
  - X can also be "function", "curve", "surface", "permutation", "graph", ..., *[insert favourite mathematical object here]*.

Why choose to do research in probability?

- Uncertainty is all around us so advancing our understanding of probability is undeniably useful.
- There is also very rich potential for research at the interface of probability with many other areas of mathematics and applications.
- Probability comes in two distinctive flavours.
  - In discrete probability, techniques are predominantly combinatorial.
  - In continuous probability, techniques are predominantly analytic.
  - Limit theorems connect discrete and continuous probability.

Squamulose lichen

Photo by Eric Guinther

Crustose lichen

Photo by James Wearn

Cancer tumor slices

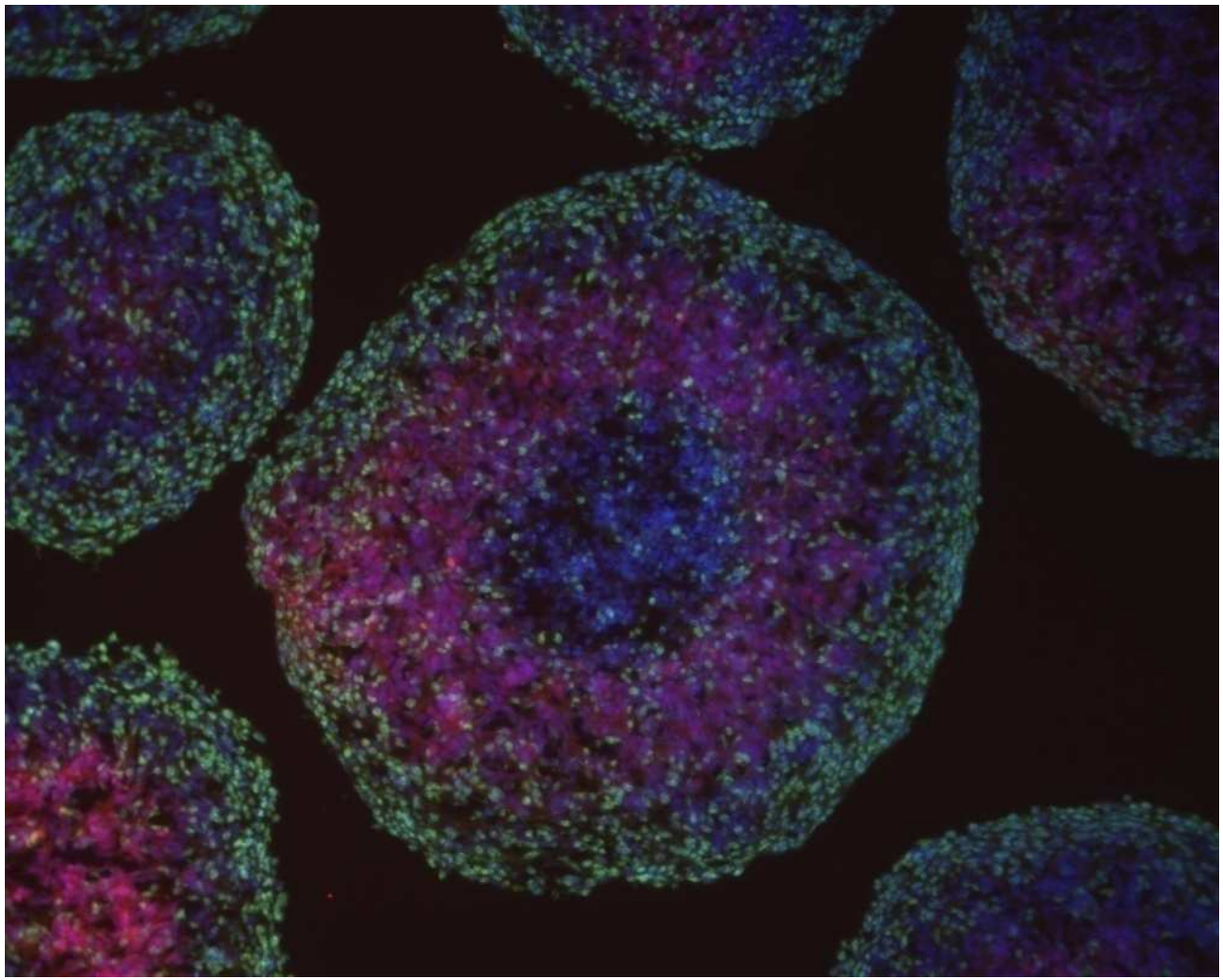


Photo by Kasia Bloch

Penicillin mould

Gift by Sir Alexander Fleming to Edinburgh University Library, Scotland

Copper aggregate formed from a copper sulphate solution in an electrode position cell

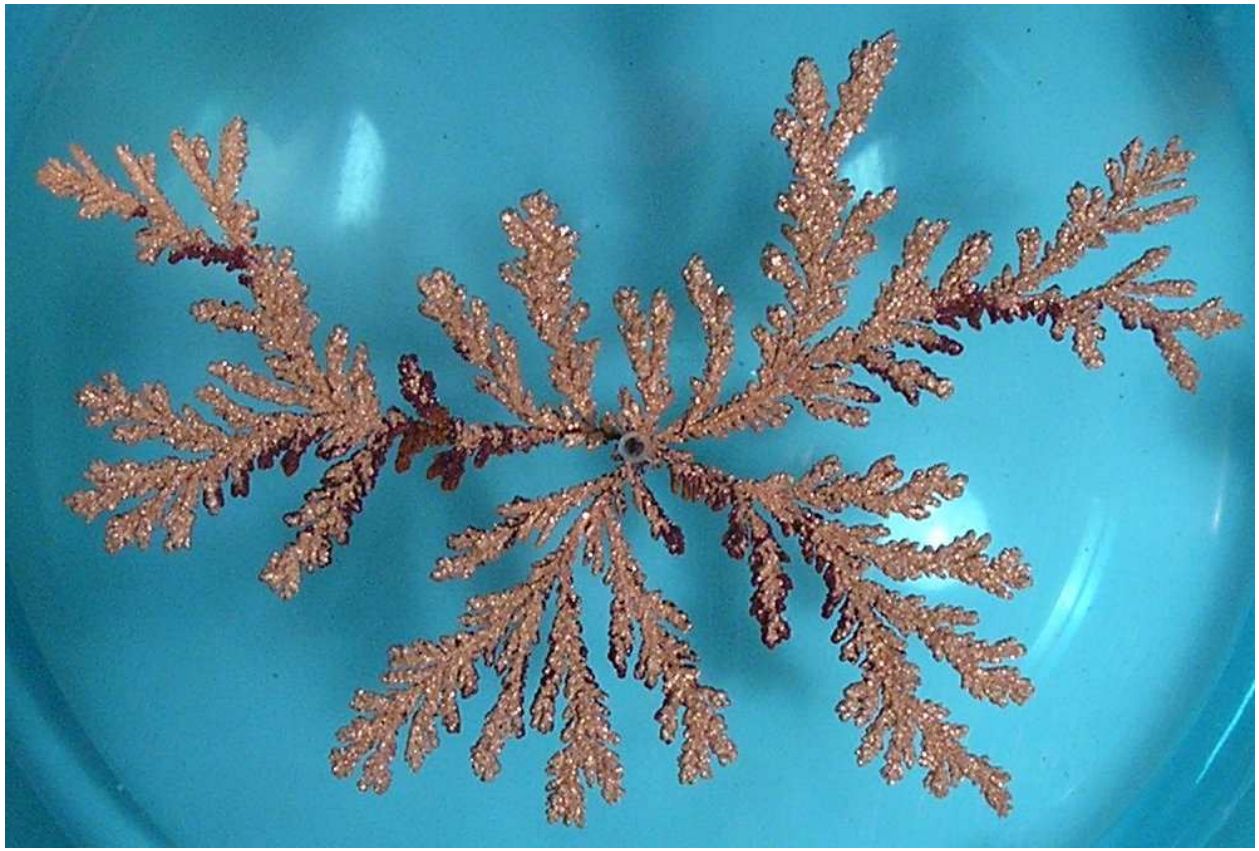


Photo by Kevin R Johnson

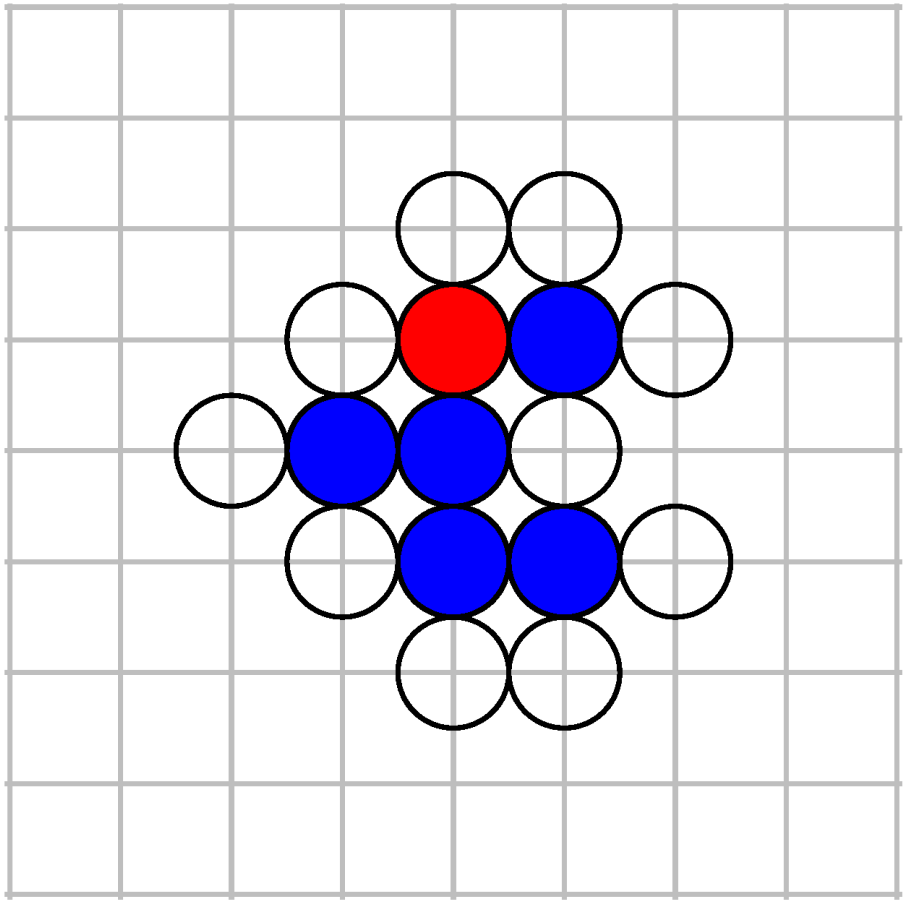
Sandstone dendrite mineral deposit

Photo by Alan Dickinson

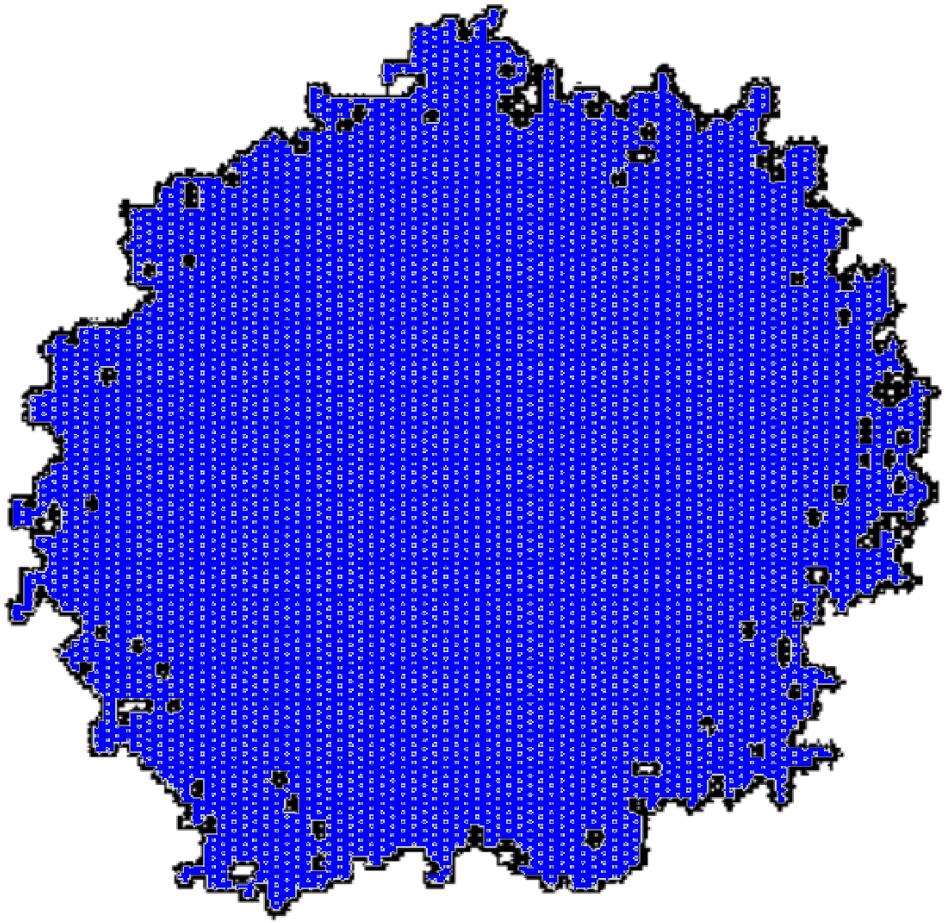
Electrical "tattoo" on survivor of lightning strike

From "Lichtenberg Figures Due to a Lightning Strike" by Yves Domart, MD, and Emmanuel Garet, MD

Lattice models for random growth

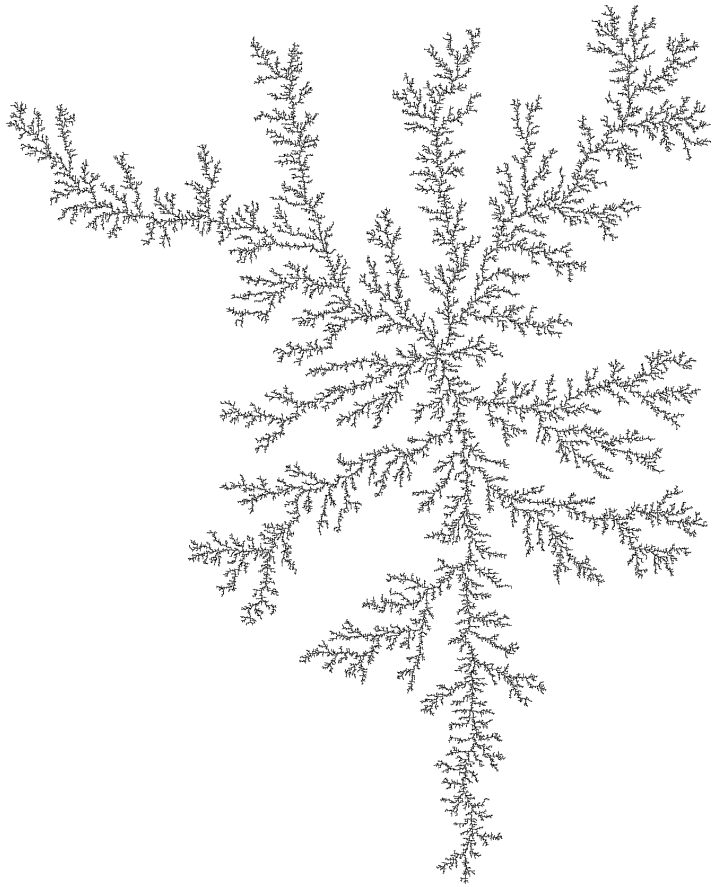


Eden model for biological growth with 1,500 particles



Simulation by H.J. Herrmann

Diffusion-limited aggregation (DLA) model with 2,000 particles

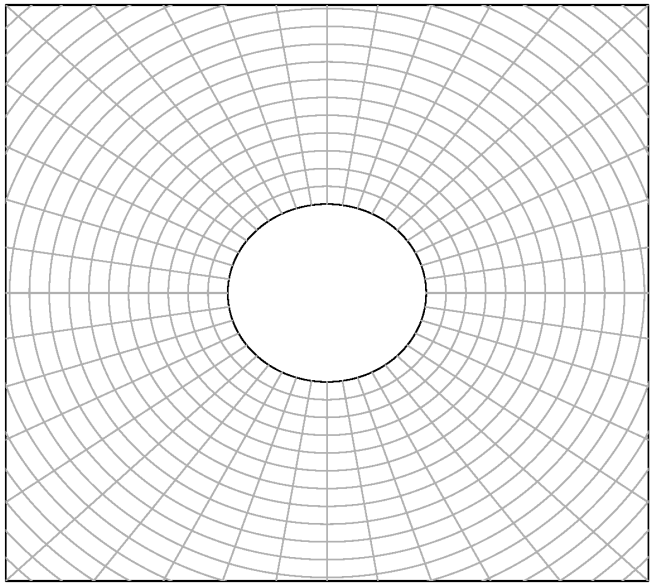


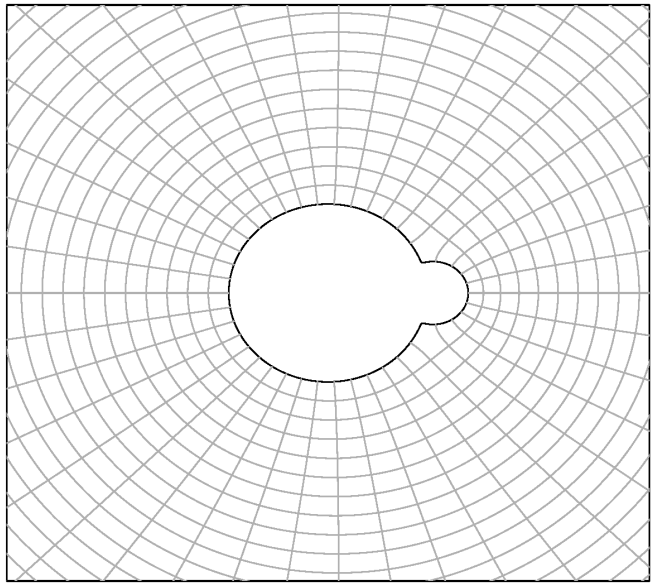
Simulation by Vincent Beffara

Conformal mappings and clusters

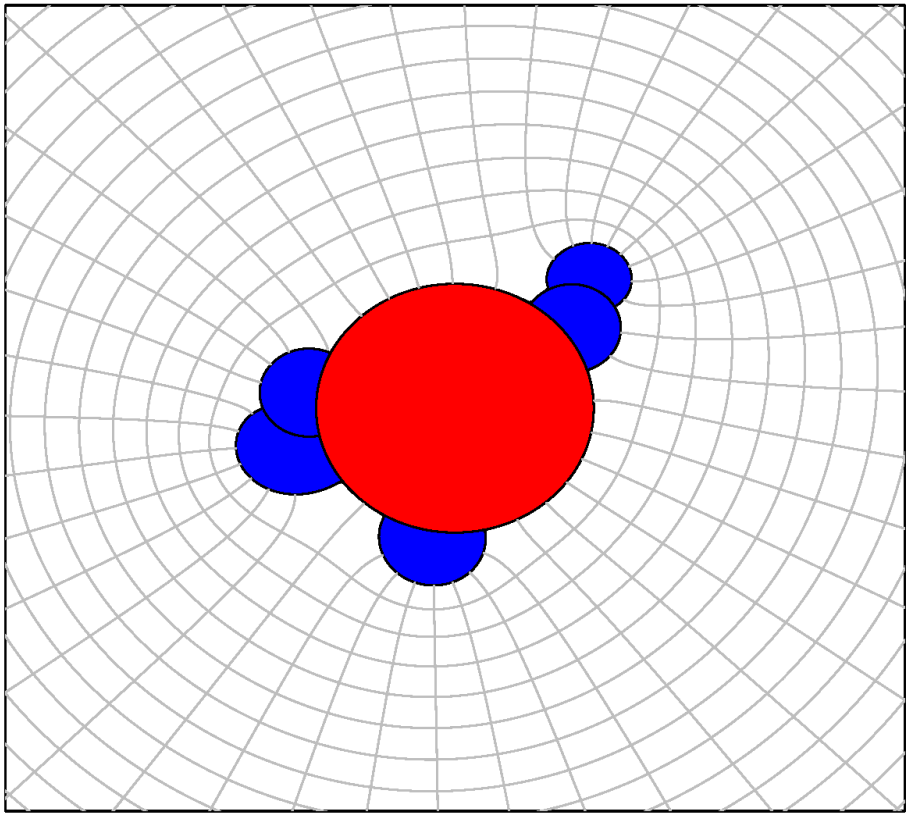
- **Riemann Mapping Theorem**: Let $K \subset \mathbb{C}$ be any connected compact subset of $\mathbb{C}$ larger than a single point and such that $\mathbb{C} \setminus K$ is connected. Then there exists a unique conformal mapping (i.e. preserves angles) from the exterior of the unit disk to $\mathbb{C} \setminus K$ that fixes infinity.
- Clusters can be uniquely represented as conformal mappings.
- Questions about geometric objects turn into questions in complex analysis.

Conformal mapping for a single particle

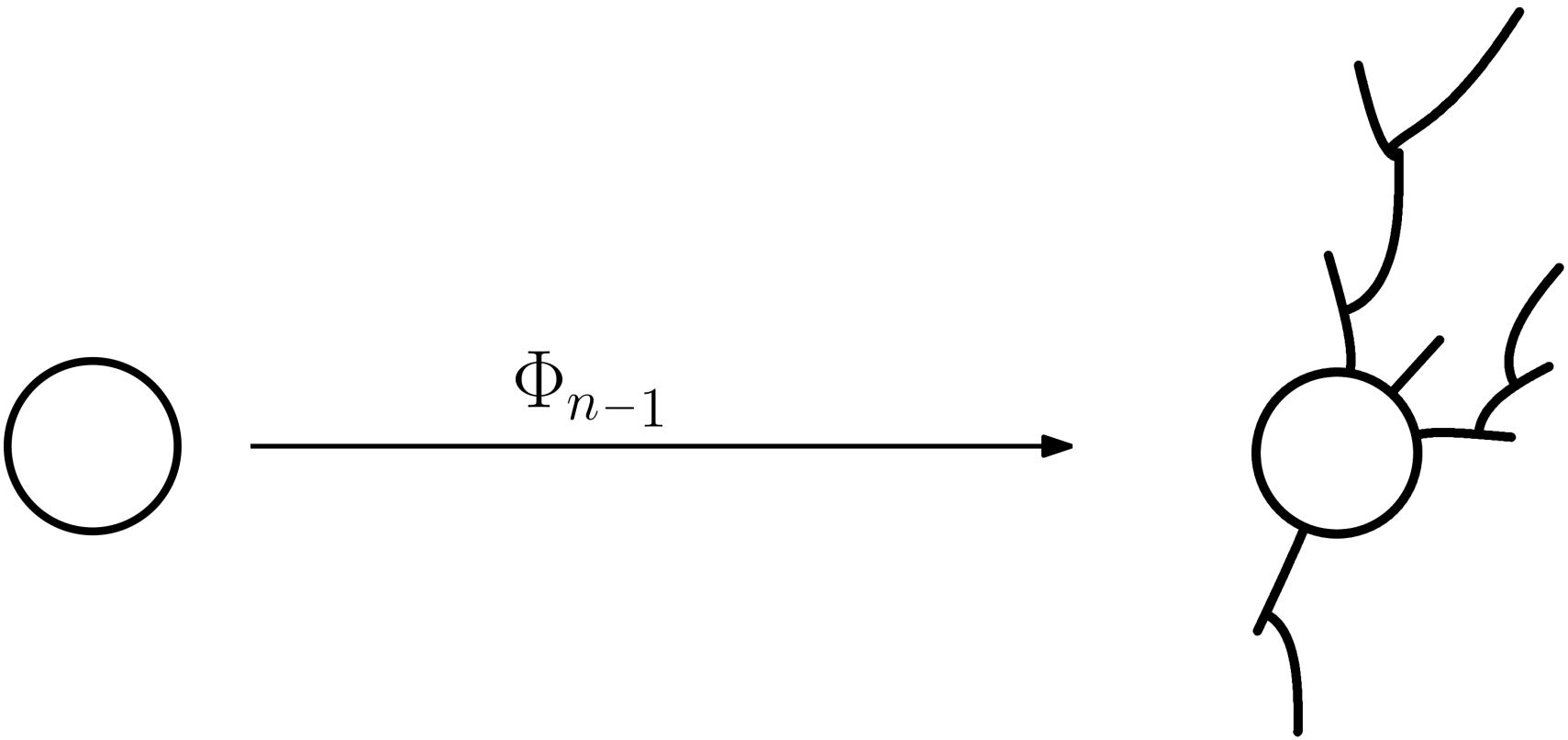




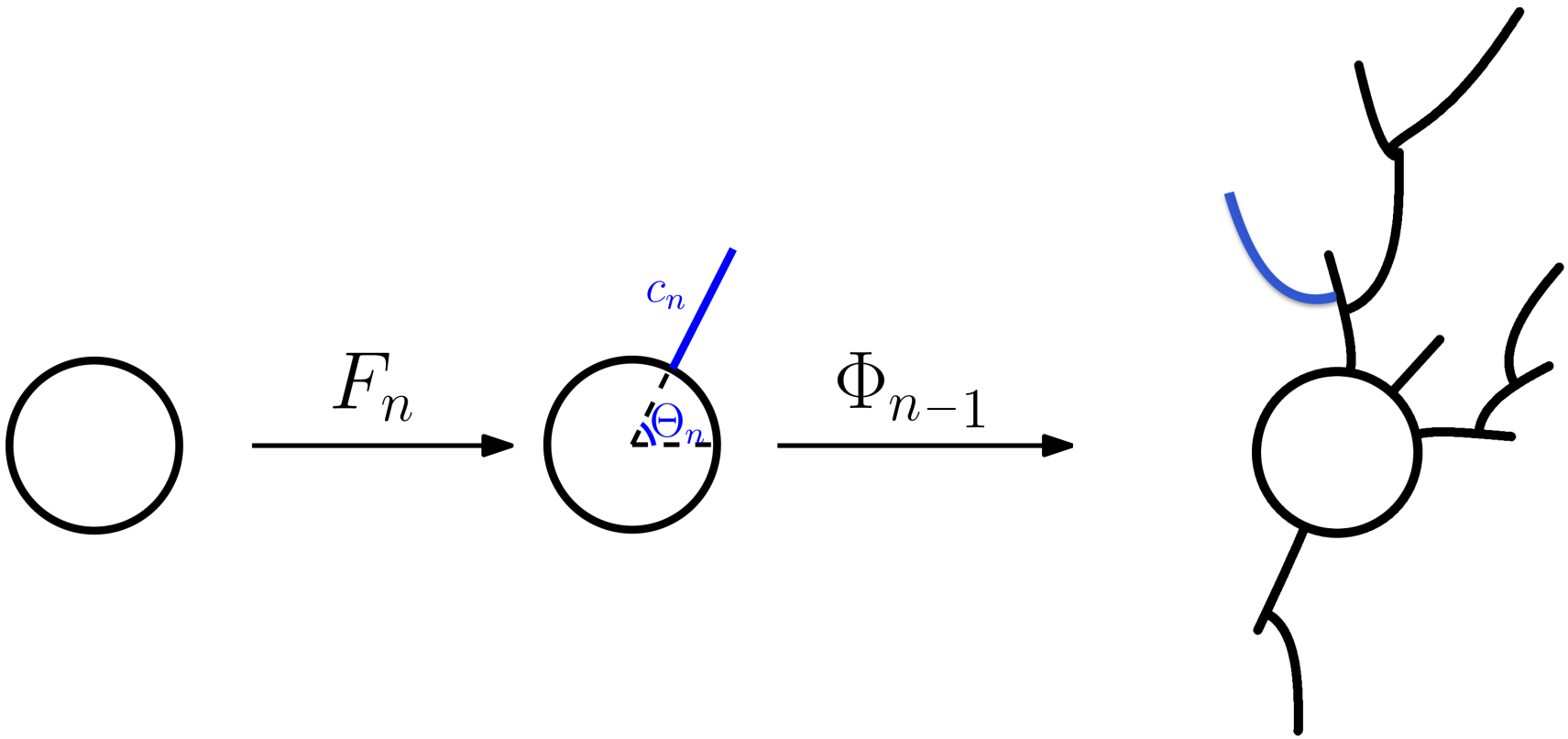
Conformal mapping for a cluster



Cluster formed by iteratively composing mappings

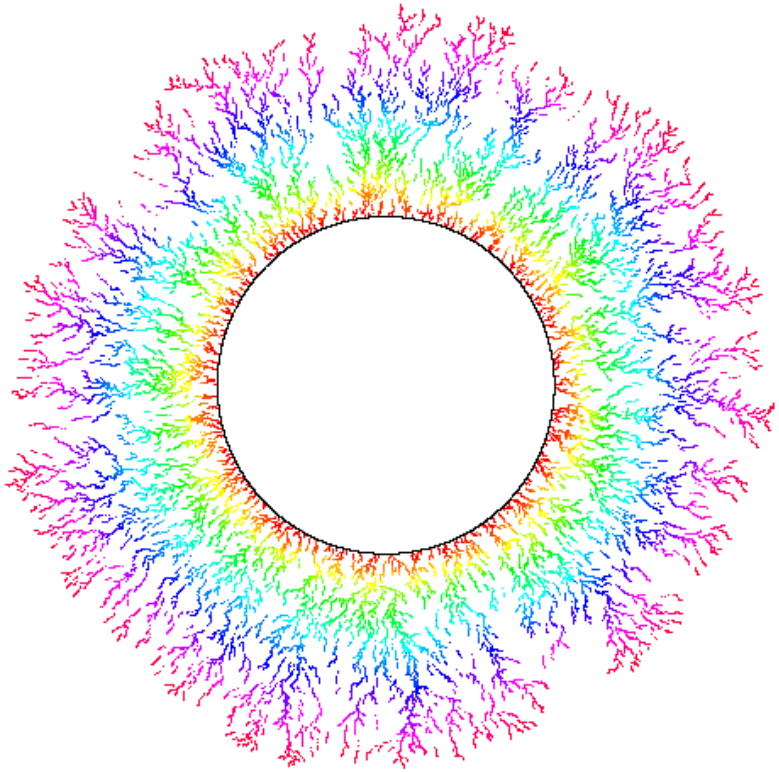


Cluster formed by iteratively composing mappings

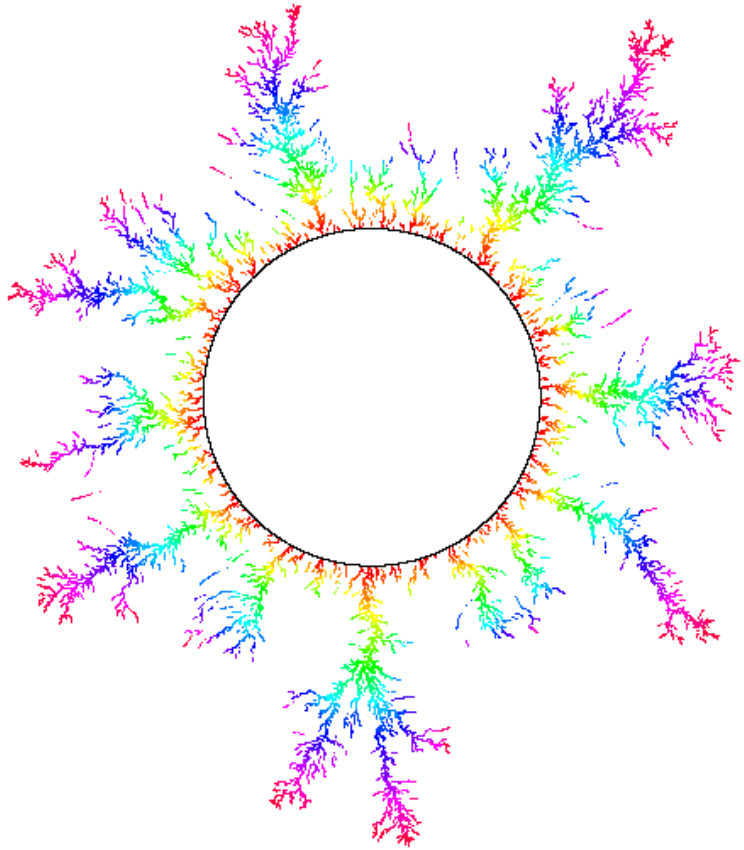


$$\Phi_n = \Phi_{n-1} \circ F_n = F_1 \circ F_2 \circ \cdots \circ F_n$$

Conformal mapping model for an Eden cluster with 8,000 particles



Conformal mapping model for a DLA cluster with 8,000 particles



Models for random growth

- Although random growth processes are completely unpredictable at the level of individual particles, large clusters often exhibit predictable or 'universal' behaviour.
- A mathematical description enables us to perform simulations and calculations that help us to establish the nature of the universal behaviour and to answer fundamental questions about the natural processes.
- By combining ideas from complex analysis and probability, understanding random growth becomes understanding random compositions of functions.

Research prospects in random growth

- Compositions of conformal maps provide a mathematical framework to describe random growth but are still very hard to analyse mathematically.
- Progress can be made by studying simplified models.
- Many good PhD-level problems involve using simplified models to identify possible mechanisms behind the universal behaviours observed in random growth.

PhD opportunities in probability

- Centres for Doctoral Training (often application focussed):
  - Bath - Statistical Applied Mathematics at Bath (SAMBa).
  - Bristol - Computational Statistics and Data Science (COMPASS).
  - Edinburgh - Mathematical Modelling, Analysis and Computation (MAC-MIGS).
  - Imperial and Oxford - Mathematics of Random Systems: Analysis, Modelling and Simulation.
  - Lancaster - Statistics and Operational Research in Partnership with Industry (STOR-i).
  - Warwick - Mathematics for Real-World Systems (MathSys).
- Universities with researchers in probability (not exhaustive):
  - Bath, Bristol, Cambridge, Durham, Edinburgh, Heriot-Watt, Imperial, Kings, Lancaster, Leeds, Liverpool, LSE, Manchester, Nottingham, Oxford, Queen Mary, Reading, Sheffield, Swansea, Surrey, UCL, Warwick, York.